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Wealth inequality through the lens of temptation preferences

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Abstract

This paper studies wealth inequality through the lens of costly self-control, modeled as Temptation Preferences, which introduce a novel term in the consumption–saving problem that acts as an effective discount factor. Relative to standard frameworks with fixed time preferences, temptation provides a structural, behaviorally grounded account of heterogeneity in discount rates and the positive association between patience and wealth, matching several empirical regularities. A stylized setup yields two mechanisms shaping intertemporal choice: the *current resources channel* (the effective discount factor increases with available resources) and *future income channel* (under standard calibration, it decreases with expected income). Embedding this mechanism in an otherwise standard OLG model, the *current resources channel* dominates, generating a *discount-factor gap* between richer and poorer agents. This in turn enables a parsimonious temptation model to match the observed wealth distribution more closely, outperforming a rational benchmark. It also shapes the distributional impact of taxation: relative to the rational baseline, wealth taxation is more effective than income taxation at reducing wealth inequality in the presence of temptation.

Keywords:
temptation preferences, wealth inequality, income inequality, wealth tax, labor income tax

JEL Classification:
D31, D90, E21

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1 Introduction

This paper studies the importance of costly self-control, modeled using temptation preferences Gul and Pesendorfer (2001), for wealth distribution and wealth inequality. Standard rational models with fixed time preferences have long struggled to account for the empirical magnitude of wealth inequality, given the relatively modest levels of income inequality observed in the data (De Nardi, 2015). A growing body of research has argued that even mild heterogeneity in discount rates can substantially improve the fit of these models to the wealth distribution (Carroll et al., 2017; Krusell and Smith, 1998). I contribute to that line of work by showing that temptation preferences offer a structural, behaviorally grounded explanation for this heterogeneity.

Indeed, temptation preferences introduce a novel term into the agent's consumption–saving problem that acts as an effective discount factor by scaling future utility. This mechanism allows the temptation model to endogenously generate substantial heterogeneity in discounting and to produce a positive correlation between an individual's discount factor and accumulated wealth. This key underlying mechanism is supported by the empirical findings of Epper et al. (2020) as temptation preferences provide a microfoundation for several stylized facts on relationship between patience and wealth, without imposing *a priori* heterogeneity in discount factors. By accounting for this *discount factor gap* between relatively wealthier and poorer individuals, a very parsimonious model with temptation preferences reproduces higher levels of wealth inequality, bringing the model-implied wealth distribution closer to the data. Moreover, this endogenous *discount factor gap* has practical policy implications: it constitutes a novel channel through which tax policy operates, shaping the extent to which wealth inequality is reduced in the temptation framework relative to the standard rational benchmark.

To account for those results, I formally show that temptation preferences give rise to two channels that shape individuals' time discount factors. First, the *current resources channel* implies that the discount factor of a tempted agent is an increasing function of current resources (either wealth or current-period income)[1]. As an immediate consequence of this relationship, tempted agents' incentive to save increase with the level of current resources, since for wealthier agents resisting temptation becomes relatively less costly on the margin. Second, the *future income channel* posits that the discount factor of a tempted agent is decreasing in the expected future income[2], because for agents who expect higher future income, the marginal cost of resisting temptation becomes greater relative to the marginal benefit of saving.

I then link the existence of these two channels to the distributional consequences of costly self-control within an otherwise standard, parsimonious overlapping generations (OLG) model, calibrated to U.S. data. In settings with persistent income dynamics, high-income individuals typically experience both elevated current income and higher expectations of future income. Such a situation gives rise to competing forces affecting the discount factor of richer agents: the positive relationship between effective patience and current resources encourages high-income agents to save more, whereas the negative relationship between effective patience and their

[1] This formalizes an intuition already noted by Kaplan and Violante (2022), who observe that temptation 'has the effect of making poor households act in a more myopic way than wealthier households' (pp. 761).

[2] This relationship holds for sufficiently persistent increases in future income, consistent with standard calibrations of quantitative models. For less persistent increases, the effect may be reversed.

elevated future income diminishes their saving incentives.

Quantitatively, I find that the *current resources channel* dominates. As a result, agents with higher current income, and therefore greater wealth, exhibit higher effective discount factors despite the countervailing force of elevated future income expectations. This generates a strong positive correlation between patience and wealth across the population. The endogenous heterogeneity in discounting leads to substantially higher wealth inequality than in the standard rational benchmark, as measured by both the Gini coefficient and the wealth share of the richest decile.

The joint impact of both channels not only yields a wealth distribution that aligns more closely with the data, but also provides novel insights into the mechanisms through which progressive income and wealth taxation affect wealth inequality. While previous work has explored the normative implications of taxation under temptation preferences (e.g., Krusell et al., 2010; Kumru and Thanopoulos, 2015), this paper takes a complementary, positive approach by analyzing, in a general equilibrium setting, the channels through which taxation impacts the wealth distribution when agents face behaviorally inspired self-control problems. In the temptation model, the redistributive effect of taxation depends critically on how policies affect the endogenous *discount factor gap* between richer and poorer individuals, that is, how they map to the *current resources* and *future income* channels.

Specifically, progressive income taxation, despite reducing disparities in available resources between richer and poorer agents, is less effective in mitigating wealth inequality in the temptation framework relative to the rational benchmark. This result arises because progressive income taxation exerts an ambiguous impact on the *discount-factor gap* between richer and poorer agents: it simultaneously reduces inequality in current income, thereby narrowing the gap through the *current resources* channel, and reduces inequality in expected future income, thereby *widening* the gap through the *future resources* channel. In contrast, even modest progressive wealth taxation, which operates solely through the *current resources channel*, directly compresses the discount factor gap, thus is more effective in mitigating wealth concentration in a model with temptation preferences. As a practical consequence of this asymmetry, a policy mix of progressive wealth tax and flat labor-income tax, judged as optimal in a model evaluated by Guvenen et al. (2023), *reduces* wealth inequality in the temptation framework relative to a baseline with progressive income taxation and no wealth tax. In a standard framework, the identical mix is predicted to *increase* wealth inequality relative to the same baseline, underscoring the pivotal role of the temptation mechanisms analyzed in this article in shaping the distributional impact of tax policy.

In summary, my contribution to the literature is fourfold:

- I show that a parsimonious model with temptation preferences (in which heterogeneity arises solely from the income process and no *ex ante* preference differences are imposed) produces a more unequal wealth distribution and, when calibrated to the same income process and aggregate wealth, substantially outperforms a standard rational model in matching empirical wealth inequality as measured by the Gini coefficient and top wealth shares.

- I identify two novel mechanisms, the *current resources channel* and the *future income* *channel*, through which temptation preferences affect intertemporal choice. Quantitatively, the *current resources channel* dominates and endogenously generates the positive correlation between wealth and effective patience.

- I show that this key mechanism, i.e. link between wealth and effective patience, not only provides a behavioral microfoundation for the observed heterogeneity in discount rates but also serves to match several[3] empirical regularities in the relationship between patience and wealth documented by (Epper et al., 2020).

- Finally, I show that these behavioral mechanisms alter the distributional impact of tax policy: within the temptation framework a policy including wealth taxation is effective at reducing inequality, reversing the predictions of standard rational models.

1.1 Related Literature

This paper speaks to two broad strands of literature. First, it contributes to the growing body of work that examines the key role of temptation and costly self-control in shaping macroeconomic outcomes. Temptation preferences, in addition to being consistent with experimental evidence (e.g. Toussaert, 2018; Bucciol et al., 2011; Beshears et al., 2020) have been shown to explain several macroeconomic regularities that are difficult to account for within a standard rational framework. These include the prevalence of "wealthy hand-to-mouth" consumers with a high marginal propensity to consume out of large income shocks (Attanasio et al., 2024), the significant relationship between consumption growth and the level of assets away from the liquidity constraint (Kovacs et al., 2021; Kovacs and Moran, 2021), the drop in consumption following retirement (Bucciol et al., 2013), and the observed demand for commitment (Kovacs and Moran, 2021; Huang et al., 2015), which enhances the wealth accumulation process and explains people's housing choices (Angelini et al., 2020).

Indeed, studies employing either linearized Euler equations (Kovacs et al., 2021; Huang et al., 2015) or structural models (Kovacs and Moran, 2021; Attanasio et al., 2024) have consistently yielded statistically significant estimates of the temptation parameter. These findings are important because many of the policy recommendations derived from standard models hinge on the assumption of cost-free self-control. For example,Kumru and Thanopoulos (2011) show that the detrimental effects of unfunded social security on overall welfare are completely reversed when temptation preferences are taken into account. Similarly, policies aimed at relaxing borrowing constraints (e.g., through home equity withdrawals) are judged to be detrimental to agents' welfare in the temptation framework, in contrast to the standard model's view (Nakajima, 2012; Kovacs and Moran, 2021). Despite this extensive literature, which provides strong empirical support for the role of self-control in shaping agents' decisions relevant both at the macro and micro level, little is known about the impact of temptation on wealth distribution and the forces shaping wealth inequality in a world where agents cannot exercise perfect, costless self-control.

[3] Specifically, temptation model predicts that (i) the patience–wealth association strengthens with age; (ii) it persists after adjusting for empirically motivated covariates such as age, education, income, and expected income; and (iii) patience measured early in life strongly predicts subsequent within-cohort wealth ranks, replicating stylized facts documented by Epper et al. (2020).

In this sense, my work connects with macroeconomic studies that employ quantitative models to disentangle the forces behind wealth inequality. As noted by De Nardi (2015), 'economic models have had difficulties in quantitatively generating the observed degree of wealth concentration from the observed income inequality'. One key mechanism that has been shown to bring the distribution of model-generated asset holdings closer to the data (thereby improving the reliability of these models as tools designed to study wealth inequality) is the incorporation of heterogeneous preferences. In particular, the heterogeneity in patience can explain the vast differences in wealth held by agents (Krusell and Smith, 1998; Hendricks, 2007). As demonstrated by Carroll et al. (2017), even a modest degree of discount rate heterogeneity can account for substantial wealth inequality between households with similar lifetime earnings.

These findings are particularly compelling given that both the survey (Falk et al., 2018) and structural (Calvet et al., 2021) estimates consistently reveal significant heterogeneity in patience within the population, thereby justifying the modeling assumption of time discounting heterogeneity. Moreover, Epper et al. (2020) document a strong association between individuals' time discounting and their position in the wealth distribution. Indeed, they show that, after adjusting for a wide range of theoretically motivated covariates, 'differences in time discounting across individuals play a significant role for wealth differences' (pp. 1202). In this paper, I demonstrate that temptation preferences offer a parsimonious way to endogenize (i) the large heterogeneity in effective patience and (ii) the association between individual time discounting and wealth rank, even when all agents share identical underlying preferences. In other words, rather than exogenously assuming heterogeneity in preferences to explain wealth inequality, the temptation model endogenously generates heterogeneity in patience, which in turn brings the model-implied distribution of wealth closer to the data.

Finally, on the policy side, this paper connects with the literature proposing tax-oriented measures to address the rise in U.S. wealth inequality observed since the late 1970s (Saez and Zucman, 2022). In particular, there has been renewed academic interest in wealth taxation, with Piketty et al. (2023) calling for a progressive wealth tax as part of an "ideal fiscal system," and others emphasizing its ability to increase tax progressivity (and, for top wealth holders, to restore it) by addressing the incompleteness of other taxes (Saez and Zucman, 2019; Bastani and Waldenström, 2023). Accordingly, I examine the channels through which a simple progressive wealth tax shapes the wealth distribution in a temptation model. Even a modest levy compresses the discount factor gap between wealthier and poorer agents and is therefore more effective at mitigating inequality in the temptation model than the rational benchmark suggests. As a stark example of the mechanism at play, I show that a policy mix combining a progressive wealth tax with a flat labor-income tax, identified by Guvenen et al. (2023) as normatively optimal, would also *reduce* wealth inequality in the temptation model, contrary to the predictions of the standard rational framework. This finding highlights the need to devote greater attention to policy solutions which include progressive wealth taxation as part of the mix designed for economies with non-rational preferences.

The paper is structured as follows. In Section 2 I formally derive the two channels in the stylized model and discuss their *ceteris paribus* interpretation. Section 3 introduces the full quantitative model calibrated for the US economy. The results are presented in section 4. Section 5 concludes with implications for future research and economic policy.

2 A stylized model

In this section, I analyze the forces that shape the consumption-savings decisions of a tempted agent and compare them to those made by a standard rational agent. I show that the effective discount factor of a tempted agent is determined by the ratio of the marginal temptation disutility to the marginal utility of increasing next-period wealth. The latter arises from the utility of next-period consumption, while the former, unique to the temptation model, reflects the temptation disutility associated with increasing the amount of tempting resources available in the next period. Whenever this ratio is high, temptation has a stronger impact on the intertemporal decisions of tempted agents, leading them to behave as though they are less patient. This dynamic is entirely absent in the rational framework, where all agents share the same discount factor.

I identify two key channels that influence the effective discount factor of tempted agents. First, the channel of *current resources* posits that, *ceteris paribus*, agents with a higher level of current resources (whether due to increased income or greater wealth) are effectively more patient. Second, the channel of *future income*, implies that agents who experience a persistent increase in the *future* income behave as if they are less patient.

2.1 The environment

Consider a partial-equilibrium economy populated by either rational or tempted agents. For clarity, in this section, I distinguish the variables corresponding to the choices of rational agents using a tilde (e.g., $\widetilde{v}$) and those corresponding to tempted agents using a hat (e.g., $\widehat{v}$).

2.2 Consumer problem

Rational and tempted agents maximize lifetime utility. For analytical convenience, in this section, I set the instantaneous utility from consumption to $u(c) = \log(c)$. Agents live for three periods, denoted by j. They discount the future with a factor $\delta < 1$ and earn an income $\mathbf{y} = [y_1, y_2, y_3] \geq 0$ for inelastically supplied labor in periods 1, 2, and 3, respectively. The agent earns an interest r on assets, let $R = (1 + r)$. The stylized model is set in partial equilibrium; thus, R is constant and exogenous. Denote the level of resources available to the agent in period j as $\omega_j = Ra_j + y_j$, where Ra_j is the level of wealth at the beginning of period j (interest accrues at the beginning of the period) and y_j is income earned income in that period. In each period, agents decide on the division of resources between consumption c_j and savings a_{j+1}. All agents face a borrowing constraint $\underline{a} = 0$.[4]

Rational agent The problem solved by a rational agent written in the recursive form reads:

$$\widetilde{V}_j\left(\omega_j\right) = \max_{\widetilde{c}_j, \widetilde{a}_{j+1}} \{\log(\widetilde{c}_j) + \delta[\widetilde{V}_{j+1}(\omega_{j+1})|\omega_j]\}$$

subject to $\widetilde{c}_j + \widetilde{a}_{j+1} \leq R\widetilde{a}_j + y_j$ and $\widetilde{a}_{j+1} \geq 0,\ \widetilde{c}_j \geq 0.$

[4]The choice of borrowing constraint $\underline{a}$ plays an important role in determining the amount of tempting resources (see the definition below). For analytical tractability, I set $\underline{a} = 0$; however, allowing a borrowing constraint $\underline{a} = c < 0$ would simply require increasing the amount of tempting resources by a constant

Tempted agent The problem solved by a tempted agent in recursive form reads:

$$\widehat{V}_j(\omega_j) = \max_{\widehat{c}_j,\widehat{a}_{j+1}} \left\{ \log(\widehat{c}_j) + \gamma\big[\log(\widehat{c}_j) - \log(R\widehat{a}_j + y_j)\big] + \delta[\widehat{V}_{j+1}(\omega_{j+1})|\omega_j] \right\}$$

subject to $\widehat{c}_j + \widehat{a}_{j+1} \leq R\widehat{a}_j + y_j$ and $\widehat{a}_{j+1} \geq 0, \widehat{c}_j \geq 0$. In the tempted agent problem $\log(\widehat{c}_j)$ is a standard, instantaneous utility from consumption, whereas $\log(R\widehat{a}_j + y_j)$ denotes the utility the agent would have experienced had she consumed all available, tempting resources (temptation utility; Gul and Pesendorfer, 2004). In this framework, the agent who decides to save and restrain herself from consuming everything bears a nontrivial utility cost of magnitude $[\log(\widehat{c}_j) - \log(R\widehat{a}_j + y_j)]$, scaled by γ: the agents' degree of temptation (greater γ implies greater temptation faced by households and higher cost of self-control). In other words, delay in consumption gratification reduces utility, which is labeled in the literature as the utility cost of self-control. Note that the tempted problem is conceptually similar to the rational problem. The key difference which drives divergent choices of rational and tempted agents (in general $\{\widehat{c}_j, \widehat{a}_{j+1}\} \neq \{\widetilde{c}_j, \widetilde{a}_{j+1}\}$) is related to the utility cost of holding non-consumed, tempting assets: $\gamma\,[\log(\widehat{c}_j) - \log(R\widehat{a}_j + y_j)]$. In the tempting case, $\widehat{V}_j(\omega_j)$ depends not only on the level of consumption in the current period and the continuation value, but also on the amount of available, tempting resources.

2.3 First order conditions

Derivation of relevant policy functions for both the rational and the tempted agent can be found in the Appendix A.1. In this section, I present the key intuitions, comparing optimal decisions of agents with temptation preferences to optimal outcomes for agents with rational preferences. By design (no bequest motive) in the last period it is optimal for both the rational and tempted agent to consume all available resources. Thus, the tempted agent bears no utility cost of self-control in the last period.

FOC in the 2nd period. FOC yield the following Euler equation in the second period:

$$\begin{aligned} \text{Rational preferences:} \quad u'(\widetilde{c}_2) &= \delta R u'(R\widetilde{a}_3 + y_3). && (1)\\ \text{Tempted preferences:} \quad u'(\widehat{c}_2) &= \delta R u'(R\widehat{a}_3 + y_3)/(1+\gamma), && (2) \end{aligned}$$

Note that the second-period Euler equation for the tempted agent's problem closely resembles that of the rational agent. The only difference is the presence of the $(1+\gamma)$ term dividing the right-hand side of equation (2), which implies that the tempted agent assigns a higher weight to second-period consumption. Intuitively, $(1+\gamma)$ reduces the marginal utility of second-period consumption, $u'(c_2)$, leading the tempted agent to consume more and save less in that period relative to the rational agent. Otherwise, the Euler equations are nearly identical, as increasing wealth for the third and final period does not incur any future self-control cost.

FOC in the 1st period. FOC for the agent with temptation preferences in the first period is characterized by the following Euler equation:

$$u'(\widehat{c}_1) = \delta R \left[u'(\widehat{c}_2) - \frac{\gamma}{1+\gamma} u'(R\widehat{a}_2 + y_2) \right]. \quad (3)$$

From the perspective of a first-period tempted decision-maker, the value of marginally increasing wealth for period $j = 2$ includes both the benefit of a marginal increase in next-period consumption utility $u'(\widehat{c}_2)$ and the marginal increase in $-u'(R\widehat{a}_2 + y_2)$, which I refer to as the "marginal temptation disutility," multiplied by $\frac{\gamma}{1+\gamma}$. This term reflects the utility cost associated with enlarging the pool of tempting resources in the subsequent (second) period. For the rational agent, with $\gamma = 0$, the equation (3) reduces to:

$$u'(\widetilde{c}_1) = \delta R u'(\widetilde{c}_2). \tag{4}$$

In comparative terms, marginal temptation disutility $-u'(R\widehat{a}_2 + y_2)$, anticipated by a tempted agent, reduces the perceived benefit of accumulating wealth for the future. As a result, $u'(\widehat{c}_1) < u'(\widetilde{c}_1)$; the tempted agent always saves less and consumes more than the rational agent. Consider rearranging the Euler equations (4) and (3), respectively:

$$\text{Rational preferences:} \quad u'(\widetilde{c}_1) = R \qquad \delta \qquad u'(\widetilde{c}_2) \tag{5}$$

$$\text{Tempted preferences:} \quad u'(\widehat{c}_1) = R \quad \delta\left[1 - \frac{\gamma}{1+\gamma}\frac{u'(R\widehat{a}_2+y_2)}{u'(\widehat{c}_2)}\right] \quad u'(\widehat{c}_2) \tag{6}$$

Accordingly, I define the first-period effective discount factor of a tempted agent as:

$$\delta_1(\widehat{a}_2, y_2, y_3) \equiv \delta\left[1 - \frac{\gamma}{1+\gamma}\frac{u'(R\widehat{a}_2 + y_2)}{u'(\widehat{c}_2)}\right]$$

The comparison of two discount factors reveals key differences between temptation preferences and rational preferences. First, the rational agent discounts the future utility at a constant rate δ, whereas the tempted agent effectively discounts future utility at a rate that depends on (a_2, y_2, y_3). Second, since agents cannot borrow, the ratio $u'(Ra_2 + y_2)/u'(c_2)$ never exceeds 1. Third, as the ratio $u'(Ra_2 + y_2)/u'(c_2)$ decreases, the effective discount factor of the tempted agent increases. Intuitively, temptation will have a smaller impact on the intertemporal choices of tempted agent (implying a higher effective "patience") whenever the marginal cost of increasing resources for the next period (i.e., temptation disutility $-u'(Ra_2 + y_2)$) is low relative to the marginal benefit $u'(c_2)$.

2.4 Discounting with temptation preferences

Observe that upon choosing the level of assets saved for $j = 2$, a rational agent discounts the future with a scalar δ. In contrast, an agent with temptation preferences discounts the future utility at a rate determined by the effective endogenous discount factor $\delta_1(\widehat{a}_2, y_2, y_3)$. This section examines the role of *current* and *future* resources in shaping $\delta_1(\widehat{a}_2, y_2, y_3)$.

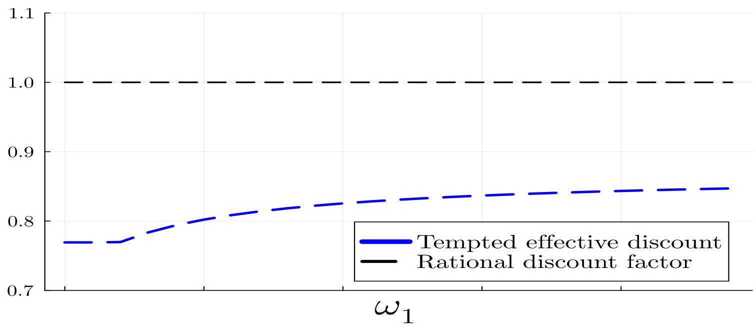
2.4.1 The current resources channel

Define the current resources, from the perspective of the agent's decision-making in period 1, as $\omega_1 = Ra_1 + y_1$. Proposition 1 explores the role of current resources, demonstrating that, *ceteris paribus*, agents with temptation preferences who have a greater level of current resources, whether due to higher first-period income or a greater first-period level of wealth, are effectively more patient in the first period.

Proposition 1 (The current resources channel ($\frac{\partial \delta_1(\cdot)}{\partial \omega_1}$)) *The effective discount factor of the tempted agent in the first period $\delta_1(\widehat{a}_2, y_2, y_3)$ is an increasing function of current resources $\omega_1 = Ra_1 + y_1$, provided the agent is unconstrained in the next (second) period (NLC case). If the agent is liquidity constrained in the next period (LC case) the $\delta_1(\widehat{a}_2, y_2, y_3)$ remains constant in ω_1.*

The proof of the Proposition 1 is relegated to the Appendix A.2.

Figure 1: Simulated effective discount factor as a function of current resources ω_1



Note: Simulated effective discount factor $\delta_1(\widehat{a}_2, y_2, y_3)$ of tempted agent in the first period. This factor doesn't increase in ω_1 if the agent is liquidity constrained in the next (second) period, as she would consume all available second-period resources anyways (LC case in the Proposition 1). Otherwise $\delta_1(\widehat{a}_2, y_2, y_3)$ increases in ω_1 because $\widehat{a_2}$ is an increasing function of ω_1. For a rational agent the discount factor δ is always a constant. The environment: $\delta = 1, R = 1, \gamma = 0.3, y_2 = y_3 = 3$, $\omega_1 \in (4, 20)$.

Proposition 1 has important implications for wealth accumulation dynamics and wealth inequality in a temptation model. Indeed, as illustrated on Figure 1, the structure of temptation preferences implies that agents with greater level of current resources will behave more "patiently" than those with fewer resources, even when all share the same degree of temptation γ and discount factor δ. Consequently, individual incentives to save are an *increasing* function of wealth. Intuitively, this dynamic arises because wealthier individuals' intertemporal choices are to a lesser extend impacted by a marginal temptation disutility. Note that the marginal temptation disutility, $-\frac{\gamma}{1+\gamma}u'(R\widehat{a}_2 + y_2)$, decreases in $\widehat{a}_2$ at a faster rate than the marginal utility of next-period consumption, $u'(\widehat{c}_2)$. As a result, the ratio of these two terms declines, diminishing the impact of marginal temptation disutility on the marginal value of increasing next-period wealth, represented by the right-hand side of the tempted Euler equation (6).

To compare consumption-saving decisions of tempted and rational agents, first observe that both types increase the level of assets saved for the next period in current resources ω_1. In the tempted model, the fact that $\frac{\partial \widehat{a_2}(\omega_1)}{\partial \omega_1} > 0$ is established by Lemma 1. For the rational model, the fact that $\frac{\partial \widetilde{a_2}(\omega_1)}{\partial \omega_1} > 0$ follows directly from the rational policy function for $\widetilde{a_2}$, derived in Appendix A.1, which increases with respect to a_1 and y_1. In this context, Proposition 1 implies that, compared to a rational agent, a tempted one with higher ω_1 will increase the level of assets saved for the next period due to: (i) the standard smoothing motive, and (ii) the additional reason: a positive relationship between effective patience and the level of current resources ω_1. This relationship implies that the incentive to save increases with ω_1, thereby amplifying the transmission of current wealth inequality into future wealth inequality. This additional impact of current wealth on the discount factor implies that saving behavior of tempted agents is more reactive to disparities in current wealth than rational agents.

2.4.2 The future income channel

Define future income, from the perspective of a decision maker in period 1, as the future income stream $[y_2, y_3]$. Consider persistent increase $[\epsilon, \rho\epsilon]$ in future income, with $\rho \geq 0$ governing the degree of persistence. Proposition 2 studies the role of future income in shaping the effective discount factor of the tempted agent. It shows that, *ceteris paribus*, increasing future income by $[\epsilon, \rho\epsilon]$ decreases the effective discount for $\rho > \frac{R\delta}{1+\gamma}$.

Proposition 2 (The Future Income Channel: $\frac{\partial \delta_1(\cdot)}{\partial \epsilon}$) *The impact of an increase of future income $[y_2, y_3]$ by $[\epsilon, \rho\epsilon]$ on the 1st period effective discount factor $\delta_1(\cdot)$ is characterized as follows:*

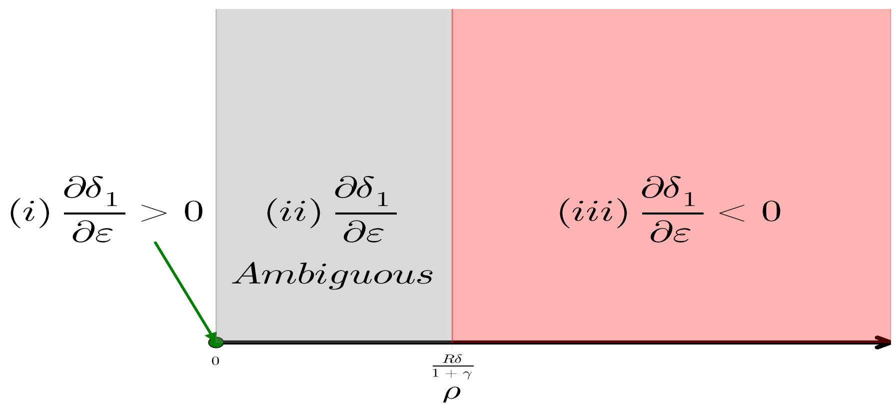
(i) for $\rho = 0$, $\frac{\partial \delta_1(\cdot)}{\partial \epsilon} > 0$;

(ii) for $\rho \in \left(0, \frac{R\delta}{1+\gamma}\right)$, the sign of $\frac{\partial \delta_1(\cdot)}{\partial \epsilon}$ cannot be determined solely by the interest rate and preference parameters but depends on the relationship between $\widehat{a}_2, y_2, y_3$;

(iii) for $\rho > \frac{R\delta}{1+\gamma}$, $\frac{\partial \delta_1(\cdot)}{\partial \epsilon} < 0$.

If the agent is liquidity constrained in the next (second) period, then $\frac{\partial \delta_1(\cdot)}{\partial \epsilon} = 0$ for all ρ.

The proof of the Proposition 2 is relegated to the Appendix A.3. Proposition 2 can be represented graphically by the following illustration. Under a typical calibration of the exogenous parameters, the impact of the *future income channel* is expected to fall within the red area; that is, $\frac{\partial \delta_1(\cdot)}{\partial \epsilon}$ is predicted to be *less than zero* in a standard computation framework which will be analyzed in Section 3.



Proposition 2 highlights the role of future income in shaping the effective endogenous discount factor of the tempted agent. The key quantity underlying the impact of an increase in future income on $\delta_1(\cdot)$ is again the ratio $-\frac{u'(Ra_2+y_2+\epsilon)}{u'(c_2)}$. Recall that whenever this ratio is low, a marginal temptation disutility has a less pronounced effect on the agent's intertemporal decision-making. Proposition 2 suggests that in Case (i), when only the next period's income is increased (i.e., $\rho = 0$), this ratio decreases because marginal temptation disutility,

$-u'(Ra_2 + y_2 + \epsilon)$, declines more rapidly than $u'(\widehat{c}_2)$, as the agent smooths consumption of the additional second-period income. In contrast, when $\rho > \frac{R\delta}{1+\gamma}$ the ratio increases because $u'(c_2)$ falls at a faster rate due to the combined rise in second- and third-period resources. For persistence levels in the range $\rho \in \left(0, \frac{R\delta}{1+\gamma}\right)$, the sign of $\frac{\partial \delta_1(\cdot)}{\partial \epsilon}$ depends on the relationship between $\widehat{a}_2, y_2, y_3$; specifically, when y_3 is low relative to $R\widehat{a}_2 + y_2$, even a moderate ρ will imply that $\frac{\partial \delta_1(\cdot)}{\partial \epsilon} < 0$. Note that under a standard calibration, $R\delta$ is typically smaller than $1 + \gamma$ (with γ often estimated around 0.3). Therefore, I expect that in a standard computational framework with ρ typically estimated to be close to one, the increase in future income will imply a decrease in the effective discount factor.

Crucially, whenever $\frac{\partial \delta_1(\cdot)}{\partial \epsilon} < 0$, the *future income* channel has clear implications for the wealth accumulation patterns of tempted agents. To see that first, note that both rational and tempted agents reduce the level of assets saved for the next period in response to increases in y_2 and y_3. In the tempted model, this behavior is established by Lemmas 2, whereas in the rational model it follows directly from the policy function for a_2 derived in Appendix A.1, which is decreasing in both variables. Next, Proposition 2 demonstrates that whenever $\rho > \frac{R\delta}{1+\gamma}$, a tempted agent with higher future income exhibits a lower level of effective level of patience. As a result, compared to a rational agent, a tempted agent facing a relatively persistent increase in income reduces asset accumulation for two reasons: (i) the standard effect arising from a diminished need for intertemporal resource transfer; and (ii) the negative relationship between the effective discount factor and future income. This additional impact of future income on the discount factor implies that tempted agents saving behavior *is more sensitive* to disparities in future income than do their rational counterparts.

Having established the impact of both channels on the effective discount factor, thus the wealth accumulation patterns, it is important to acknowledge the ambiguity in their interplay when the *ceteris paribus* condition is relaxed. The ultimate impact of current and future income on effective patience depends on the relative strength of each channel. In a standard framework agent simultaneously have a higher levels of current resources due to a high current income realization, while also anticipating greater future income. Such a setup introduces two competing forces: the positive relationship between effective patience and current resources promotes higher savings, whereas the negative relationship between effective patience and future income reduces the incentive to save. Their net effect on wealth accumulation is determined by the balance between these opposing forces and the broader dynamics of the model, thus introduces a puzzle warranting a more comprehensive quantitative assessment. In the following section, I analyze the interaction of the two channels within overlapping generations (OLG) model with temptation preference.

3 Quantitative model and calibration

In this section, I place the intuitions from the stylized theoretical setup into an empirical context. I incorporate temptation preferences into an otherwise standard overlapping generations (OLG) framework with uncertain lifetimes and income shocks, and I calibrate the model to reflect the U.S. economy in 2016. The degree of temptation (γ) is set in accordance with the empirical evidence provided by Kovacs and Moran (2021).

3.1 The model

3.1.1 Population

Households enter the labor market at age $j = 1$, corresponding to a biological age of 20 years. Each agent below the mandatory retirement age $\bar{j} = 45$ (biological age 65) inelastically supplies one unit of labor. Households live for $J = 70$ periods, facing age-specific survival probabilities π_j. At the biological age of 90, households face certain death.

3.1.2 Budget constraint

Households younger than $j < \bar{j}$ face a pre-tax labor income $y_{e,j} = wh_{e,j}$, where w represents the wage per unit of efficiency labor and $h_{e,j}$, idiosyncratic productivity profile that follows the following equation:

$$\log h_{e,j} = \beta_e + \epsilon_{e,j} + \eta \tag{7}$$

Fixed effects β_e are drawn at the beginning of life and characterize all households of e educational type, where $e \in \{u, h\}$. Here u denotes the university degree (college or more) and h high school or less. I normalize β_h to one, thus β_u can be interpreted as a skill premium of college-educated individuals. The deterministic age profile specific to educational type is denoted by $\epsilon_{e,j}$. The stochastic process η follows $AR(1)$, with persistence ρ and an idiosyncratic productivity shock $u \sim N(0, \sigma^2)$, thus η evolves according to: $\eta' = \rho\eta + u$, where η' is the next period η conditional on the current period η.

Households pay a simple progressive labor income tax $\mathcal{T}(y_{e,j}, \bar{y})$ given by $\mathcal{T}(y_{e,j}, \bar{y}) = y_{e,j} - (1 - \lambda_\ell)\left(\frac{y_{e,j}}{\bar{y}}\right)^{1-\tau_\ell} \bar{y}$, where $\bar{y}$ denotes the average pre-tax labor income, $1-\tau_\ell$ the elasticity of after-tax to pre-tax income, and $\lambda_{\ell,t}$ the average labor income tax rate. Household after-tax labor income is denoted by $\widehat{y}_{e,j}$.

Households aged $j \geq \bar{j}$ receive a pension $pen_e = p \cdot \widehat{y}_{e,\bar{j}-1}$ (for households younger than $\bar{j}$, $pen_e = 0$). To avoid the computationally costly procedure of tracking individual lifetime income, I assume that the pension is proportional to the last after-tax income income realization $\widehat{y}_{e,\bar{j}-1}$; p represents the replacement rate. Productivity $h_{e,j} = 0$ for households aged $j \geq \bar{j}$.

Households leave accidental bequests which are distributed in a framework stylized after Blanchard (1985). Households surviving between periods $j - 1$ and j will inherit $b_j \cdot a_{e,j}$ in period j , where $a_{e,j}$ denotes the level of assets saved in period $j - 1$ and b_j is:

$$b_j = \begin{cases} 0 & \text{if } j = 1 \\ \frac{\sum_{e\in\{u,h\}} \int_\Omega 1 d\mathbb{P}_{e,j-1}}{\sum_{e\in\{u,h\}} \int_\Omega 1 d\mathbb{P}_{e,j}} & \text{else} \end{cases} \tag{8}$$

Overall, for any period j, the household budget constraint reads:

$$a_{e,j+1} + c_{e,j} = b_j (1 + r) a_{e,j} + y_{e,j} - \mathcal{T}(y_{e,j}, \bar{y}) + pen_e \tag{9}$$

3.1.3 Households

The individual state vector for household of age j and type e is $\mathbf{x}_{e,j} = [a_{e,j}, h_{e,j}, b_j] \in \Omega$, where Ω is the state space, $a_{e,j}$ denotes the level of assets at the beginning of the period, $h_{e,j}$ the level of productivity and b_j the inheritance fator (see equation 8). Households of age

$j = 1$ enter without assets (that is, $a_{e,1} = 0$). Households discount the future utility with δ and the conditional survival probabilities π_j. The instantaneous utility of consumption is: $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$.

Rational agent problem The optimization problem of the rational agent in a recursive form is:

$$V_{e,j}\left(\mathbf{x}_{e,j}\right) = \max_{c_j, a_{j+1}} \left\{u(c_{e,j}) + \delta\pi_{j+1}\mathbb{E}_{h_{e,j+1}|h_{e,j}}[V_{e,j+1}(\mathbf{x}_{e,j+1})|\mathbf{x}_{e,j}]\right\} \tag{10}$$

Subject to the budget constraint (9).

Tempted agent problem As described in more detail in Section 2.2, tempted agent who decides to save and does not consume all available resources $(R\widehat{a}_j + y_j)$ has to bear a utility cost of self-control $\gamma[u(\widehat{c}_j) - u(R\widehat{a}_j + y_j)]$, where γ governs the degree of temptation.
The optimization problem of the tempted agent in a recursive form is:

$$V_{e,j}(\mathbf{x}_{e,j}) = \max_{c_j, a_{j+1}} \left\{u(c_{e,j}) + \gamma\big[u(c_{e,j}) - u(Ra_{e,j} + y_{e,j})\big] + \pi_{j+1}\delta\mathbb{E}_{h_{e,j+1}|h_{e,j}}[V_{e,j+1}(\mathbf{x}_{e,j+1})|\mathbf{x}_{e,j}]\right\} \tag{11}$$

subject to the budget constraint (9).

Effective discount factor of the tempted agent: Consider the full-model analog of the effective endogenous discount factor, introduced in Section 2. For compactness purposes, define the amount of resources available to the agent of type e in period $j+1$ as: $\omega_{e,j+1} = b_j\left(1+r\right)a_{e,j} + y_{e,j} - \mathcal{T}\left(y_{e,j}, \bar{y}\right) + pen_e$. The first order conditions yield the following Euler equation characterizing the interior solution to the tempted agent maximization problem:

$$u'\left(c_{e,j}\right) = R\delta\left(1 - \frac{\gamma}{1+\gamma}\frac{\mathbb{E}_{h_{e,j+1}|h_{e,j}}\left[u'\left(\omega_{e,j+1}\right)\right]}{\mathbb{E}_{h_{e,j+1}|h_{e,j}}\left[u'\left(c_{e,j+1}\right)\right]}\right)\mathbb{E}_{h_{e,j+1}|h_{e,j}}\left[u'\left(c_{e,j+1}\right)\right] \tag{12}$$

Accordingly, observe that tempted agent discounts $j+1$ expected marginal utility of increasing $j+1$ resources with the following effective discount factor:

$$\delta_{e,j+1} = \delta\left(1 - \frac{\gamma}{1+\gamma}\frac{\mathbb{E}_{h_{e,j+1}|h_{e,j}}\left[u'\left(\omega_{e,j+1}\right)\right]}{\mathbb{E}_{h_{e,j+1}|h_{e,j}}\left[u'\left(c_{e,j}\right)\right]}\right) \tag{13}$$

With the degree of temptation $\gamma = 0$, the effective discount factor just collapses to the rational agent discount factor δ.

3.1.4 Firm

There is one representative and competitive firm which uses capital K and labor L to produce the final output according to the Cobb-Douglas production function:

$$Y = K^{\alpha}L^{1-\alpha} \tag{14}$$

where α is an output elasticity with respect to capital. The capital depreciation rate is denoted as d. The profit maximization problem yields the standard formula for the net rate of return on capital r and the wage per unit of efficiency labor w:

$$r = \alpha K^{\alpha-1}L^{1-\alpha} - d \text{ and } w = (1-\alpha)\,K^{\alpha}L^{1-\alpha} \tag{15}$$

3.1.5 Government

The tax revenue T is spent to finance the social security system S and the government purchases G.

$$T = S + G \tag{16}$$

where

$$T = \sum_{e\in\{u,h\}} \sum_{j=1}^{\bar{j}-1} N_{e,j} \int_{\Omega} \mathcal{T}\left(y_{e,j}, \bar{y}\right) d\mathbb{P}_{e,j} \tag{17}$$

$\mathbb{P}$ is the probability measure which describes the distribution of agents with the educational level e and age j over the state space Ω. The resources used to finance the social security system are equal to:

$$S = \sum_{e\in\{u,h\}} \sum_{j=\widehat{j}}^{J} N_j \int_{\Omega} pen_{j,t} d\mathbb{P}_{e,j} \tag{18}$$

3.1.6 Equilibrium

Given government policies $\{\tau_l, p\}$, a competitive equilibrium is a sequence of: i) value functions: $\{V_{e,j}\left(\mathbf{x}_{e,j}\right)\}_{j=1}^{J}$ for $e \in \{u,h\}$; ii) policy functions $\{c_{e,j}, a_{e,j+1}\}_{j=1}^{J}$ for $e \in \{u,h\}$, iii) prices $\{r, w\}$; iv) aggregate quantities $\{L, K, Y, C, S, G, T\}$; v) measure of households $\left(\mathbb{P}_{e,j}\right)_{j=1}^{J}$ for $e \in \{u,h\}$;

- **Household problem:** For each e, j, value functions $V_{e,j}$ and policy functions $c_{e,j}, a_{e,j+1}$ solve the problem for rational and tempted agents, respectively equations: (10) and (11) hold.
- **Firm problem:** Prices $\{r, w\}$ satisfy equations (15).
- **Government:** The government budget and social security constraints are satisfied, i.e., equations (16) and (18) hold.
- **Markets clear:**
  - **Labor market:** $L = \sum_{e\in\{u,h\}} \sum_{j=1}^{\bar{j}-1} N_{e,j} \int_{\Omega} h_{e,j} d\mathbb{P}_{e,j}$
  - **Asset market:** $K = \sum_{e\in\{u,h\}} \sum_{j=1}^{J} N_{e,j} \int_{\Omega} a_{e,j+1} d\mathbb{P}_{e,j}$
  - **Goods market:** $Y = C + K - (1-d)\,K + G$
- **Probability measure:** measure of households $\left(\mathbb{P}_{e,j}\right)_{j=1}^{J}$ for $e \in \{u,h\}$ is consistent with exogenous processes for productivity and policy functions.

3.2 Calibration

Preferences The degree of temptation is set to $\gamma = 0.339$, following the estimates of Kovacs and Moran (2021), who use the simulated method of moments to match a set of life-cycle moments. This value of $\gamma = 0.339$ also falls within the range of estimates obtained from the linearized Euler equation in Kovacs et al. (2021). The coefficient of relative risk aversion is set to a standard value of $\sigma = 1.5$ (e.g., Cagetti and De Nardi (2006); Attanasio et al. (1999)).

The discount factor δ is calibrated to target a private wealth-to-income ratio of 4.2 in both the rational and tempted economies. This calibration results in a slightly higher δ for tempted agents relative to rational ones: $\delta = 0.955$ for rational agents and $\delta = 0.977$ for tempted agents.

Production The parameter α in Equation (14) is set to 0.367, following the calibration of Straub (2019) who relied on US non-financial corporate sector data for 2014 to calibrate the capital share. Similarly, the private wealth/Y ratio is targeted at 4.2, based on the Straub (2019) estimates. The private wealth/Y ratio is maintained across rational and tempted economies by manipulating the discount factor δ. The depreciation rate d (Eq. 15) is set to 0.038 targeting the interest rate $r = 0.05$.

Income process & taxes Households productivity profile (Eq. 7) is calibrated as follows. I take the value of college wage premium (β_e) from (Autor et al., 2020). Labor income average tax rate is set to $\lambda_\ell = 0.63$, a level high enough to finance Social Security spending. Finally, labor income tax progressivity is set to $\tau_\ell = 0.123$, based on the 2015 value for the U.S. reported by Makarski et al. (2025), who obtain year-specific progressivity parameters using average marginal and average tax rates.

I calibrate incomes using SCF data for working-age population (using the distribution of this data developed by Kuhn et al., 2020). I estimate the education-specific age-profiles $\epsilon_{e,j}$ using Deaton and Paxson (2000) decomposition. The age profiles are introduced to the model as deterministic, adjusting for cohort and time effects. The idiosyncratic income shocks are calibrated using the strategy proposed by Cagetti and De Nardi (2006). Specifically, I adopt the persistence parameter $\rho = 0.985$ and calibrate the variance of the idiosyncratic productivity shock σ^2 to match the U.S. top 10% earnings share in 2016. Taking education premium, deterministic age profiles and income shocks persistence as given, the implied variance of income matchig the empirical counterparts is $\sigma^2 = 0.0473$. This income process calibration yields data-consistent net income distribution as illustrated in Table 1.

Table 1: Matching income distribution

	Income Inequality	
	Top 10% share	Gini coefficient
Data	**48%**	**58%**
Model: Rational	48%	61.4%
Model: Tempted	48%	61.4%

Note: Income inequality indices for the USA are calculated based on the Survey of Consumer Finance using data for 2016 (in a version harmonized and distributed by Kuhn et al., 2017).

Survival probabilities The age specific survival probabilities π_j are calculated based on 2015 Centers for Disease Control and Prevention (CDC) life table.

4 Results

4.1 Wealth inequality

Standard rational models with homogeneous time preferences struggle to match the observed degree of wealth concentration, given the observed concentration of income (De Nardi, 2015). Adding behaviorally motivated costly self-control (temptation preferences) to this parsimonious benchmark, in which heterogeneity arises solely from the income process and no *ex ante* preference differences are imposed, substantially improves the model's fit to observed wealth concentration. Table 2 reports the levels of wealth inequality generated in both the rational and temptation models, each calibrated to the same aggregate private wealth-to-income ratio of 4.2. Using identical inputs, the temptation model matches U.S. wealth inequality much more closely, deviating from the data by only 6.6 percentage points for the top 10% wealth share and 2.5 points for the Gini coefficient.

Table 2: Wealth inequality measures models vs data

	Wealth Inequality	
	Top 10% share	Gini coefficient
Data	**76.7%**	**85**
Model: Rational	58.1%	73.3
Model: Tempted	70.1%	83.4

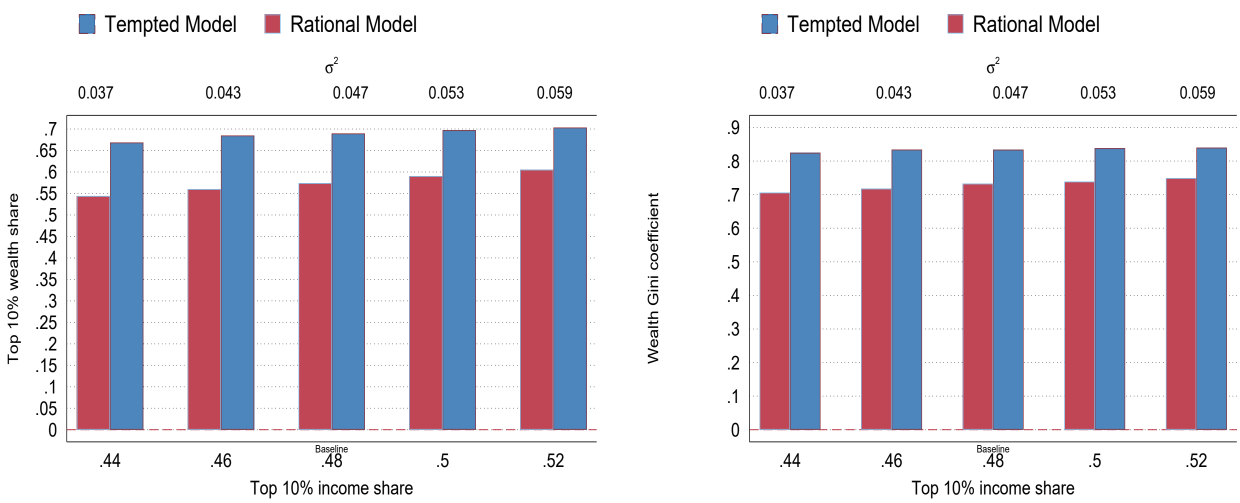
Note: Wealth inequality indices for the USA are calculated based on the Survey of Consumer Finance using data for 2016 (in a version harmonized and distributed by Kuhn et al., 2017).

Importantly, the higher levels of wealth concentration generated by the temptation model are not an artifact of a particular calibration of the income process; it is a feature of temptation model and arises across a range of values of the idiosyncratic productivity variance σ^2 (see Figure 2). The key mechanism that allows the model to account for higher wealth inequality is the endogenously generated positive correlation between patience and wealth. In Section 4.2, I re-examine the determinants of tempted agents' effective discount factor and show that both the *current resources* and *future income* channels derived in the stylized model are present in the full OLG model. Quantitatively, the *current resources* channel dominates, generating the positive correlation between patience and wealth. In Section 4.3, I show that this mechanism is consistent with recent empirical evidence and enables the temptation model to reproduce several stylized facts on the relationship between patience and wealth documented by (Epper et al., 2020), without imposing *a priori* heterogeneity in discount factors.

In the temptation model, the key driver of the wealth distribution is thus the endogenously generated *discount-factor gap* between richer and poorer individuals. This mechanism has direct policy implications: the redistributive effect of taxation depends on how policies affect the *discount-factor gap*, that is, on how they map into the *current resources* and *future income* channels. I examine these implications in detail in Section 4.4.

Figure 2: Comparison of wealth inequality for different values of idiosyncratic productivity shock variance

(a) Top 10% wealth share (b) Gini coefficient on wealth



Note: The top of the figure displays value of σ^2, the bottom displays the corresponding income inequality measure (top 10% income share)

4.2 Effective patience in the full model: the two channels revisited

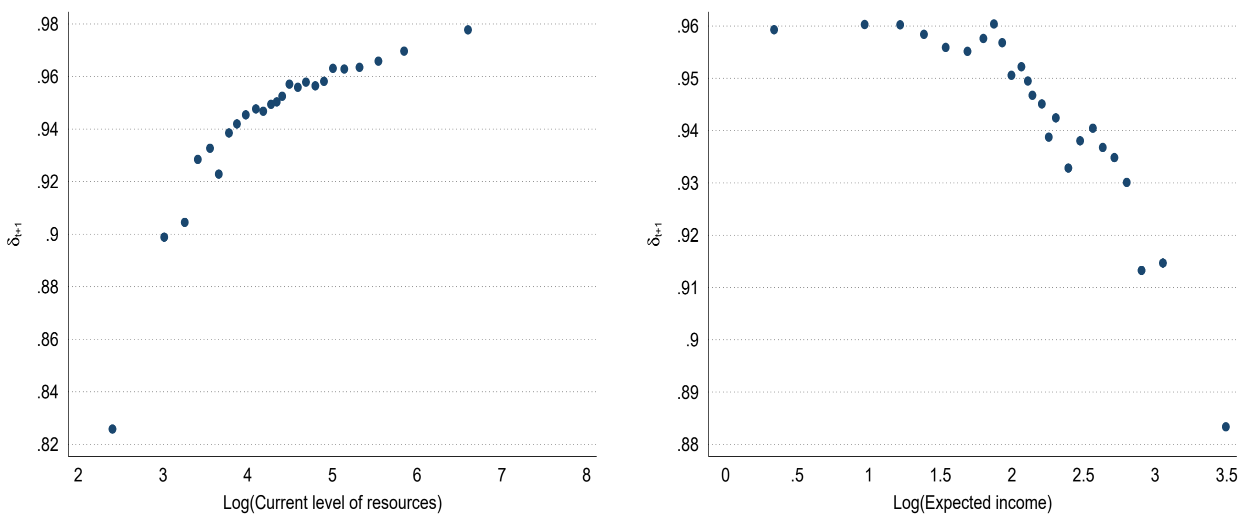
In Section 2, I derived two channels which shape the effective discount factor of tempted agents in a stylized model. In this section I examine the impact of these channels on $\delta_{e,j+1}$, the full-model stochastic analog of the effective discount factor analyzed in a stylized setup (see Equation 13 for the derivation). To this end, Figure 3 presents both channels, maintaining the *ceteris paribus* assumption as in Section 2. Panel (a) illustrates the influence of the current level of resources, $\omega_{e,j}$, on $\delta_{e,j+1}$, adjusting for the expected income level; whereas Panel (b) presents the full-model counterpart of the future income channel, displaying the impact of expected future income on $\delta_{e,j+1}$ while adjusting for the current level of resources.

The impact of both channels remains consistent with the analysis in Section 2. In particular, the positive relationship between effective patience and the current level of resources (Panel a) suggests that, *ceteris paribus*, the incentive to save increases with the resources available to the tempted agent. Furthermore, the negative relationship between effective patience and expected future income (Panel b) indicates that, controlling for current resources, a tempted agent anticipating higher future income reduces their savings for two reasons: (i) the standard effect of a diminished need for intertemporal resource transfer; and (ii) the inverse relationship between effective patience and expected income. This additional influence of future income on the discount factor implies that the saving behavior of tempted agents is more sensitive to disparities in anticipated future income than that of the rational agent. Similarly, the positive impact of current resources on the effective discount factor suggests that tempted agents saving choices are more responsive to inequality in current resources.

Figure 3: Full-model analog of the two channels derived in Section 2

(a) The current resources channel

(b) The future income channel



Note: Both panels present the forces that shape tempted agents' effective discount factors in a full OLG model augmented with temptation preferences. Panel (a) replicates the *current resources* channel derived in Section 2.4. It illustrates the impact of the current level of resources (i.e. wealth and income or pension realization) on the tempted agent's effective discount factor, $\delta_{e,j+1}$, while adjusting for the impact of expected income on $\delta_{e,j+1}$. Panel (b) replicates the *future income* channel derived in Section 2.4. It presents the association between the level of expected income and $\delta_{e,j+1}$, while adjusting for the impact of current resources.

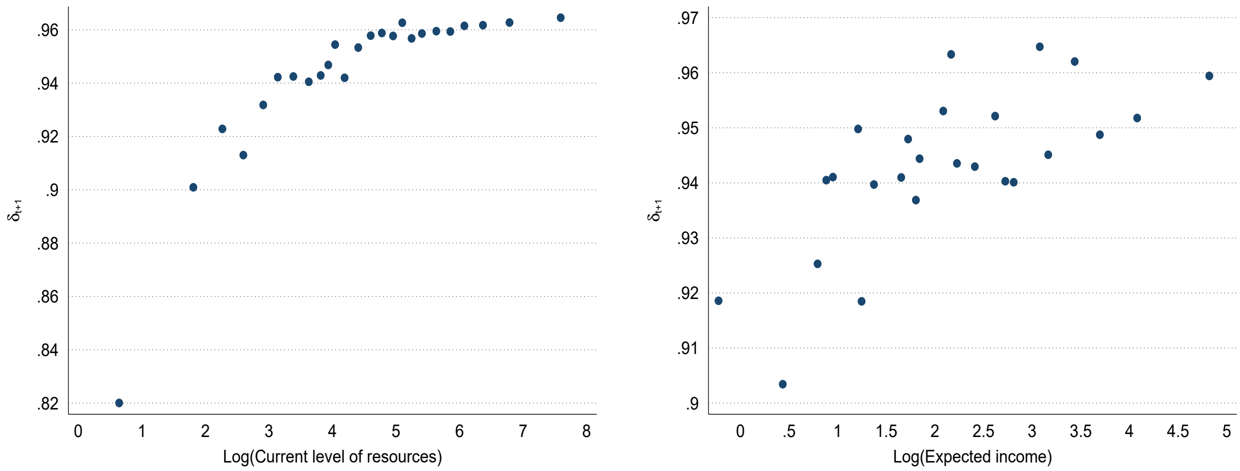
Relaxing the *ceteris paribus* assumption: the key mechanism which $\Uparrow$ wealth inequality in the tempted model

Relaxing the *ceteris paribus* assumption introduces ambiguity regarding the net effect of both channels on the correlation between effective patience and income or wealth. Due to income persistence, an expectation of high future income is typically associated with a higher level of current resources due to (i) greater current income realization and (ii) greater nominal level of already accumulated wealth. Therefore, agents with elevated expected income will typically have a higher level of current resources. If the *future income* channel dominates, then agents with a higher level of current resources, who also tend to expect higher future income, will exhibit lower effective patience. In this scenario, the prospect of high future income overwhelms the influence of current resources, thereby reducing the effective discount factor of wealthier agents. Conversely, if the *current resources* channel is the primary driver of the effective discount factor, wealthier agents will exhibit greater discount factors despite their relatively elevated expected future income.

Quantitatively, the evidence supports the dominance of the *current resources* channel in shaping the effective discount factor across agents of all ages and income shock histories. Panel(a) of Figure 4 illustrates that, even when the *ceteris paribus* assumption is relaxed, a strong positive relationship remains between an agent's current level of resources and δ_{t+1}, despite the elevated levels of expected future income typically associated with wealthier agents. Panel (b) shows that the relationship between expected income and the effective discount factor, although more dispersed, is overall positive, reversing the negative association that holds under the *ceteris paribus* assumption. This reversal arises due to the positive association between expected income and current resources, as illustrated in Figure 5. Overall, agents who expect greater income in the future tend to have greater level of current resources, which in turn raises their effective discount factor. This mechanism implies a positive association between wealth, expected income, and effective patience in the temptation model, thus it generates a *discount factor gap* between relatively wealthier and poorer agents.

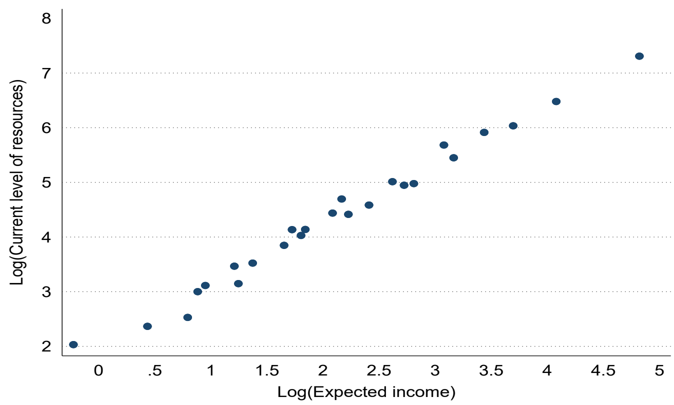
Figure 4: Relaxing the *ceteris paribus* assumption

(a) Current wealth and $\delta_{e,j+1}$ (b) Expected income and $\delta_{e,j+1}$



Note: Both panels illustrate the forces that shape tempted agents' effective discount factors in a full OLG model augmented with temptation preferences. Panel (a) presents the association between the tempted agent's effective discount factor, $\delta_{e,j+1}$, and the current level of resources without adjusting for expected income. The relationship remains strong, suggesting that quantitatively current resources are a primary driver of $\delta_{e,j+1}$. Panel (b) displays the association between $\delta_{e,j+1}$ and expected income, without adjusting for the impact of current resources. Compared to the *ceteris paribus* analysis, the direction of this correlation is reversed, indicating that agents with greater expected income also tend to be more patient. This reversal occurs due to the positive correlation between expected income and the current level of resources, as shown in Figure 5.

Figure 5: Association between the expected income and the current level of resources



Note: This figure presents the model-generated positive correlation between expected income and the current level of resources. Overall, agents who expect higher income also tend to be wealthier, a force which drives the postive relationhsip between expected income and patience presented in Panel (b) of Figure 4.

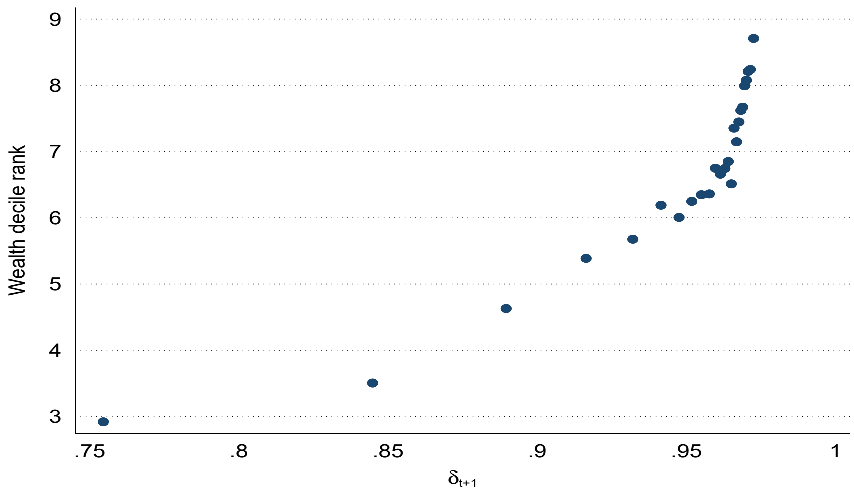
4.3 The underlying mechanism is consistent with empirical findings

Given the dominance of the *current resources* channel in the temptation model, on average, wealthier individuals also tend to be more patient. This key mechanism, which enables the model to replicate high levels of wealth concentration, is also consistent with available empirical evidence. In this section, I show that the temptation model reproduces several stylized facts about the link between patience and wealth documented by Epper et al. (2020). First, as in the data, the relationship between patience and wealth persists after controlling for life-cycle effects and examining the within-cohort wealth distribution. Second, consistent with the data, the model predicts that the association between patience and wealth increases (almost linearly) with age. Third, in a regression analysis that additionally adjusts for educational attainment, income, and expected income, the relationship persists as reported in Epper et al. (2020), though its magnitude is somewhat larger. Finally, effective patience measured as early as period $j = 2$ is highly predictive of an agent's within-cohort wealth rank in all subsequent periods, replicating the lifetime impact of time discounting on wealth rank observed in Epper et al. (2020)[5].

Figure 6 illustrates the relationship between the effective discount factor $\delta_{e,j+1}$ and the individual decile rank in the within-cohort distribution of assets saved. Note that this measure inherently accounts for life-cycle effects in saving behavior by comparing only individuals within the same age cohort. Crucially despite the fact that all agents share the same discount factor δ and temptation parameter γ, the model with temptation preferences can endogenously generate a strong association between patience (understood as effective discount factor $\delta_{e,j+1}$ applied to discount future expected marginal utility) and wealth rank. While Epper et al. (2020) interpret such empirically observed associations as an evidence supporting models with heterogeneous discount factors (which assume that individuals wealth ranks are driven in part by preferences heterogeneity), the results of the temptation model suggest that this associations can emerge

[5]For the last two analyses, see Appendix A.7.

Figure 6: Association between $\delta_{e,j+1}$ and (within-cohort) rank in the distribution of current resources



Note: The individual effective discount factor is highly predictive of an agent's savings level. This figure examines the model-implied relationship between a tempted agent's individual effective discount factor, $\delta_{e,j+1}$, and their position in the within-cohort distribution of assets saved for the next period. This analysis includes only agents who are not liquidity constrained.

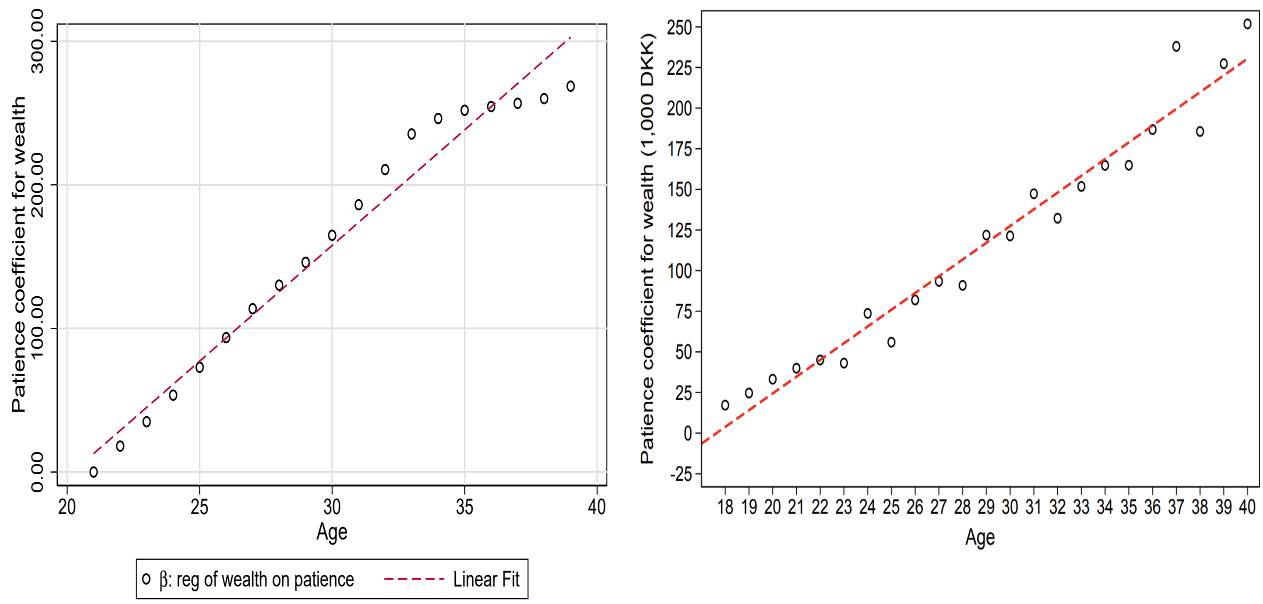
endogenously, even when agents share identical underlying preferences.

The temptation model not only replicates the general stylized fact that individuals with higher within-cohort wealth rank are more patient, but also predicts that the association between patience and wealth strengthens with age. This pattern is documented in Figure 7, which juxtaposes the association generated by the model with the one reported by Epper et al. (2020). The points in the figure denote the coefficients from age-specific regressions of wealth (in levels) on patience, and the dashed red line shows the fitted linear trend. As in the data, the coefficients rise with age and closely follow a linear pattern. Despite its parsimony, the temptation model closely replicates the age gradient in the patience–wealth relationship.

Figure 7: Relationship between patience and wealth over age

(a) Model-generated association

(b) Association in Epper et al. (2020)



Note: Each dot represents the coefficient from age specific regression of wealth on patience measure. The slope of the red line can be interpreted as the coefficient on the interaction between patience and age in a regression of wealth on patience and age. The Figure (b) is the Figure A7 which can be found in the Appendix to Epper et al. (2020).

4.4 Temptation & wealth inequality: progressive taxation case

Whereas the previous sections underscored the central role of the *current resources* channel in driving wealth inequality in the temptation model, this section demonstrates that *both* channels are practically relevant in moderating the impact of progressive taxation on the wealth distribution. Taken together, my findings indicate that wealth concentration in the temptation model is less sensitive to changes in income taxation, yet highly sensitive to changes in wealth taxation. Progressive income taxes affect both the *current resources* and *future income* channels simultaneously, but in opposite directions, and therefore have an ambiguous effect on the discount-factor gap between richer and poorer agents. In contrast, wealth taxes act solely through the *current resources* channel and thus directly compress that gap. Against this background, a policy mix combining a progressive wealth tax with a flat labor-income tax (judged as optimal in a model analyzed by Guvenen et al., 2023) *reduces* wealth inequality in the temptation model, contrary to the predictions of the standard rational framework.

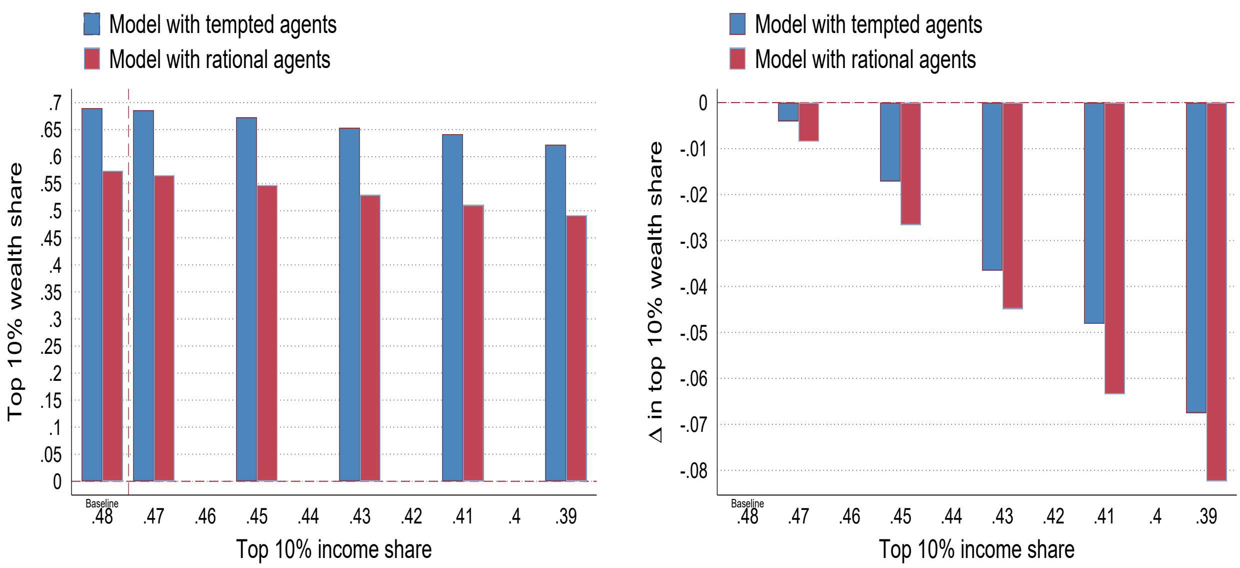
Temptation & progressive income taxation: To study the link between income and wealth inequality, I conducted several experiments in which I gradually *increase* the progressivity of income taxes in both the temptation and rational economies. In each experiment, the model is recalculated and a new market-clearing interest rate r is determined, while all other parameters remain at their baseline values.

Panel (a) of Figure 9 illustrates the impact of increasing income tax progressivity on wealth inequality in terms of *levels*, panel (b) shows the same relationship in terms of *changes* relative to baseline calibration. For each level of income tax progressivity parameter, τ_ℓ (see Section 3.1.2 for income tax schedule), the corresponding measure of income inequality (i.e., the top 10% income share) is calculated and displayed on the x-axis. Notably, reducing income inequality in the tempted framework results in a smaller reduction in wealth inequality than that predicted by the standard rational model. This is a practical consequence of the interplay between the *current resources channel* and the *future income channel,* both of which are affected by changes in income inequality.

Figure 8: Change in top 10% wealth share due to shifts in progressive taxation

(a) Progressive income taxation

(b) Progressive wealth taxation



Note: Panel (a) illustrates the level of wealth concentration, measured by the top 10% wealth share, for each level of income concentration implied by tax progressivity parameter, τ_ℓ. Panel (b) illustrates she same relationship but as a *change* relative to the baseline calibration. Overall, reducing income inequality is less effective at reducing wealth inequality in the temptation model, due to the juxtaposing impact of income taxation on the discount factor gap between income-poorer and income-richer agents.

Indeed, changing income inequality affects both the dispersion of realized post-tax income (i.e. $y_{e,j} - \mathcal{T}(y_{e,j}, \bar{y})$ in the budget constraint, Equation 9) and the dispersion of expected post-tax income (i.e. the income over which agents form expectations in the recursive agent problem, Equation 11). The former one implies, reduced inequality between 'good' and 'bad' income realizations which, through the *current resources channel,* translates into a reduced gap in effective patience between high- and low-income individuals. This force makes the temptation model more reactive to progressive income taxation, suggesting greater wealth inequality reduction. However, progressive income taxation also *increases* the discount factor of income-rich agents through the "future income channel." When the *expected* future income of high-income agents is reduced, through progressive taxation, their discount factor rises; similarly, when the expected income of low-income agents increases, their discount factor falls (recall that the *future income channel* implies a negative relationship between expected income and the discount factor)[6]. In sum, progressive income taxation, due to its impact through the *future income channel*, can

[6]For a full decomposition, showing that wealth inequality in the temptation model responds strongly to

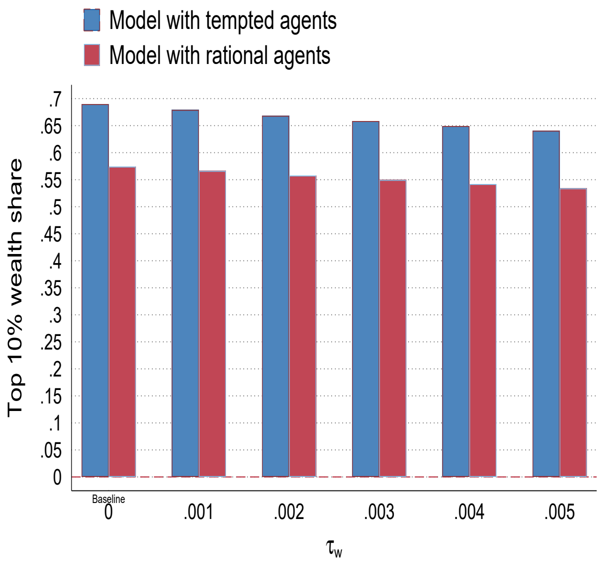
actually serve to increase the gap in effective patience between relatively richer and relatively poorer agents. As a result of this ambiguous effect, the effectiveness of progressive income taxation as a tool for reducing wealth inequality is diminished in the temptation model relative to the rational benchmark, which does not feature these behavioral dynamics.

Temptation & progressive wealth taxation: Figure 9 compares the impact of progressive wealth taxation on wealth concentration in both models, where again Panel (a) expresses this relationship in *levels* of wealth concentration; whereas Panel (b) expresses it in terms of changes in wealth concentration relative to the baseline calibration. In this experiment, wealth is taxed according to a simple progressive tax schedule: $\mathcal{T}_w(a_{e,j}) = b_j(1+r)a_{e,j} - [b_j(1+r)a_{e,j}]^{1-\tau_w}$, where τ_w denotes the progressivity of the wealth tax ($\tau_w = 0$ corresponds to the no-tax baseline). Note that the progressive wealth tax directly affects only the *current resources channel.* Even a modest reduction in wealth inequality through progressive wealth taxation narrows the gap in effective patience between wealthier and poorer individuals, which in turn amplifies the initial impact of progressive wealth tax on the wealth distribution. Unlike progressive income taxation, which has an ambiguous effect on the magnitude of the patience gap between income-rich and income-poor individuals (operating through both the *current resources channel* and the *future income channel* in opposing directions), progressive wealth taxation has a unidirectional and unambiguous impact. By acting solely through the *current resources channel,* it consistently narrows disparities in effective discount factors between wealthier and poorer agents, thereby amplifying the initial impact of the policy on the wealth distribution in the tempted model.

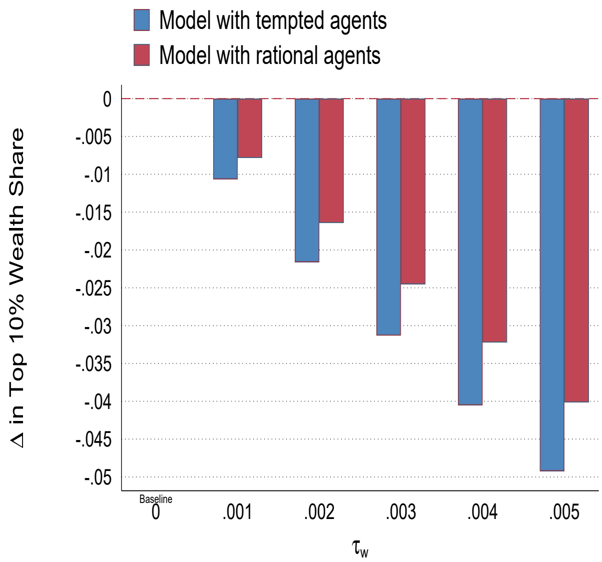
both realised and expected income inequality, yet the two effects offset each other and thereby mute the overall impact of progressive taxation, see Appendix A.8.

Figure 9: Change in top 10% wealth share due to shifts in progressive taxation

(a) Progressive income taxation



(b) Progressive wealth taxation



Note: Panel (a) illustrates the level of wealth concentration implied by a model with particular τ_w (i.e. progressivity of the wealth tax, where $\tau_w = 0$ implies no wealth tax baseline). Panel (b) depicts the same relationship in changes relative to the baseline calibration. Progressive wealth taxation directly affects the *current resources channel,* narrows the disparity in discount factors between wealthier and poorer agents, and thus its impact on wealth inequality is amplified in the temptation model.

A distributional impact of a joint change in wealth & income taxes. Table 3 reports the distributional impact of replacing the baseline progressive income tax with a flat labor-income tax ($\tau_\ell : 0.123 \to 0$) while introducing a modest progressive wealth tax ($\tau_w : 0 \to 0.006$) in both the temptation and rational frameworks. The resulting changes in wealth concentration differ markedly:

- **Temptation model.** Because wealth inequality is *less sensitive* to income taxation (progressive income taxes operate through both the *current resources* and *future resources* channels and thus have an ambiguous effect on the discount-factor gap) and *more sensitive* to wealth taxation, the net effect of the reform is a *decrease* in wealth inequality.

- **Rational model.** In the absence of these behavioral channels, the small wealth tax cannot offset the redistribution lost when the income tax is flattened, thus the same policy mix produces an *increase* in wealth inequality.

Table 3: Comparison of the policy impact across both models

	Tempted Model			**Rational Model**		
	Top 10% share	Gini	$\Sigma\tau/Y$	Top 10% share	Gini	$\Sigma\tau/Y$
B: $\tau_\ell = 0.123; \tau_w = 0$	**69.3%**	**83.7**	**0.092**	**57.5%**	**72.8**	**0.092**
S2: $\tau_\ell = 0.000; \tau_w = 0.006$	68.1%	82.1	0.147	58.7%	73.1	0.142

Note: The first row presents the baseline scenario calibrate to the U.S. economy in 2016. The second row mimics the optimal policy structure analyzed by Guvenen et al. (2023) with flat income tax rate and progressive wealth taxation. The ratio of sum of resources collected through taxes to GDP is denoted in the $\Sigma\tau/Y$ column.

5 Conclusions

Models that incorporate imperfect self-control in the form of temptation preferences have received broad experimental support and have proven capable of explaining several macroeconomic regularities that are otherwise difficult to reconcile with standard models (e.g. Toussaert, 2018; Bucciol et al., 2011; Beshears et al., 2020; Attanasio et al., 2024; Kovacs et al., 2021). In fact, the available evidence suggests that perfect and costless self-control is a rare phenomenon. Against this background, the present paper shows that accounting for imperfect self-control is not only consistent with available evidence, but also provides a structural, behaviorally grounded microfoundation for a wide range of stylized facts regarding the patience–wealth association observed in the data (Epper et al., 2020), without assuming any ex-ante differences in preferences. Consistent with the data, the temptation model predicts that such association (i) strengthens with age, (ii) persists after controlling for empirically relevant covariates such as education, income, and expected income, and (iii) when measured early in life, strongly predicts subsequent within-cohort wealth ranks.

The endogenous *discount-factor gap* between poorer and wealthier agents is a key force governing the distributional impact of temptation preferences. It allows a parsimonious OLG model with temptation preferences to generate a wealth distribution that closely matches the data (outperforming the standard model). Importantly, the positive correlation between patience and wealth that emerges in the quantitative model masks two opposing forces that shape tempted agents' discount factors. In addition to the *current-resources channel* (the discount factor increasing in current resources), there exists a *future-income channel* (the discount factor decreasing in expected future income under standard calibrations). Quantitatively, the future-income channel is modest and typically dominated by the current-resources channel in a cross-section.

However, both channels, are central to understanding the impact of taxation on the wealth distribution. Progressive income taxation reduces inequality in current resources, narrowing the discount-factor gap through the current-resources channel. At the same time, it lowers the expected income of high-income agents, which raises their discount factor through the future-income channel. The net effect is therefore ambiguous, making wealth concentration in the temptation model less responsive to income-tax progressivity than in the rational benchmark, which lacks these behavioral dynamics. By contrast, progressive wealth taxation operates solely through the current-resources channel and compresses the discount-factor gap; consequently, even a modest levy reduces wealth inequality more strongly in the temptation model than in the standard rational framework.

Taken together, my results demonstrate that incorporating self-control problems into macroeconomic models (i) improves the fit of the wealth distribution, (ii) allows the model to endogenously generate the positive correlation between patience and wealth without assuming any ex-ante differences in preferences, and (iii) offers novel insights into the mechanisms through which tax policy can affect wealth inequality in the presence of behavioral frictions.

References

Angelini, V., Bucciol, A., Wakefield, M., and Weber, G. (2020). Can temptation explain housing choices over the life cycle? *The Manchester School*, 88(2):229–261.

Attanasio, O., Kovacs, A., and Moran, P. (2024). Temptation and commitment: A model of hand-to-mouth behavior. *Journal of the European Economic Association*, page jvae016.

Attanasio, O. P., Banks, J., Meghir, C., and Weber, G. (1999). Humps and bumps in lifetime consumption. *Journal of Business & Economic Statistics*, 17(1):22–35.

Autor, D., Goldin, C., and Katz, L. F. (2020). Extending the race between education and technology. In *AEA papers and proceedings*, volume 110, pages 347–351. American Economic Association 2014 Broadway, Suite 305, Nashville, TN 37203.

Bastani, S. and Waldenström, D. (2023). Taxing the wealthy: the choice between wealth and capital income taxation. *Oxford review of economic policy*, 39(3):604–616.

Beshears, J., Choi, J. J., Harris, C., Laibson, D., Madrian, B. C., and Sakong, J. (2020). Which early withdrawal penalty attracts the most deposits to a commitment savings account? *Journal of public economics*, 183:104144.

Blanchard, O. J. (1985). Debt, deficits, and finite horizons. *Journal of political economy*, 93(2):223–247.

Bucciol, A. et al. (2013). *The roles of temptation and social security in explaining individual behavior*. SSRN.

Bucciol, A., Houser, D., and Piovesan, M. (2011). Temptation and productivity: A field experiment with children. *Journal of Economic Behavior & Organization*, 78(1-2):126–136.

Cagetti, M. and De Nardi, M. (2006). Entrepreneurship, frictions, and wealth. *Journal of political Economy*, 114(5):835–870.

Calvet, L. E., Campbell, J. Y., Gomes, F., and Sodini, P. (2021). The cross-section of household preferences. Technical report, National Bureau of Economic Research.

Carroll, C., Slacalek, J., Tokuoka, K., and White, M. N. (2017). The distribution of wealth and the marginal propensity to consume. *Quantitative Economics*, 8(3):977–1020.

De Nardi, M. (2015). Quantitative models of wealth inequality: A survey.

Deaton, A. and Paxson, C. (2000). Growth and saving among individuals and households. *Review of Economics and Statistics*, 82(2):212–225.

Epper, T., Fehr, E., Fehr-Duda, H., Kreiner, C. T., Lassen, D. D., Leth-Petersen, S., and Rasmussen, G. N. (2020). Time discounting and wealth inequality. *American Economic Review*, 110(4):1177–1205.

Falk, A., Becker, A., Dohmen, T., Enke, B., Huffman, D., and Sunde, U. (2018). Global evidence on economic preferences. *The quarterly journal of economics*, 133(4):1645–1692.

Gul, F. and Pesendorfer, W. (2001). Temptation and self-control. *Econometrica*, 69(6):1403–1435.

Gul, F. and Pesendorfer, W. (2004). Self-control, revealed preference and consumption choice. *Review of Economic Dynamics*, 7(2):243–264.

Guvenen, F., Kambourov, G., Kuruscu, B., Ocampo, S., and Chen, D. (2023). Use it or lose it: Efficiency and redistributional effects of wealth taxation. *The Quarterly Journal of Economics*, 138(2):835–894.

Hendricks, L. (2007). How important is discount rate heterogeneity for wealth inequality? *Journal of Economic Dynamics and Control*, 31(9):3042–3068.

Huang, K. X., Liu, Z., and Zhu, J. Q. (2015). Temptation and self-control: some evidence and applications. *Journal of Money, Credit and Banking*, 47(4):581–615.

Kaplan, G. and Violante, G. L. (2022). The marginal propensity to consume in heterogeneous agent models. *Annual Review of Economics*, 14(1):747–775.

Kovacs, A., Low, H., and Moran, P. (2021). Estimating temptation and commitment over the life cycle. *International Economic Review*, 62(1):101–139.

Kovacs, A. and Moran, P. (2021). Breaking the commitment device: The effect of home equity withdrawal on consumption, saving, and welfare.

Krusell, P., Kuruşçu, B., and Smith Jr, A. A. (2010). Temptation and taxation. *Econometrica*, 78(6):2063–2084.

Krusell, P. and Smith, Jr, A. A. (1998). Income and wealth heterogeneity in the macroeconomy. *Journal of political Economy*, 106(5):867–896.

Kuhn, M., Schularick, M., and Steins, U. I. (2017). Income and wealth inequality in america. *Journal of Political Economy*, 4.

Kuhn, M., Schularick, M., and Steins, U. I. (2020). Income and wealth inequality in america, 1949–2016. *Journal of Political Economy*, 128(9):3469–3519.

Kumru, C. S. and Thanopoulos, A. C. (2011). Social security reform with self-control preferences. *Journal of Public Economics*, 95(7-8):886–899.

Kumru, C. S. and Thanopoulos, A. C. (2015). Temptation-driven behavior and taxation: A quantitative analysis in a life-cycle model. *Economic Inquiry*, 53(3):1470–1490.

Makarski, K., Tyrowicz, J., and Żoch, P. (2025). Demographic transition and the rise of wealth inequality.

Nakajima, M. (2012). Rising indebtedness and temptation: A welfare analysis. *Quantitative Economics*, 3(2):257–288.

Piketty, T., Saez, E., and Zucman, G. (2023). Rethinking capital and wealth taxation. *Oxford Review of Economic Policy*, 39(3):575–591.

Saez, E. and Zucman, G. (2019). Progressive wealth taxation. *Brookings Papers on Economic Activity*, 2019(2):437–533.

Saez, E. and Zucman, G. (2022). Wealth taxation: lessons from history and recent developments. In *AEA Papers and Proceedings*, volume 112, pages 58–62. American Economic Association 2014 Broadway, Suite 305, Nashville, TN 37203.

Straub, L. (2019). Consumption, savings, and the distribution of permanent income. *Unpublished manuscript, Harvard University.*

Toussaert, S. (2018). Eliciting temptation and self-control through menu choices: A lab experiment. *Econometrica*, 86(3):859–889.

A Appendix

A.1 Derivation of relevant policy functions

Recall that rational and tempted begin their life with the initial wealth Ra_1 and receive income $\mathbf{y} = [y_1, y_{,2}, y_{,3}] \neq 0$ in each period. Each period the rational agent derives a standard, time separable instantaneous utility from consumption $u(c) = log(\widetilde{c})$. The period utility of the tempted agent is specified as follows: $u(\widehat{c}_j, a_j, y_j) = log(\widehat{c}_j) + \gamma[log(\widehat{c}_j) - log(R\widehat{a}_j + y_j)]$, where γ governs agent's degree of temptation. Here I present derivation of policy functions: a_2, c_2, c_3 needed for the purposes of propositions and lemmas presented in Section 2.

Deriving $\widetilde{a}_2, \widetilde{c}_2, \widetilde{c}_3$ for the rational agent

$$\begin{aligned}
\textbf{3rd period: } & \widetilde{V}_3(R\widetilde{a}_3, y_3) = \log(R\widetilde{a}_3 + y_3) \\
\text{subject to: } & \widetilde{c}_3 = y_3 + R\widetilde{a}_3 \\
\textbf{2nd period: } & \widetilde{V}_2(R\widetilde{a}_2, y_2, y_3) = \max_{(\widetilde{c}_2, \widetilde{a}_3)} \left\{ \log(\widetilde{c}_2) + \delta \widetilde{V}_3(R\widetilde{a}_3, y_3) \right\} \\
\text{subject to: } & \widetilde{c}_2 + \widetilde{a}_3 = y_2 + R\widetilde{a}_2 \\
\text{FOC : } & \frac{1}{\widetilde{c}_2} = \delta R \frac{1}{y_3 + R\widetilde{a}_3} \Rightarrow \widetilde{c}_2 = \frac{1}{1+\delta}\left(R\widetilde{a}_2 + y_2 + \frac{y_3}{R}\right) \\
\textbf{1st period: } & \widetilde{V}_1(Ra_1, y_1) = \left\{ \max_{(\widetilde{c}_1, \widetilde{a}_2)} \log(\widetilde{c}_1) + \delta \widetilde{V}_2(R\widetilde{a}_2, y_2, y_3) \right\} \\
\text{subject to: } & \widetilde{c}_1 + \widetilde{a}_2 = Ra_1 + y_1 \\
\text{FOC: } & \frac{1}{\widetilde{c}_1} = \delta R \left(\frac{1}{\frac{1}{1+\delta}\left(R\widetilde{a}_2 + y_2 + \frac{y_3}{R}\right)} \right)
\end{aligned}$$

which yields:

$$\widetilde{a}_2 = \frac{\delta + \delta^2}{(1+\delta+\delta^2)}(a_1 R + y_1) - \frac{1}{(1+\delta+\delta^2)}\left(\frac{y_2}{R} + \frac{y_3}{R^2}\right)$$

Deriving $\widehat{a}_2, \widehat{c}_2, \widehat{c}_3$ for the tempted agent

$$\begin{aligned}
\textbf{3rd period: } & \widehat{V}_3(R\widehat{a}_3, y_3) = \log(R\widehat{a}_3 + y_3) \\
\text{subject to: } & \widehat{c}_3 = y_3 + R\widehat{a}_3 \\
\textbf{2nd period: } & \widehat{V}_2(R\widehat{a}_2, y_2) = \max_{(\widehat{c}_2, \widehat{a}_3)} \left(\log(\widehat{c}_2) + \gamma \Big[\log(\widehat{c}_2) - \log(R\widehat{a}_2 + y_2) \Big] \right) + \delta \widehat{V}_3(R\widehat{a}_3, y_3) \\
\text{subject to: } & \widehat{c}_2 + \widehat{a}_3 = y_2 + R\widehat{a}_2 \\
\text{FOC : } & (1+\gamma)\frac{1}{\widehat{c}_2} = \delta R \frac{1}{y_3 + R\widehat{a}_3} \Rightarrow \widehat{c}_2 = \frac{1}{1+\delta}\left(R\widehat{a}_2 + y_2 + \frac{y_3}{R}\right) \\
\textbf{1st period: } & \widehat{V}_1(Ra_1, y_1) = \max_{(\widehat{c}_1, \widehat{a}_2)} \left(\log(\widehat{c}_1) + \gamma \Big[\log(\widehat{c}_1) - \log(Ra_1 + y_1) \Big] \right) + \delta \widehat{V}_2(R\widehat{a}_2, y_2) \\
\text{subject to: } & \widehat{c}_1 + \widehat{a}_2 = Ra_1 + y_1 \\
\text{FOC: } & (1+\gamma)\frac{1}{\widehat{c}_1} = \delta R \left(\frac{(1+\gamma)}{\frac{1}{\delta+1}\left(R\widehat{a}_2 + y_2 + \frac{y_3}{R}\right)} - \frac{\gamma}{R\widehat{a}_2 + y_2} \right) \qquad \text{(A.1)}
\end{aligned}$$

Label the initial, period 1 resources as $\omega_1 \equiv Ra_1 + y_1$. The first order condition A.1 yields:

$$\widehat{a}_2 = \frac{\delta+\delta^2}{2\left(1+\gamma+\delta+\delta^2\right)}\omega_1 - \frac{1}{2\left(1+\gamma+\delta+\delta^2\right)}\left[\frac{y_2\left(2+2\gamma+\delta+\delta^2\right)}{R} - \frac{y_3\left(1+\gamma\left(\delta-1\right)\right)}{R^2}\right]$$
$$+\frac{\sqrt{\left(\omega_1+\frac{y_2}{R}\right)^2\delta^2\left(1+\delta\right)^2+\left(\frac{y_3}{R^2}\right)^2\left(1+\gamma-\gamma\delta\right)^2-2\left(\omega_1+\frac{y_2}{R}\right)\frac{y_3}{R^2}\delta\left(1+\delta+\gamma\left(3+2\gamma+\delta\left(2+\delta\right)\right)\right)}}{2\left(1+\gamma+\delta+\delta^2\right)}$$

A.2 Proof of Proposition 1

Recall the Proposition 1:

The current resources channel ($\frac{\partial\delta_1(\cdot)}{\partial\omega_1}$): *The effective discount factor of the tempted agent in the first period $\delta_1\left(\widehat{a}_2, y_2, y_3\right)$ is an increasing function of current resources $\omega_1 = Ra_1 + y_1$, provided the agent is unconstrained in the next (second) period (NLC case). If the agent is liquidity constrained in the next period (LC case) the $\delta_1\left(\widehat{a}_2, y_2, y_3\right)$ remains constant in ω_1.*

The proof of Proposition 1 relies on the Lemma 1 which is proved in the Appendix A.4[7].

Lemma 1 *The level of assets saved by the tempted agent in the first period is an increasing function of the first period resources. Formally:* $\frac{\partial\widehat{a}_2(\omega_1)}{\partial\omega_1} > 0$.

Proof of Proposition 1:

$$\frac{\partial\delta_1\left(\cdot\right)}{\partial\omega_1} = \frac{\partial\left(\delta\left[1-\frac{\gamma}{1+\gamma}\frac{u'\left(R\widehat{a}_2(\omega_1)+y_2\right)}{u'\left(\widehat{c}_2(\omega_1)\right)}\right]\right)}{\partial\omega_1} = \frac{\partial\left(\delta\left[1-\frac{\gamma}{1+\gamma}\frac{\widehat{c}_2(\omega_1)}{R\widehat{a}_2(\omega_1)+y_2}\right]\right)}{\partial\omega_1} \tag{A.2}$$

Consider two cases of consumption policy function c_2 which arise due to non-borrowing constraint:

$$\widehat{c}_2\left(\omega_1\right) = \begin{cases} R\widehat{a}_2+y_2 & \text{if } \frac{1}{\left(\frac{\delta}{1+\gamma}+1\right)}\left(R\widehat{a}_2+y_2+\frac{y_3}{R}\right) \geq R\widehat{a}_2+y_2 \quad \Leftarrow \text{liquidity-constrained} \\ \frac{1}{\left(\frac{\delta}{1+\gamma}+1\right)}\left(R\widehat{a}_2+y_2+\frac{y_3}{R}\right) & \text{if } \frac{1}{\left(\frac{\delta}{1+\gamma}+1\right)}\left(R\widehat{a}_2+y_2+\frac{y_3}{R}\right) < R\widehat{a}_2+y_2 \quad \Leftarrow \text{non-constrained} \end{cases}$$

LC case: substitute $\widehat{c}_2 = R\widehat{a}_2 + y_2$ into the $\dfrac{\partial\delta_1\left(\cdot\right)}{\partial\omega}$ (Equation A.2)

$$\frac{\partial\delta_1\left(\cdot\right)}{\partial\omega_1} = \frac{\partial\left(\delta\left[1-\frac{\gamma}{1+\gamma}\frac{R\widehat{a}_2(\omega_1)+y_2}{R\widehat{a}_2(\omega_1)+y_2}\right]\right)}{\partial\omega_1} = 0 \tag{A.3}$$

NLC case: substitute: $\widehat{c}_2 = \dfrac{1}{\left(\frac{\delta}{1+\gamma}+1\right)}\left(R\widehat{a}_2+y_2+\dfrac{y_3}{R}\right)$

$$\frac{\partial\delta_1\left(\cdot\right)}{\partial\omega_1} = \frac{\partial\left(\delta\left[1-\frac{\gamma}{1+\gamma}\frac{1}{\frac{\delta}{1+\gamma}+1}\frac{\left(R\widehat{a}_2(\omega_1)+y_2+\frac{y_3}{R}\right)}{R\widehat{a}_2+y_2}\right]\right)}{\partial\omega_1} = \frac{\delta\gamma}{\delta+1+\gamma}\cdot\frac{R\frac{\partial\widehat{a}_2(\omega_1)}{\partial\omega_1}y_3}{\left(R\widehat{a}_2+y_2\right)^2} > 0 \tag{A.4}$$

[7] The proof relies on Lemma 3, formulated in Appendix A.5, which states that the second period value function of the tempted agent is strictly concave under the assumption $\delta < (1+\gamma)/\gamma$. Note that by assumption $\delta < 1$, thus the latter assumption is not restrictive.

Given that $\frac{\partial \widehat{a}_2(\omega_1)}{\partial \omega_1} > 0$ due to Lemma 1, the object analyzed in equation (A.4) is strictly positive. Thus, provided the agent is non-liqudidity constrained in the second period, the $\delta_1(\cdot)$ increases in the level of initial resources ω_1. If the agent is liquidity constrained in the second period the $\delta_1(\widehat{a}_2, y_2, y_3)$ remains constant in ω_1.[8]

■

A.3 Proof of Proposition 2

Recall the Proposition 2:

The Future Income Channel: $\frac{\partial \delta_1(\cdot)}{\partial \epsilon}$**:** *The impact of an increase of future income* $[y_2, y_3]$ *by* $[\epsilon, \rho\epsilon]$ *on the 1st period effective discount factor* $\delta_1(\cdot)$ *is characterized as follows:*

(i) for $\rho = 0$, $\frac{\partial \delta_1(\cdot)}{\partial \epsilon} > 0$*;*

(ii) for $\rho \in \left(0, \frac{R\delta}{1+\gamma}\right)$*, the sign of* $\frac{\partial \delta_1(\cdot)}{\partial \epsilon}$ *cannot be determined solely by the interest rate and preference parameters but depends on the relationship between* $\widehat{a}_2, y_2, y_3$*;*

(iii) for $\rho > \frac{R\delta}{1+\gamma}$, $\frac{\partial \delta_1(\cdot)}{\partial \epsilon} < 0$.

If the agent is liquidity constrained in the next (second) period, then $\frac{\partial \delta_1(\cdot)}{\partial \epsilon} = 0$ *for all* ρ*.*

For the proof presented in this section and the subsequent discussion, it is useful to first establish the following lemma:

Lemma 2 *Let:*

(i) $\rho > 0$*. The level of assets saved by the tempted agent in the first is a decreasing function of* ϵ*. Formally:* $\frac{\partial \widehat{a}_2(\epsilon)}{\partial \epsilon} < 0$

(ii) $\rho = 0$*. The level of assets saved by the tempted agent in the first period is a decreasing function of* ϵ*, however the decrease in* $\widehat{a}_2$ *due to the change in* ϵ *does not exceed the* $\frac{1}{R}$ *rate. Formally:* $-1 < \frac{\partial R\widehat{a}_2(\epsilon)}{\partial \epsilon} < 0$.

The proofs of (i) and (ii) are provided in Appendix A.6.

Proof of Proposition 2:

$$\frac{\partial \delta_1(\cdot)}{\partial \epsilon} = \frac{\partial\left(\delta\left[1 - \frac{\gamma}{1+\gamma}\frac{\widehat{c}_2(\epsilon)}{R\widehat{a}_2(\epsilon)+y_2+\epsilon}\right]\right)}{\partial \epsilon}$$

For brevity, define $\Xi = \widehat{a}_2 + y_2 + \epsilon + \frac{y_3}{R} + \frac{\rho\epsilon}{R}$ Consider two cases of policy function for consumption in period 2:

$$\widehat{c}_2 = \begin{cases} R\widehat{a}_2 + y_2 + \epsilon & \text{if } \frac{1+\gamma}{\delta+1+\gamma}\Xi > R\widehat{a}_2 + y_2 + \epsilon \Leftarrow \text{2nd period liquidity-constrained (LC)} \\ \frac{1+\gamma}{\delta+1+\gamma}\Xi & \text{if } \frac{1+\gamma}{\delta+1+\gamma}\Xi < R\widehat{a}_2 + y_2 + \epsilon \Leftarrow \text{2nd period non-constrained (NLC)} \end{cases}$$

[8] Intuitively $\delta_1(\widehat{a}_2, y_2, y_3)$ is constant in ω_1 if the agent is liquidity constrained in the next (second) period, because she will consume all available second-period resources anyways (therefore there exist no cost of self-control in the second period, which would impact the agent first-period decision-making).

LC case: $\frac{\partial \delta_1(\cdot)}{\partial \epsilon} = \frac{\partial\left(\delta\left[1-\frac{\gamma}{1+\gamma}\frac{R\widehat{a}_2+y_2+\epsilon}{R\widehat{a}_2+y_2+\epsilon}\right]\right)}{\partial \epsilon} = 0 \quad \blacksquare$

NLC case:

$$\widehat{c}_2 = \frac{1+\gamma}{\delta+1+\gamma}\left(R\widehat{a}_2 + y_2 + \epsilon + \frac{y_3}{R} + \frac{\rho\epsilon}{R}\right)$$

$$\frac{\partial \delta_1(\cdot)}{\partial \epsilon} = \frac{\gamma\delta}{1+\gamma+\delta}\frac{\left(\frac{\partial R\widehat{a}_2(\epsilon)}{\partial \epsilon}+1\right)\left(\frac{y_3}{R}+\frac{\rho\epsilon}{R}\right) - \frac{\rho}{R}\left(R\widehat{a}_2(\epsilon)+y_2+\epsilon\right)}{\left(R\widehat{a}_2(\epsilon)+y_2+\epsilon\right)^2} \tag{A.5}$$

If $\rho = 0$, the $\frac{\partial \delta_1(\cdot)}{\partial \epsilon}$ derived in equation (A.5) collapses to $\left(\frac{\partial R\widehat{a}_2(\epsilon)}{\partial \epsilon}+1\right)\frac{y_3}{R}$. Observe that for $\rho = 0$ Lemma 2 implies: $-1 < \frac{\partial R\widehat{a}_2(\cdot)}{\partial \epsilon} < 0$. Therefore, $\left(\frac{\partial R\widehat{a}_2(\epsilon)}{\partial \epsilon}+1\right)$ is certainly greater than 0, implying $\frac{\partial \delta_1(\cdot)}{\partial \epsilon} > 0$. Hence, if the tempted agent experiences an income increase only in the next (second) period, its effective discount factor $\delta_1(\cdot)$ will increase, which proves Case (i).

Assume $\rho \neq 0$ then $\frac{\partial \delta_1(\cdot)}{\partial \epsilon} < 0$ if the nominator of (A.5) is smaller than 0, that is:

$$\left(\frac{\partial R\widehat{a}_2(\epsilon)}{\partial \epsilon}+1\right)\left(\frac{y_3}{R}+\frac{\rho\epsilon}{R}\right) - \frac{\rho}{R}\left(R\widehat{a}_2(\epsilon)+y_2+\epsilon\right) < 0$$

Therefore, $\frac{\partial \delta_1(\cdot)}{\partial \epsilon} < 0$ whenever:

$$\rho > \frac{\left(\frac{\partial R\widehat{a}_2}{\partial \epsilon}+1\right)y_3}{\left(R\widehat{a}_2+y_2+\epsilon\right) - \epsilon\left(\frac{\partial R\widehat{a}_2}{\partial \epsilon}+1\right)} \tag{A.6}$$

Observe that in general the RHS of (A.6) grows in y_3 and falls in $R\widehat{a}_2 + y_2$. Therefore for the lower value of y_3 or the higher $R\widehat{a}_2 + y_2$, the lower ρ has to be to imply $\frac{\partial \delta_1(\cdot)}{\partial \epsilon} < 0$.

To get a bound on the minimum value of $\rho > 0$ which certainly yields a decrease in effective discount factor observe that the right hand side of equation (A.6) increases in $\left(\frac{\partial R\widehat{a}_2}{\partial \epsilon}+1\right)$ and (2) that $\frac{\partial R\widehat{a}_2}{\partial \epsilon} < 0$ due to Lemma 2. Therefore the RHS of (A.6) can be bounded with:

$$\frac{y_3}{\left(R\widehat{a}_2+y_2+\cancel{\epsilon}\right)-\cancel{\epsilon}} > \frac{\left(\frac{\partial R\widehat{a}_2}{\partial \epsilon}+1\right)y_3}{\left(R\widehat{a}_2+y_2+\epsilon\right) - \epsilon\left(\frac{\partial R\widehat{a}_2}{\partial \epsilon}+1\right)}$$

Thus, whenever $\rho > \frac{y_3}{R\widehat{a}_2+y_2}$, an increase in future income will lead to a decrease in the tempted agent's first-period effective discount factor. This lower bound on ρ, which ensures that $\frac{\partial \delta_1(\cdot)}{\partial \epsilon} < 0$, can be expressed in terms of exogenous preference parameters and the interest rate, provided that the largest possible value of y_3 allowed in the NLC case is assumed. In the non-constrained case, maximum $y_3 < R\left(\frac{\delta}{1+\gamma}\right)\left(R\widehat{a}_2+y_2+\epsilon\right) - \rho\epsilon$, therefore:

$$\frac{R\left(\frac{\delta}{1+\gamma}\right)\left(R\widehat{a}_2+y_2+\epsilon\right)-\rho\epsilon}{R\widehat{a}_2+y_2} > \frac{y_3}{R\widehat{a}_2+y_2}$$

Hence, for any permissible values of $\widehat{a}_2, y_2, y_3$ in the non-constrained case, $\frac{\partial \delta_1(\cdot)}{\partial \epsilon} < 0$ if:

$$\rho > \frac{R\left(\frac{\delta}{1+\gamma}\right)\left(R\widehat{a}_2+y_2+\epsilon\right)-\rho\epsilon}{R\widehat{a}_2+y_2}$$

$$\rho > \frac{R\delta}{1+\gamma}$$

Therefore as in the case (ii) when $\rho \in \left(0, \frac{R\delta}{1+\gamma}\right)$, the sign of $\frac{\partial \delta_1(\cdot)}{\partial \epsilon}$ cannot be determined solely by the interest rate and preferences parameters. In contrast, whenever $\rho > \frac{R\delta}{1+\gamma}$, as in case (iii) the sign of $\frac{\partial \delta_1(\cdot)}{\partial \epsilon}$ can be bounded such that: $\frac{\partial \delta_1(\cdot)}{\partial \epsilon} < 0$.
■

A.4 Proof of Lemma 1

Proof of Lemma 1
I use the FOC in the first period (derived in the full form in equation (A.1), see above) to define a function H:

$$\begin{aligned} H : &(1+\gamma)\frac{\partial u\left(Ra_1 + y_1 - \widehat{a}_2\right)}{\partial a_2} + \frac{\partial V_2\left(\widehat{a}_2, y_2, y_3\right)}{\partial a_2} \\ = &-(1+\gamma)\frac{\partial u\left(c_1\right)}{\partial c_1} + \frac{\partial V_2\left(\widehat{a}_2, y_2, y_3\right)}{\partial a_2} \end{aligned}$$

The object $\frac{\partial \widehat{a}_2}{\partial \omega_1}$ can be obtained from the implicit function theorem, that is:

$$\frac{\partial \widehat{a}_2}{\partial \widehat{a}_1} = -\frac{\frac{\partial H}{\partial \widehat{a}_1}}{\frac{\partial H}{\partial \widehat{a}_2}} = \frac{(1+\gamma)\frac{\partial^2 u(c_1)}{\partial c_1^2}}{(1+\gamma)\frac{\partial^2 u(c_1)}{\partial c_1^2} + \delta\frac{\partial^2 V_2(\widehat{a}_2, y_2, y_3)}{\partial a_2^2}} > 0 \tag{A.7}$$

The inequality A.7 holds because $\frac{\partial^2 u(c_1)}{\partial c_1^2} < 0$ due to the concavity of the utility function, $\frac{\partial^2 V_2(\widehat{a}_2, y_2, y_3)}{\partial a_2^2} < 0$ due to Lemma 3. Overall, the object defined in equation (A.7) is positive, thus $\frac{\partial \widehat{a}_2}{\partial \widehat{a}_1} > 0$.
■

A.5 Proof of Lemma 3

Lemma 3 *The value function of the tempted agent in period 2 is increasing in assets and strictly concave. Formally:*

$$\frac{\partial V_2(\cdot)}{\partial \widehat{a}_2} > 0, \quad \textit{and} \quad \frac{\partial^2 V_2(\cdot)}{\partial \widehat{a}_2^2} < 0.$$

Proof of Lemma 3.
Consider two cases of consumption policy function in period 2 for the agent with temptation preferences. These two cases arise due to the non-borrowing constraint.

$$\widehat{c}_2 = \begin{cases} R\widehat{a}_2 + y_2 & \text{if } \frac{1}{\left(\frac{\delta}{1+\gamma}+1\right)}\left(R\widehat{a}_2 + y_2 + \frac{y_3}{R}\right) \geq R\widehat{a}_2 + y_2 \quad (\Leftarrow \text{liquidity-constrained agent}) \\ \frac{1}{\left(\frac{\delta}{1+\gamma}+1\right)}\left(R\widehat{a}_2 + y_2 + \frac{y_3}{R}\right) & \text{if } \frac{1}{\left(\frac{\delta}{1+\gamma}+1\right)}\left(R\widehat{a}_2 + y_2 + \frac{y_3}{R}\right) < R\widehat{a}_2 + y_2 \quad (\Leftarrow \text{non-constrained agent}) \end{cases}$$

Note that the function for c_2 is continuous. At the maximum value of $y_3 = \delta R/(1+\gamma)\left(R\widehat{a}_2 + y_2\right)$, which implies that the agent has *just* became liquidity constrained the $\frac{1}{\left(\frac{\delta}{1+\gamma}+1\right)}\left(R\widehat{a}_2 + y_2 + \frac{y_3}{R}\right) = R\widehat{a}_2 + y_2$.

First, consider the liquidity-constrained agent in period 2:

$$\frac{\partial V_2(\cdot)}{\partial a_2} = \frac{(1+\gamma)R}{Ra_2+y_2} - \gamma\frac{R}{Ra_2+y_2} = \frac{R}{Ra_2+y_2} > 0$$
$$\frac{\partial^2 \widehat{V}_2(\cdot)}{\partial \widehat{a}_2^2} = -\frac{R^2}{\left(R\widehat{a}_2+y_2\right)^2} < 0$$

Thus, for the liquidity-constrained agent the value function is concave in assets in period 2.

For the non-constrained agent in period 2:

$$\frac{\partial \widehat{V}_2(\cdot)}{\partial a_2} = \frac{R(1+\gamma)}{c_2} - \gamma\frac{R}{a_2R+y_2} > 0 \quad \text{if}$$
$$R\left(\delta+1+\gamma\right)\left(a_2R+y_2\right) - \gamma R\left(R\widehat{a}_2+y_2+\frac{y_3}{R}\right) > 0 \tag{A.8}$$

In equation A.8, the policy function for consumption c_2 was inserted. This inequality must hold unless the agent becomes liquidity constrained (in which case, the analysis shifts to the first scenario). Substituting the maximum possible level of $y_3 = \frac{\delta R}{1+\gamma}\left(R\widehat{a}_2+y_2\right)$ into equation A.8 bounds it from below, therefore $\frac{\partial \widehat{V}_2(\cdot)}{\partial a_2} > 0$ if :

$$\frac{(\delta+1+\gamma)}{(1+\gamma)}\left(R\widehat{a}_2+y_2\right) > 0$$

which is always true, therefore the $\frac{\partial \widehat{V}_2(\cdot)}{\partial a_2} > 0$.

Consider the second derivative of the value function:

$$\frac{\partial^2 \widehat{V}_2(\cdot)}{\partial a_2^2} = -\frac{R^2(1+\gamma)}{c_2^2} + \gamma\frac{R^2}{\left(a_2R+y_2\right)^2} < 0 \quad \text{if}$$
$$\left(R^2\gamma\right)\left(R\widehat{a}_2+y_2+\frac{y_3}{R}\right)^2 - \left(R^2(1+\gamma)\left(1+\frac{\delta}{1+\gamma}\right)\right)\left(R\widehat{a}_2+y_2\right)^2 < 0.$$

Again, note that agent is non-liquidity constrained if $y_3 < \delta R/(1+\gamma)\left(R\widehat{a}_2+y_2\right)$. Plugging in the maximum possible level of y_3 which fulfills the non-constrained case bounds this expression from above. Therefore, $\frac{\partial^2 \widehat{V}_2(\cdot)}{\partial a_2^2} < 0$ if:

$$-\frac{R^2\left(\widehat{a}_2R+y_2\right)^2\left(\gamma\left(1-\delta\right)+1\right)\left(1+\gamma+\delta\right)}{(1+\gamma)^2} < 0$$

The condition $\delta < (1+\gamma)/\gamma$ is always fulfilled under the conventional assumption that $\delta < 1$. Summarizing, $V_2(\cdot)$ of the tempted agent is a concave function of a_2.

■

$(1+\gamma)\frac{\partial u(Ra_1+y_1+\epsilon-\widehat{a}_2)}{\partial a_2 \partial \epsilon}$

A.6 Proofs of Lemma 2

Recall the setup of Proposition 2. The a_1, y_1 are assumed to be constant and the future income y_2, y_3 stream is increased respectively by $\epsilon, \rho\epsilon$.

To proof both lemmas first define a function $H : (1+\gamma)\frac{\partial u(Ra_1+y_1-\widehat{a}_2)}{\partial a_2} + \delta\frac{\partial V_2(\widehat{a}_2,y_2+\epsilon,y_3+\rho\epsilon)}{\partial a_2}$. The derivative $\frac{\partial a_2}{\partial \epsilon}$ will be obtained from the implicit function theorem.

$$\frac{\partial a_2}{\partial \epsilon} = -\frac{\frac{\partial H}{\partial \epsilon}}{\frac{\partial H}{\partial a_2}} = -\frac{\delta\frac{\partial^2 V_2(\widehat{a}_2,y_2+\epsilon,y_3+\rho\epsilon)}{\partial\epsilon\partial a_2}}{(1+\gamma)\frac{\partial^2 u(Ra_1+y_1-\widehat{a}_2)}{\partial a_2\partial a_2} + \delta\frac{\partial^2 V_2(\widehat{a}_2,y_2+\epsilon,y_3+\rho\epsilon)}{\partial a_2\partial a_2}} \tag{A.9}$$

where

$$\begin{aligned}
\frac{\partial^2 V_2(a_2,y_2+\epsilon,y_3+\rho\epsilon)}{\partial\epsilon\partial a_2} &= -(1+\rho)\frac{R(1+\gamma)}{c_2^2} + \frac{R\gamma}{(a_2R+y_2+\epsilon)^2} \\
\frac{\partial^2 V_2(a_2,y_2+\epsilon,y_3+\rho\epsilon)}{\partial a_2\partial a_2} &= -\frac{R^2(1+\gamma)}{c_2^2} + \frac{R^2\gamma}{(a_2R+y_2+\epsilon)^2} \\
\frac{\partial^2 u(Ra_1+y_1-a_2)}{\partial a_2\partial a_2} &= -\left(\frac{1}{Ra_1+y_1-a_2}\right)^2
\end{aligned}$$

Proof of Lemma 2 point (i): Substituting the relevant derivatives into the equation (A.9) yields:

$$\frac{\partial a_2}{\partial \epsilon} = -\frac{\overbrace{-\frac{\delta R(1+\gamma)}{c_2^2} + \frac{\delta R\gamma}{(a_2R+y_2+\epsilon)^2}}^{\star} - \rho\frac{\delta R(1+\gamma)}{c_2^2}}{-(1+\gamma)\left(\frac{1}{Ra_1+y_1-a_2}\right)^2 \underbrace{-\frac{\delta R^2(1+\gamma)}{c_2^2} + \frac{\delta R^2\gamma}{(a_2R+y_2+\epsilon)^2}}_{\star}} \tag{A.10}$$

Note that the second derivatives of the value function marked with $\star$ have been shown in Appendix A.5 to be negative under the assumption $\delta < (1+\gamma)/\gamma$. Additionally, the term $-\rho\frac{R(1+\gamma)}{c_2^2}$ is negative by construction, thus $\frac{R\partial a_2}{\partial \epsilon} < 0$.

■

Proof of Lemma 2 point (ii): Assume $\rho = 0$. Substituting the relevant derivatives into the equation (A.9) and multiplying (A.9) by R to obtain $\frac{R\partial a_2}{\partial y_2}$ yields:

$$\frac{R\partial a_2}{\partial \epsilon} = -\frac{\overbrace{-\frac{\delta R^2(1+\gamma)}{c_2^2} + \frac{\delta R^2\gamma}{(a_2R+y_2+\epsilon)^2}}^{\star}}{-(1+\gamma)\left(\frac{1}{Ra_1+y_1-a_2}\right)^2 \underbrace{-\frac{\delta R^2(1+\gamma)}{c_2^2} + \frac{\delta R^2\gamma}{(a_2R+y_2+\epsilon)^2}}_{\star}} \tag{A.11}$$

Note that the second derivatives of the value function marked with $\star$ have been shown in Appendix A.5 to be negative under the assumption $\delta < (1+\gamma)/\gamma$. Additionally, the term $-(1+\gamma)\left(\frac{1}{Ra_1+y_1-a_2}\right)^2 < 0$ is negative by construction, which implies $\frac{R\partial a_2}{\partial \epsilon} < 0$. Furthermore, observe that the denominator of the expression defined in equation A.11 is equal to the $\star$ term minus a strictly positive value. Consequently, the denominator is always more negative than the numerator, leading to the result $-1 < \frac{R\partial a_2}{\partial y_2} < 0 \Rightarrow -\frac{1}{R} < \frac{\partial a_2}{\partial y_2} < 0$.

■

A.7 Replicating stylized facts from Epper et al. (2020)

Temptation preferences can replicate two additional facts regarding the relationship between patience and wealth, not discussed in detail in Section 4.3.

First, the model-implied association persists after adjusting for covariates inspired by the empirical analysis. To replicate Epper et al. (2020) as closely as possible in the context of the structural model output, I control for age, education, income, and expected income—that is, all applicable covariates in this stylized setting[9]. Table 4 reports the estimated coefficient relating patience to wealth (individual wealth is expressed relative to the median wealth). The estimated coefficient of 1.29 implies that moving from the lowest to the highest patience is associated, on average, with an increase in wealth equal to 1.29 times the median wealth. Epper et al. (2020) estimate a model with a similar set of controls in their sample and find that the corresponding increase is equal to 0.32 times the median wealth. Therefore, the endogenous relationship implied by the temptation model is robust to the inclusion of empirically motivated controls, with the magnitude of the effect being somewhat larger than the empirical one.

Table 4: OLS regression: relationship between patience & wealth

	Dependent variable: $\frac{\text{wealth}_i}{\textbf{median wealth}}$
Patience	1.29***
	(.055)
Age FE	Yes
Education FE	Yes
Income Deciles FE	Yes
Expected income	Yes

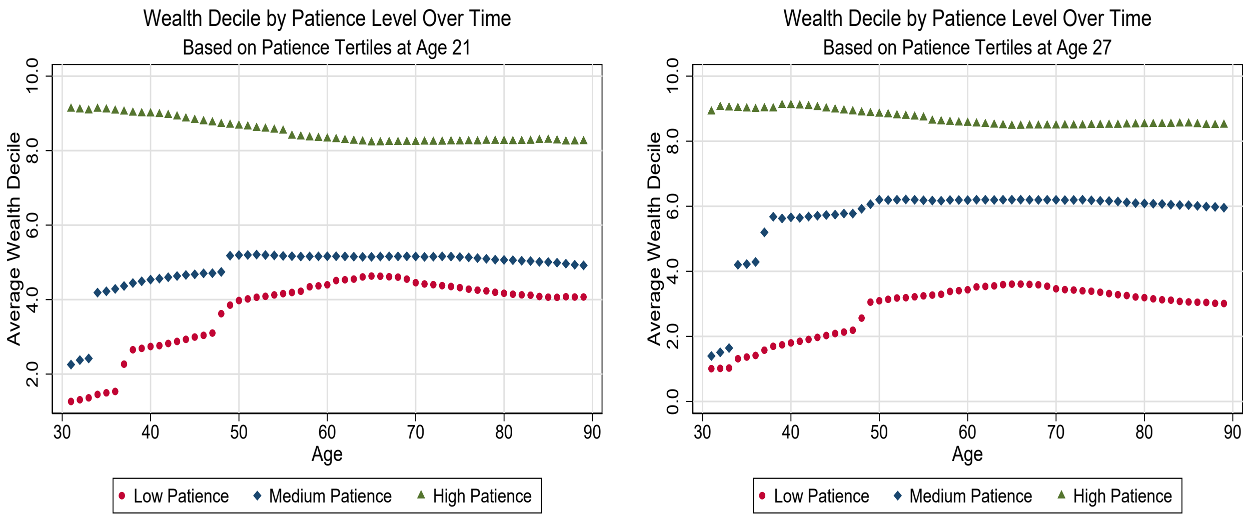
Note: The dependent variable is constructed as individual wealth divided by median wealth. The coefficient of 1.29 implies that moving from the lowest to the highest patience is associated, on average, with an increase in wealth equal to 1.29 times the median wealth. In line with the empirical analysis, only agents younger than 53 are included. Income is coded using within-cohort income deciles which are included as fixed effects. Education fixed effects in the model correspond to an indicator for educational type $e = h$. Due to strong collinearity between income decile and expected-income decile in the model, expected income is included as a linear control.

Second, temptation mode implies that effective patience measured as early as in period $j = 2$ is highly predictive of an agent's within-cohort wealth rank in all subsequent periods, replicating the lifetime impact of time discounting on wealth rank observed in Epper et al. (2020). In their retrospective analysis retrospective Epper et al. (2020), used simple survey question administered to young individuals to proxy their time discounting in 1973. They used this crude measure to examine the association between time discounting in 1973 and individual positions within the wealth distribution during the 2001-2015 period. Following a similar approach, I simulate 15,000 agents to measure their effective discount factors at $j = 2$ or $j = 8$, group them into three effective patience quantiles, and track their wealth rank over the life cycle. Figure 10 presents the results of this simulation. Interestingly, though consistent with the empirical findings of Epper et al. (2020), effective patience measured early in life consistently predicts the average wealth rank of agents in the future.

[9] Consider this output to be analog of Table 2; Column 3; Panel B in Epper et al. (2020)

Figure 10: Replicating the lifetime impact of time discounting on wealth rank observed in Epper et al. (2020)

(a) Patience measured at $j = 2$ (b) Patience measured at $j = 8$



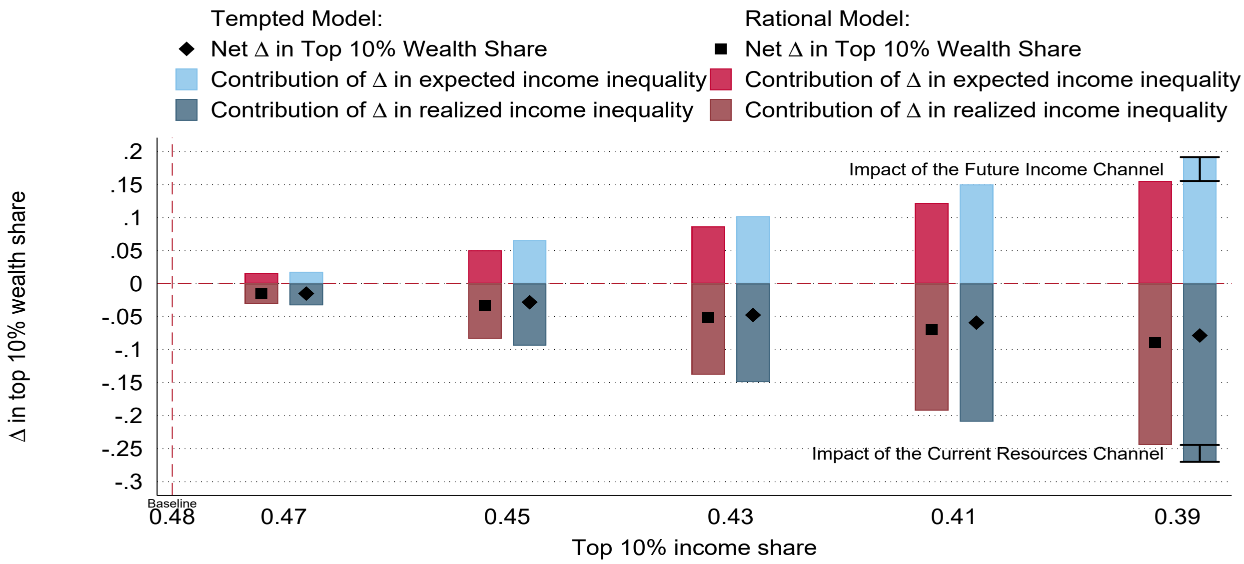
Note: The lives of 20,000 agents who enter the model with no initial assets ($a_{e,1} = 0$) are simulated based on the exogenous productivity process (Equation 7). Agents' consumption-saving choices follow the policy functions defined in Section 3.1.6. For each agent, the effective discount factor $\delta_{e,j}$ (Equation 13) is measured at $j = 2$ (left figure) or $j = 7$ (right figure). Agents are subsequently grouped into one of three patience tertiles based on the $\delta_{e,j}$ values. In each subsequent period $j \in (10, 70)$, agents are assigned to wealth deciles based on the level of assets saved for the next period (wealth is measured at the end of the period).

A.8 Decomposing the impact of Δ in current and future income inequality

Changing income inequality, through progressive income taxation, affects both the dispersion of realized post-tax income (i.e. $y_{e,j} - \mathcal{T}(y_{e,j}, \bar{y})$ in the budget constraint, Equation 9) and the dispersion of expected post-tax income (i.e. the income over which agents form expectations in the recursive agent problem, Equation 11). Figure 11 presents a decomposition of the overall (net) effect of reducing income inequality on wealth inequality into contributions that result separately from a reduction in realized income inequality and a reduction in expected income inequality. All changes are expressed relative to the baseline calibration, which yields the top 10% income share equal to 0.48. The wealth inequality in alternative scenarios with greater progressivity of labor income taxes (thus lower income inequality as expressed by the top 10% of income share) are presented on the right hand side of the figure.

Note that in both the rational and temptation models, a decrease in *realized* income inequality alone (holding expected income inequality fixed) leads to a large reduction in wealth inequality, due to narrowing of the gap in available resources between relatively income-poorer and income-richer agents (darker red and blue bars). In contrast, reducing *expected* income inequality narrows the gap in anticipated income between income groups. Given persistence in the income process, richer agents come to expect relatively lower levels of future income, which increases their incentive to save, while poorer agents expect relatively higher future incomes,

Figure 11: Change in top 10% wealth share shifts in expected or realized income inequality



Note: This figure illustrates the relationship between changes in wealth inequality and (i) changes in *expected* post-tax income inequality (i.e., the income over which agents form expectations in the recursive agent problem, Equation 11) or (ii) changes in *realized* post-tax income inequality (i.e., $y_{e,j} - \mathcal{T}(y_{e,j}, \bar{y})$ in the budget constraint, Equation 9). Changes in wealth inequality are calculated relative to a baseline scenario in which expected and realized income processes coincide and generate a top 10% income share of 0.48. Changes in either expected, realized, or joint income inequality are achieved through adjustments in progressive income taxation. The lighter red and blue bars denote the impact of a decrease in the dispersion of expected income on wealth inequality, for the rational and temptation models respectively. The darker red and blue bars denote the impact of a decrease in the dispersion of realized income on wealth inequality, holding expected income inequality fixed at its baseline level. The diamonds and squares represent the net change in wealth inequality resulting from the combined impact of changes in expected and realized income inequality. Note that the effects of changes in either expected or realized income inequality on wealth inequality are amplified in the temptation model, due to the reinforcing roles of the *current resources channel* (which reduces the effective patience gap between income-rich and income-poor agents following a reduction in realized income dispersion) and the *future income channel* (which increases the effective patience gap between income-rich and income-poor agents following a reduction in expected income dispersion). This ambiguous effect of reduced income inequality on the patience gap implies that the overall net reduction in wealth inequality is smaller than in the standard rational model.

thus reducing their incentive to save [10]. Therefore, a decrease in expected income inequality results in an *increase* in wealth inequality (light red and blue bars), by strengthening the saving incentives of relatively richer agents and weakening those of relatively poorer agents.

Note that although the directional impact of these mechanisms is similar in both the rational and temptation models, the magnitude of the effects of changes in realized and expected income is greater in the temptation model (as illustrated by the relative size of the blue and red bars in

[10] As an analog one can consider a case of greater taxes progressivity which would be introdcued next-year. Relatively richer agents would expect a decrease in their labor income which would increase their incentives to save in this period. Relaively poorer agents would expect an elevated nex-year income which wouyld decrease their incentives to save. The (expected) reduction of inequality in the next-year income would therefore raise wealth inequality. This logic is consistent withthe impact of expected income inequality reduction on the wealth inequality in both models

Figure 11). This amplification arises because decreased inequality in both realized and expected income triggers the *current resources channel* and the *future income channel,* thereby affecting tempted agents' effective discount factors.

First, the reduced inequality between 'good' and 'bad' income realizations, through the *current resources channel,* translates into a reduced gap in effective patience between high- and low-income individuals. This force makes the temptation model more reactive to progressive income taxation, suggesting greater wealth inequality reduction. However, progressive income taxation also *increases* the discount factor of income-rich agents through the "future income channel." Specifically, when the *expected* future income of high-income agents is reduced, through progressive taxation, their discount factor rises; similarly, when the expected income of low-income agents increases, their discount factor falls (recall that the *future income channel* under standard assumptions implies a negative relationship between expected income and the discount factor). This mechanism reinforces the negative link between expected income inequality and wealth inequality already present in the rational model, making wealth inequality in the temptation model more sensitive to changes in expected income inequality (as illustrated by the relative size of the light blue bars compared to the light red bars in Figure 11). In sum, progressive income taxation, by simultaneously impacting both channels, has an ambiguous effect on the gap in effective patience between relatively richer and relatively poorer agents. As a result, the effectiveness of progressive income taxation as a tool for reducing wealth inequality is diminished in the temptation model relative to the rational benchmark, which does not feature these behavioral dynamics.