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Productivity-Rent Effects in Intergenerational Transfers: Education versus Bequests

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Abstract

This paper studies optimal intergenerational transfers when altruistic parents can transfer resources to their children through financial bequests or through investment in human capital. The key distinction is that education is a productive transfer: it changes the mapping from privately observed ability to output, whereas bequests are budgetary transfers. The paper characterizes how this distinction affects information rents in a dynastic Mirrlees environment.

The main result is a decomposition of the informational effect of education relative to bequests. With endogenous labor supply, both instruments affect incentive provision through marginal-utility and labor-supply responses. Education, however, generates an additional productivity-rent component governed by the cross-partial between ability and human capital in production. This component is absent for purely budgetary transfers. When ability and human capital are sufficiently complementary, the productivity-rent component can dominate the standard labor-requirement channel, so that education may be optimally distorted downward relative to the bequest margin.

The analysis clarifies why education and bequests are not equivalent instruments of intergenerational redistribution. The difference is not only that education has risky returns, but also that it changes the sensitivity of output to privately observed ability. This distinction provides a force that can work against the standard education-subsidy logic in Mirrlees models.

Keywords:
optimal taxation; intergenerational transfers; human capital; financial bequests; private information; education wedge; ability risk.

JEL Classification:
D64, D82, H21, H24, I26, J24

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1 Introduction

How should resources be transferred across generations when parents can use both financial bequests and investments in human capital? A large literature studies parental investment, human-capital formation, and intergenerational transfers, but these instruments are often treated as alternative ways of moving resources from parents to children. This paper argues that this equivalence breaks down once ability risk and private information are taken seriously. Education and financial bequests are not mechanism-equivalent intergenerational transfer instruments.

The reason is simple but consequential. A financial bequest transfers resources without changing the child's productivity. Educational investment, by contrast, changes the mapping from child ability to output. This technological distinction implies that the two instruments interact differently with uncertainty and incentive constraints. Education is chosen before child ability is realized, so its return is risky from the ex ante perspective. Moreover, when child ability is privately observed, education changes the labor requirement and the slope of information rents. Financial bequests may affect incentives indirectly through wealth effects and labor-supply responses, but they do not generate the same direct productivity-based informational channel.

The paper develops this argument in a dynastic Mirrlees environment. Altruistic parents can transfer resources to their children through financial bequests or through educational investment. Children differ in ability, and ability affects the return to human capital. The model allows us to compare education with a purely budgetary transfer margin within the same planner's problem. This comparison is essential: the relevant question is not only whether education is distorted, but whether education is distorted differently from financial transfers.

The central formal result is the endogenous-labor decomposition. Once labor supply responds to transfers, bequests are no longer informationally neutral: they affect information rents through marginal-utility and labor-supply channels. However, education remains distinct because it changes the productivity derivative with respect to ability. The resulting productivity-rent component, governed by $Y_{\theta h}$, is present for education and absent for bequests. This is the sense in which the asymmetry between education and bequests survives endogenous behavioral responses.

The *first contribution* is to establish a direct informational asymmetry between the two instruments. Under private information, educational investment enters the incentive problem because it changes the labor required to implement a given output assignment. Productive education reduces the labor required to implement assigned output levels, thereby changing the attractiveness of mimicking across types. Financial bequests do not enter this labor requirement mapping. As a result, education generates a direct productivity-based informational term that has no counterpart for purely budgetary transfers. As a result, education generates a direct productivity-based informational term that has no counterpart for financial bequests. More generally, the mechanism applies to budgetary transfer instruments that do not alter the production technology: such instruments may affect allocations through resources, but they do not generate the direct productivity-rent component created by education.

The *second contribution* is to decompose the education wedge into three components. The first is a *risk component*: education is chosen before ability is realized, whereas its payoff depends on the realized type. The average risk wedge equals minus the covariance between marginal utility and the return to education, normalized by expected marginal utility. Under standard monotone allocations, higher-ability children both receive higher consumption and have higher returns to education, so this covariance is negative and the average risk wedge is positive. The second is a *technological component*: human capital is produced through a costly investment technology, while financial wealth is transferred through the intertemporal budget constraint. The third is an *informational component*: education affects incentive compatibility because it changes the productivity schedule. This decomposition shows that the education wedge is not a single distortion. It is the sum of distinct economic forces that operate through different margins and apply asymmetrically to education and financial transfers.

The *third contribution* is to show that the sign of the informational wedge is not predetermined. In the baseline labor-requirement representation, productive education reduces the labor required to implement assigned outputs and relaxes one dimension of incentive provision. This corresponds to a non-positive informational wedge, consistent with the standard logic that education should be subsidized. With endogenous labor supply, however, education also changes the slope of information rents through a productivity-rent component governed by $Y_{\theta' h'}$, the cross-partial of output in ability and human capital. When human capital and ability are complements in production, $Y_{\theta' h'} > 0$, this component is positive and may dominate the baseline effect. In that case, the total informational wedge becomes positive: the planner requires a higher expected marginal return to education than under full information, and if this return is decreasing in educational investment, the private-information allocation features less education than the full-information benchmark. Optimal policy may then tax rather than subsidize education relative to the financial bequest margin. This provides a conditional reversal of the standard Bovenberg–Jacobs subsidy logic and arises precisely because financial bequests provide a purely budgetary alternative transfer margin against which the informational cost of education can be measured.

Related literature. This paper contributes to several strands of the literature.

First, it relates to dynastic models of intergenerational transfers and parental investment, starting with Becker and Tomes (1979, 1986) and Loury (1981), and to models in which the composition of intergenerational transfers matters for inequality and mobility, such as Galor and Zeira (1993). In these models, education and bequests are natural instruments through which parents shape children's outcomes. We show that these instruments cease to be equivalent once ability risk and private information are introduced: education changes the production and incentive structure, while financial bequests do not directly alter the production technology.

Second, the paper builds on the optimal taxation literature with human capital. Bovenberg and Jacobs (2005) establish that education subsidies and income taxation are complementary in a static Mirrlees model, and Jacobs and Bovenberg (2010) emphasize the asymmetry between human and financial capital. Stantcheva (2017) studies optimal human capital policy over the life cycle, while Koeniger and Prat (2018) and Kapička and Neira (2019) characterize human-capital wedges in dynamic and dynastic environments. Existing work shows that human capital can interact with incentive provision. The present paper shows that this interaction is instrument-specific: relative to financial bequests, education generates a productivity-based informational wedge that has no counterpart for purely budgetary transfers.

The closest antecedent is Stantcheva (2015), who studies optimal income, education, and bequest taxation jointly in a dynastic Barro–Becker framework. Our contribution relative to her analysis is threefold. First, we work in a nonlinear Mirrlees environment with explicit incentive compatibility constraints. Second, we decompose the wedge between education and bequests into risk, technological, and informational components. Third, we identify a productivity-rent channel, governed by $Y_{\theta' h'}$, that can conditionally reverse the standard education-subsidy logic when human capital and ability are complements. A particularly close contribution is Koeniger and Prat (2018), who study human capital and redistribution in a dynastic Mirrlees economy with intergenerational transfers and privately observed ability. Their analysis shows that human-capital investment and financial bequests generally face different wedges, and that the sign of the human-capital wedge depends on risk and incentive effects. Relative to this work, the present paper focuses on a narrower analytical question: which part of the education wedge is specific to education as a productive transfer, rather than common to budgetary intergenerational transfers. The paper separates the informational effect of education into components that are also present for bequests—the marginal-utility and labor-supply channels—and a direct productivity-rent component governed by $Y_{\theta h}$, which is absent for purely budgetary transfers. This decomposition clarifies the mechanism through which complementarity between ability and human capital can work against the standard education-subsidy logic.

Third, the paper relates to the literature on risky human capital investment, including Levhari and Weiss (1974), Eaton and Rosen (1980), and Krebs (2003). This literature studies how uncertainty about future earnings and human-capital returns affects investment, taxation, and growth. We embed this risk in a dynastic environment in which education is chosen before child ability is realized and compare it with financial bequests as a risk-free transfer margin. This generates a gap between ex ante and ex post marginal returns to education, characterized by a covariance between marginal utility and the marginal return to human capital.

Finally, the paper connects to optimal bequest taxation and to empirical work on within-family information frictions. Farhi and Werning (2010) and Piketty and Saez (2013) study the optimal taxation of financial bequests; we complement this literature by comparing bequests with education as an alternative intergenerational transfer instrument. Empirical evidence by Bursztyn and Coffman (2012) and Bergman (2021) suggests that education choices within families are shaped by informational frictions. Our framework provides a theoretical mechanism for such patterns: education, unlike financial bequests, directly affects incentive constraints through the production technology.

Organisation. The remainder of the paper is organised as follows. Section 2 presents the dynastic environment and the full-information benchmark. Section 3 studies ability risk and derives the wedge between ex ante and ex post returns to education. Section 4 introduces private information and establishes the direct informational asymmetry between education and financial bequests. Section 5 decomposes the education wedge into risk, technological, and informational components and presents the main decomposition theorem. Section 6 extends the model to endogenous labor supply and derives the productivity-rent channel that can reverse the sign of the informational wedge. Section 7 discusses implications and comparative statics. Section 8 concludes.

2 Model

We begin with a benchmark environment without informational frictions or uncertainty. In this setting, the parent allocates resources between own consumption, financial transfers, and investment in the child's human capital. Ability is assumed to be deterministic and commonly observed in the benchmark analysis.

2.1 Environment

Consider a dynastic household consisting of a parent and a child. The parent derives utility from own consumption and labor supply and cares about the child's welfare through altruistic preferences:

$$U^P = u(c) - v(l) + \beta \left[u(c') - v(l') \right], \tag{1}$$

where u is increasing and concave, v is increasing and convex, and $\beta \in (0, 1)$ measures parental altruism.

Parents can transfer resources to their children through two instruments: financial bequests b' and investment in the child's human capital h'. The intergenerational budget constraints are

$$c + b' + g(h', h) = (1 + r)b + Y(l, h, \theta), \tag{2}$$

$$c' = (1 + r)b' + Y(l', h', \theta'). \tag{3}$$

Here b denotes inherited wealth, r is the interest rate, h is the parent's stock of human capital, and θ and θ' denote parental and child ability, respectively. More generally, child ability may depend on parental ability through a joint distribution over (θ, θ').

In the benchmark analysis, ability is treated as deterministic and commonly observed. Subsequent sections introduce uncertainty and intergenerational dependence in abilities. The education cost function $g(h', h)$ captures the resources required to attain human capital h' given parental background h, with $g_{h'}(h', h) > 0$.

Output is produced according to a function $Y(l, h, \theta)$, which is increasing in labor l, human capital h, and ability θ, and concave in (l, h). In addition, we assume that the marginal product of human capital, $Y_h(l, h, \theta)$, is decreasing in h, and that the education cost function $g(h', h)$ is convex in h'. For concreteness, one may consider a Cobb–Douglas specification $Y(l, h, \theta) = \theta^{\xi} h^{1-\xi} l$ with $\xi \in (0, 1)$, although the main results do not depend on this functional form.

The parent chooses (c, l, b', h') taking into account the child's future behavior.

2.2 Efficient Resource Benchmark

To evaluate whether parental education choices are aligned with the child's interests, it is useful to characterize the allocation that the child would choose if she could directly allocate intergenerational resources between financial transfers and education.

An additional unit of human capital requires $g_{h'}(h', h)$ units of resources, while one unit of financial wealth yields a gross return of $1 + r$. The relevant comparison is therefore the return obtained from reallocating one unit of resources from financial wealth to education.

Lemma 1 (Resource benchmark)**.** *Holding the child's labor supply fixed, education and financial transfers are locally equivalent if and only if*

$$\frac{\partial Y(l', h', \theta')}{\partial h'} = (1 + r) g_{h'}(h', h). \tag{4}$$

Proof. See Appendix. □

The left-hand side is the marginal output gain from higher human capital, while the right-hand side is the corresponding opportunity cost in terms of foregone financial wealth. The condition therefore defines the efficient resource benchmark: education is chosen so that its marginal productivity gain equals its marginal resource cost evaluated at the return on financial assets.

2.3 Intergenerational Allocation under Full Information

We now compare the parent's optimal education choice with the child's benchmark allocation.

In standard dynastic models following Becker and Tomes (1979, 1986), education and bequests are often treated as interchangeable instruments for transferring resources across generations. In such environments, parents can replicate any desired allocation through financial transfers alone.

In the present framework, the two instruments play distinct roles. Financial bequests are pure transfers that shift resources across generations, whereas education changes the child's productivity by increasing the marginal return to labor. Despite this technological difference, in the absence of uncertainty and informational frictions both parents and children evaluate education using the same marginal resource trade-off. The efficient allocation therefore satisfies

$$\frac{\partial Y(l', h', \theta')}{\partial h'} = (1 + r)g_{h'}(h', h). \tag{5}$$

Proposition 1 (No conflict under full information)**.** *Suppose ability and labor supply are commonly observed and there is no uncertainty about future productivity. Then the parent's optimal choice of education coincides with the child's resource benchmark.*

Proof. See Appendix. □

Because parents are altruistic and internalize the child's welfare, their education choice reflects the same marginal trade-off between education and financial transfers as in the child's benchmark. As a result, there is no intrinsic conflict between parents and children in this environment.

Any departure from this benchmark must therefore be driven by additional frictions, which we introduce in the next sections.

Remark (Endogenous labor supply). This benchmark characterizes the efficient intergenerational allocation in the absence of uncertainty and informational frictions. It is a resource-based allocation criterion: it compares the marginal output gain from education to the marginal resource cost of attaining it, evaluated at the return on financial wealth.

If labor supply is endogenous, both instruments affect labor supply, but through fundamentally different channels. Financial bequests operate through a standard wealth effect, whereas education alters labor supply by changing the marginal productivity of labor. Through these channels, both instruments indirectly affect the realized return to education and therefore interact with incentive constraints through their effect on labor supply.

However, this interaction is qualitatively asymmetric. Financial bequests affect incentives only indirectly through their impact on labor supply, whereas education additionally affects incentives directly through its impact on the production technology. As a result, financial bequests are no longer informationally neutral once labor supply is endogenous, but they do not generate a direct informational wedge. By contrast, education continues to create a direct productivity-based informational distortion. Section 6 formalizes this distinction.

3 Ability Risk

We introduce ability risk while maintaining the continuum formulation of child ability $\theta' \in [\underline{\theta}, \overline{\theta}]$ and abstracting from informational frictions. Educational investment is chosen before ability is realized, so that the return to education is uncertain at the time of investment. As a result, education is evaluated based on expected marginal returns.

Throughout this section, we use a resource-based benchmark that compares the marginal productivity gain from education to its marginal resource cost. In Section 5, we introduce an alternative return-based benchmark that will be useful for decomposing the education wedge.

In contrast, financial bequests provide a budgetary transfer across generations and yield the risk-free intertemporal return. This asymmetry implies that the two instruments are no longer equivalent for transferring resources: while financial wealth delivers a return that does not depend on the child's realized ability, the return to education depends on the realized productivity of the child.

Consequently, ability risk generates a wedge between ex ante expected and ex post realized marginal returns to education, evaluated relative to its marginal resource cost in terms of financial wealth.

3.1 Independent Ability

Let θ denote parental ability and θ' the child's ability. We assume that child ability is independently drawn from a fixed distribution:

$$\theta' \sim F(\theta').$$

Parents choose human capital investment h' before the realization of θ'. Because the return to education depends on realized ability, education is a risky investment, whereas financial bequests yield a risk-free intertemporal return $(1+r)$ that does not depend on the child's type.

The parent's optimality condition for education is therefore based on expected marginal returns:

$$\mathbb{E}\left[u'(c'(\theta'))Y_{h'}\big(\ell(\theta';h'),h',\theta'\big)\right] = (1+r)\, g_{h'}(h',h)\, \mathbb{E}[u'(c'(\theta'))]. \tag{6}$$

Dividing by $\mathbb{E}[u'(c'(\theta'))]$ yields

$$\overline{Y}_{h'} = (1+r)\, g_{h'}(h',h), \tag{7}$$

where

$$\overline{Y}_{h'} = \frac{\mathbb{E}\left[u'(c'(\theta'))Y_{h'}\big(\ell(\theta';h'),h',\theta'\big)\right]}{\mathbb{E}[u'(c'(\theta'))]}$$

denotes the expected marginal return to education, weighted by marginal utility.

Economically, this object captures how parents evaluate education before the child's ability is realized. In contrast to financial bequests, which deliver a risk-free intertemporal return $(1+r)$, the return to education depends on realized productivity and is therefore uncertain.

3.2 Ex Post Evaluation

Once ability is realized, the marginal return to education can be evaluated ex post. Conditional on type θ', the relevant benchmark compares the realized marginal return to education with its marginal resource cost:

$$Y_{h'}\big(\ell(\theta';h'),h',\theta'\big) = (1+r)\, g_{h'}(h',h). \tag{8}$$

This equality should be read as an ex post benchmark condition, not as an actual choice condition. Because the marginal product of human capital depends on realized ability, the ex post valuation of education differs across children.

Proposition 2 (Risk wedge under ability uncertainty)**.** *The parent's optimal choice satisfies*

$$\overline{Y}_{h'} = (1+r)\, g_{h'}(h', h).$$

Ex post, the realized marginal return differs from its ex ante expected value, so that the wedge

$$Y_{h'}\big(\ell(\theta'; h'), h', \theta'\big) - (1+r)\, g_{h'}(h', h)$$

depends on realized ability θ'.

The proof is provided in Appendix B.

For notational simplicity, write

$$Y_{h'}(\theta') \equiv Y_{h'}\big(\ell(\theta'; h'), h', \theta'\big).$$

Taking expectations across types, the average risk wedge admits a covariance representation:

$$\mathbb{E}\big[Y_{h'}(\theta') - \overline{Y}_{h'}\big] = -\frac{\mathrm{Cov}(u'(c'(\theta')), Y_{h'}(\theta'))}{\mathbb{E}[u'(c'(\theta'))]}.$$

Thus, when higher-return states are associated with lower marginal utility, the average risk wedge is positive. The derivation is provided in Appendix B.

Ability risk therefore generates disagreement between ex ante and ex post valuations of education even in the absence of informational frictions. This disagreement arises because education is chosen based on expected marginal returns, while its payoff is realized ex post.

Under independent ability, there is no systematic bias in educational investment across dynasties. Parents choose education to be optimal in expectation, but ex post valuations differ across realizations of ability, so that the wedge is purely idiosyncratic.

Importantly, this risk-based wedge arises only for education. Financial bequests yield the risk-free intertemporal return and are therefore not subject to this return-risk distortion. This asymmetry will play a central role in the analysis of optimal policy under informational frictions.

3.3 Intergenerational Ability Persistence

We now allow ability to be correlated across generations. Let ability evolve according to

$$\theta' \sim F(\theta' \mid \theta),$$

where higher parental ability shifts the distribution of θ' toward higher values.

In this environment, parental ability becomes informative about the expected return to education. Since education affects the child's productivity, parents condition their investment decision on θ, which determines the distribution of future returns.

The parent's first-order condition for education becomes

$$\mathbb{E}\left[u'(c'(\theta'))Y_{h'}\big(\ell(\theta';h'),h',\theta'\big) \;\middle|\; \theta\right] = (1+r)\,g_{h'}(h',h)\,\mathbb{E}[u'(c'(\theta')) \mid \theta]. \quad (9)$$

Dividing by $\mathbb{E}[u'(c'(\theta')) \mid \theta]$ yields

$$\overline{Y}_{h'}(\theta) = (1+r)\,g_{h'}(h',h),$$

where

$$\overline{Y}_{h'}(\theta) = \frac{\mathbb{E}\left[u'(c'(\theta'))Y_{h'}\big(\ell(\theta';h'),h',\theta'\big) \;\middle|\; \theta\right]}{\mathbb{E}[u'(c'(\theta')) \mid \theta]}$$

denotes the expected marginal return to education conditional on parental ability.

Because the conditional distribution $F(\theta' \mid \theta)$ depends on θ, expected returns vary systematically across dynasties.

Proposition 3 (Ability-dependent investment)**.** *Suppose that the marginal-utility-weighted expected return to education $\overline{Y}_{h'}(\theta, h')$ is increasing in parental ability θ, and that $\overline{Y}_{h'}(\theta, h')$ is decreasing in h'. If $g_{h'}(h', h)$ is weakly increasing in h', then higher-ability parents choose weakly higher levels of human capital investment.*

A sufficient condition for the first assumption is that $F(\theta' \mid \theta)$ shifts to the right with θ, $Y_{h'}(l', h', \theta')$ is increasing in θ', and the induced marginal-utility weights do not overturn this monotonicity.

The proof is provided in Appendix B.

Ability persistence therefore generates systematic heterogeneity in educational investment across dynasties. Because education affects productivity, parental information about ability directly shifts the expected return to education.

Importantly, the resulting wedge remains purely driven by uncertainty. It reflects the fact that education is chosen based on expected returns, while its realized payoff depends on ex post ability. In contrast to the informational wedge studied later, it does not arise from incentive considerations.

In contrast, financial bequests yield the risk-free intertemporal return $(1+r)$, which does not depend directly on the child's ability. As a result, they do not respond to parental information about future productivity through this return channel.

3.4 Independent versus Correlated Ability

The comparison between independent and correlated ability highlights how expectations shape the education wedge.

When ability is independent across generations, all parents face the same distribution of future productivity. As a result, they choose education based on a common expected marginal return, and the wedge between ex ante and ex post returns is purely idiosyncratic.

When ability is correlated across generations, parental ability becomes informative about the distribution of the child's productivity. In this case, expected returns vary across dynasties, leading to systematic differences in educational investment.

In both environments, the underlying mechanism is the same: education is chosen based on expected returns, while its payoff is realized ex post. However, ability persistence transforms the wedge from idiosyncratic to systematic. Importantly, this mechanism operates only through education. Because financial bequests yield the risk-free intertemporal return, they are not exposed to the same return-risk channel.

Ability risk therefore introduces a distinct source of distortion in intergenerational allocation, arising from the gap between ex ante expected returns and ex post realized productivity. This risk-based wedge will interact with informational frictions in the next section, where we show that education is subject to an additional, qualitatively distinct distortion arising from incentive provision.

4 Private Information and Instrument-Specific Incentive Effects

This section introduces private information about child ability and derives the main asymmetry between intergenerational transfer instruments. The key distinction is not that education and bequests have different returns, but that they enter the incentive problem differently. Education changes the productivity schedule and therefore the labor cost of mimicking across types. Financial bequests shift resources but do not alter this mapping.

We show that this difference generates a direct informational effect for education and no analogous direct effect for financial bequests. This is the central mechanism of the paper.

4.1 Environment

We consider a dynastic economy with a continuum of child ability types $\theta' \in [\underline{\theta}, \overline{\theta}]$, distributed with density f. The parent chooses consumption c, labor supply l, a financial bequest b', and educational investment h', as well as type-contingent allocations $\{c'(\theta'), y'(\theta')\}_{\theta'}$.

Preferences are given by

$$U^P = u(c) - v(l) + \beta \int_{\underline{\theta}}^{\overline{\theta}} \Big[u\big(c'(\theta')\big) - v\big(\ell(y'(\theta'); h', \theta')\big) \Big] f(\theta')\, d\theta', \tag{10}$$

where $\ell(y'(\theta'); h', \theta')$ denotes the labor required to produce output $y'(\theta')$ for a child of ability θ' and human capital h', implicitly defined by

$$Y\big(\ell(y'(\theta'); h', \theta'), h', \theta'\big) = y'(\theta'). \tag{11}$$

The parent's budget constraint is

$$c + b' + g(h', h) = (1 + r)b + Y(l, h, \theta), \tag{12}$$

and child consumption is given by

$$c'(\theta') = (1 + r)b' + y'(\theta'). \tag{13}$$

As standard in Mirrlees models, labor supply is not chosen directly by the child but is implicitly determined by the output assignment through the production technology. Financial bequests are budgetary transfers: they shift resources across generations without entering the production function. Educational investment, by contrast, enters $Y(l', h', \theta')$ and changes the productivity schedule.

The information structure is as follows. The parent and the planner share the same information ex ante: neither observes the realization of child ability θ' when b' and h' are chosen. After these choices are made, θ' is realized and privately observed by the child. The planner therefore designs an incentive-compatible direct mechanism assigning type-contingent allocations $(c'(\theta'), y'(\theta'))$.

Throughout this section, parental ability θ is taken as given and is assumed to be commonly observed. We abstract from private information at the parental level and from intergenerational correlation in types in order to isolate the role of the child's private information in shaping incentive provision. Allowing for parental private information or ability persistence would introduce additional screening motives, but would not eliminate the instrument-specific channel emphasized here.

Under the Spence–Mirrlees condition and monotonicity, the first-order approach applies.

4.2 Full-Information Benchmark

Under full information, the planner observes child ability and faces no incentive constraints. The first-order condition for education is

$$\overline{Y}_{h'} = (1+r)g_{h'}(h', h), \tag{14}$$

where

$$\overline{Y}_{h'} = \frac{\int_{\underline{\theta}}^{\overline{\theta}} u'\big(c'(\theta')\big) Y_{h'}\big(\ell(y'(\theta'); h', \theta'), h', \theta'\big) f(\theta')\, d\theta'}{\int_{\underline{\theta}}^{\overline{\theta}} u'\big(c'(\theta')\big) f(\theta')\, d\theta'}$$

denotes the expected marginal product of education weighted by marginal utility.

This condition coincides with the resource-based benchmark derived in the absence of informational frictions. Thus, under full information, there is no informational distortion in the education margin, and education and financial bequests remain equivalent at the level of the marginal resource trade-off. Any departure from this benchmark must therefore arise from the informational friction introduced below.

4.3 Private Information and Instrument-Specific Distortions

We now introduce private information about child ability. The planner must offer type-contingent allocations that satisfy incentive compatibility.

The key observation is that educational investment affects the productivity schedule and therefore changes the labor required to implement a given output assignment. As a result, it changes the attractiveness of mimicking across types and hence the informational rents required to sustain truthful revelation. Financial bequests, by contrast, do not affect productivity and therefore do not enter the incentive problem through this channel directly.

Let λ denote the multiplier on the resource constraint and let $\mu(\theta') \geq 0$ denote the shadow value of the local incentive constraint. The planner's first-order conditions imply

$$\lambda = \beta(1+r) \int_{\underline{\theta}}^{\overline{\theta}} u'\big(c'(\theta')\big) f(\theta')\, d\theta', \tag{15}$$

$$\beta \int_{\underline{\theta}}^{\overline{\theta}} u'\big(c'(\theta')\big) Y_{h'}\big(\ell(y'(\theta'); h', \theta'), h', \theta'\big) f(\theta')\, d\theta' = \lambda g_{h'}(h', h) + R_{h'}, \tag{16}$$

where

$$R_{h'} = -\int_{\underline{\theta}}^{\overline{\theta}} \mu(\theta') \frac{\partial}{\partial h'} \left[u\big(c'(\theta')\big) - v\big(\ell(y'(\theta'); h', \theta')\big) \right] f(\theta')\, d\theta'. \tag{17}$$

The first-order conditions reveal a direct asymmetry across instruments in the local-envelope representation. The bequest condition contains no direct productivity-based informational term, while the education condition contains $R_{h'}$. The next result characterizes this direct channel and the sign of the baseline labor-requirement component. The sign result should be interpreted as applying to this component, rather than to the total effect of education on all global incentive constraints.

Theorem 1 (Direct labor-requirement effect of education)**.** *Consider the private-information environment described above and suppose that the first-order approach is valid. The first-order conditions* (15)–(17) *have the following properties.*

First, in this baseline representation, financial bequests generate no direct productivity-based term in the local incentive component, because they do not alter the labor requirement mapping. By contrast, educational investment generates the term $R_{h'}$, because it affects $\ell(y; h', \theta')$, which enters the local incentive constraints.

Moreover, by the implicit function theorem applied to

$$Y\big(\ell(y; h', \theta'), h', \theta'\big) = y,$$

the labor requirement mapping satisfies

$$\ell_{h'}(y; h', \theta') = -\frac{Y_{h'}\big(\ell(y; h', \theta'), h', \theta'\big)}{Y_l\big(\ell(y; h', \theta'), h', \theta'\big)}. \tag{18}$$

Thus, the sign of the direct labor-requirement component is determined by how educational investment changes the labor required to implement a given output assignment.

Define

$$M_{h'}(\theta') \equiv -\frac{\partial}{\partial h'}\left[u\big(c'(\theta')\big) - v\big(\ell(y'(\theta'); h', \theta')\big)\right].$$

Then

$$M_{h'}(\theta') = v'\big(\ell(y'(\theta'); h', \theta')\big)\ell_{h'}(y'(\theta'); h', \theta'), \tag{19}$$

and hence

$$R_{h'} = \int_{\underline{\theta}}^{\overline{\theta}} \mu(\theta') M_{h'}(\theta') f(\theta')\, d\theta'. \tag{20}$$

If $Y_{h'} > 0$ and $Y_l > 0$, then $\ell_{h'}(y; h', \theta') < 0$. Hence $M_{h'}(\theta') < 0$, and $R_{h'} \leq 0$, with strict inequality whenever the local incentive component binds on a set of positive measure.

Financial bequests generate no analogous direct term because they enter the planner's problem through the resource constraint and do not affect the production function or the labor requirement mapping $\ell(y; h', \theta')$.

Proof. The first-order conditions (15) and (16) follow from differentiating the planner's Lagrangian with respect to b' and h', respectively. Since financial bequests do not enter the production function or the labor requirement mapping, no direct productivity-based informational term appears in the bequest condition.

Educational investment affects the marginal product of human capital and also changes the labor requirement associated with any assigned output level. The relevant channel is

$$h' \longrightarrow \ell(y'(\theta'); h', \theta') \longrightarrow v\big(\ell(y'(\theta'); h', \theta')\big) \longrightarrow \text{local incentive constraints.}$$

Thus, differentiating the Lagrangian with respect to h' brings in the multiplier on the local incentive constraint and generates the term $R_{h'}$ in (17).

Equation (18) follows from the implicit function theorem applied to $Y(\ell(y; h', \theta'), h', \theta') = y$, using $Y_l > 0$. Equation (19) follows by differentiating $u(c'(\theta')) - v(\ell(y'(\theta'); h', \theta'))$ with respect to h', treating $(c'(\theta'), y'(\theta'))$ as separate control variables in the Lagrangian, as in the standard envelope step. The sign statements follow from $v' > 0$, $Y_l > 0$, and $\mu(\theta') \geq 0$. □

The sign result in Theorem 1 concerns the direct labor-requirement component obtained from the local-envelope representation. It should not be read as a statement that education unambiguously relaxes every global incentive constraint. In a finite-type representation, a change in education may affect both the utility from the truthful allocation and the utility from mimicking another type's allocation. The contribution of the theorem is to isolate the direct component that arises because education changes the labor requirement associated with a given output assignment. Section 6 shows that additional marginal-utility, labor-supply, and productivity-rent components appear once labor supply is endogenous.

A simple example illustrates the sign of the baseline labor-requirement channel. Under the Cobb–Douglas technology

$$Y(l, h', \theta') = \theta'^{\xi}(h')^{1-\xi} l, \qquad \xi \in (0, 1),$$

the labor requirement is

$$\ell(y; h', \theta') = \frac{y}{\theta'^{\xi}(h')^{1-\xi}},$$

so

$$\ell_{h'}(y; h', \theta') = -\frac{1-\xi}{h'}\ell(y; h', \theta') < 0.$$

Therefore $M_{h'}(\theta') < 0$ and, under the sign convention adopted in (17), $R_{h'} < 0$ whenever incentive constraints bind. In this baseline representation, productive education relaxes the direct labor-requirement component of incentive provision. This example does not sign the total informational effect once labor supply and the slope of information rents are endogenous; that case is analyzed in Section 6.

Combining the two first-order conditions gives the wedge representation for education.

Corollary 1 (Education wedge under private information)**.** *Let $\overline{Y}_{h'}$ denote the expected marginal product of education defined in* (14)*. Then the education condition can be written as*

$$\overline{Y}_{h'} = (1+r)g_{h'}(h', h) + \Delta_{h'}, \tag{21}$$

where

$$\Delta_{h'} = \frac{R_{h'}}{\beta \int_{\underline{\theta}}^{\overline{\theta}} u'\big(c'(\theta')\big) f(\theta')\, d\theta'}. \tag{22}$$

Proof. Divide (16) by

$$\beta \int_{\underline{\theta}}^{\overline{\theta}} u'\big(c'(\theta')\big) f(\theta')\, d\theta'$$

and use (15) to substitute for the resource multiplier λ. This substitution uses the bequest optimality condition to express the resource multiplier in terms of the marginal utility value of financial transfers. The wedge $\Delta_{h'}$ is therefore measured relative to the bequest margin.

Under full information, $\mu(\theta') \equiv 0$, so $R_{h'} = 0$ and $\Delta_{h'} = 0$, recovering (14). □

Under the sign convention in (17), a negative informational wedge means that the planner requires a lower expected marginal product of education relative to the bequest margin. If $\overline{Y}_{h'}$ is decreasing in h', this corresponds to higher education than under the full-information benchmark, or equivalently to a subsidy-like distortion of education relative to financial bequests. A positive wedge has the opposite interpretation.

Remark 1 (Productive and budgetary transfer margins)**.** *This observation is mechanical but useful for organizing the comparison. A transfer instrument that enters only through the resource constraint cannot generate a derivative of the production mapping with respect to ability. Hence it cannot generate the direct productivity-rent term identified below. The substantive question is not this absence per se, but how large the education-specific productivity-rent term is relative to the indirect channels that affect both instruments.*

4.4 Interpretation

Theorem 1 identifies the central asymmetry across intergenerational transfer instruments. Educational investment alters the productivity schedule and therefore affects the labor requirement associated with assigned output levels. Financial bequests shift resources across generations but do not modify the production technology.

As a result, education is the only instrument that generates a direct productivity-based informational term in this baseline environment. This mechanism is absent for financial bequests, which affect the allocation through the resource constraint but do not alter the labor requirement schedule.

The sign of the baseline informational term depends on how education changes the labor required to implement assigned outputs. Under the maintained assumption that education raises productivity, the labor requirement falls, so $R_{h'} < 0$ under the sign convention adopted in (17). Thus, in the baseline labor-requirement representation, productive education relaxes this dimension of incentive provision.

This conclusion should not be interpreted as saying that education is always distorted upward. Once labor supply and the slope of information rents are endogenous, education also affects how ability is translated into output. Section 6 shows that the direct productivity-rent component governed by $Y_{\theta' h'}$ may offset or reverse the baseline effect.

Importantly, this result complements the analysis of ability risk. While risk generates a wedge between ex ante and ex post returns to education, informational frictions generate an additional wedge through incentive provision. The two wedges may interact through their joint dependence on the production technology, but only education directly changes the mapping from private information to implemented outcomes.

This asymmetry is the key force behind the education wedge studied in the next section and distinguishes human capital investment from purely budgetary forms of intergenerational transfers.

5 The Education Wedge

This section provides the central decomposition of the paper. The objective is to compare education with financial bequests as alternative instruments for transferring resources across generations. The key distinction is that education changes the child's productivity, whereas financial bequests are budgetary transfers. As a result, education interacts with both ability risk and incentive constraints in ways that financial bequests do not.

The decomposition below separates three forces: a risk component, a technological component, and an informational component. The risk component arises because education is chosen before ability is realized. The technological component reflects the resource cost of producing human capital. The informational component is the direct incentive effect derived in Section 4.

Throughout this section, write

$$Y_{h'}(\theta') \equiv Y_{h'}\big(\ell(y'(\theta'); h', \theta'), h', \theta'\big)$$

for the realized marginal product of education for type θ'.

5.1 Child's ex post benchmark

From the child's perspective, education is evaluated after ability has been realized. Conditional on θ', the child compares the realized marginal return to human capital with the return on financial wealth. This yields the return-based benchmark

$$Y_{h'}(\theta') = 1 + r. \tag{23}$$

This condition is not an optimal education choice by the child. Rather, it is an ex post evaluation criterion that compares human capital with financial wealth after the uncertainty about ability has been resolved. The resource cost of education is incurred at the investment stage and is internalized by the parent or planner.

Equation (23) therefore provides a useful reference point: in the absence of risk, technological differences, and incentive problems, education and financial wealth would be perfect substitutes at the margin.

5.2 Full-information allocation

Under full information, the planner observes child ability and faces no incentive constraints. Let $\overline{Y}_{h'}$ denote the expected marginal product of education weighted by marginal utility, as defined in Section 4.2. The full-information optimality condition is

$$\overline{Y}_{h'} = (1 + r) g_{h'}(h', h). \tag{24}$$

Relative to the child's return-based benchmark (23), the full-information allocation differs because the planner internalizes the marginal resource cost of producing education. The term

$$(1+r)\big(g_{h'}(h',h)-1\big)$$

therefore captures the difference between evaluating education as a return and evaluating it as a costly investment technology.

5.3 Private information

Under private information, the planner must satisfy incentive compatibility. As shown in Section 4, educational investment affects the labor requirement mapping and therefore enters the incentive constraints directly. Financial bequests do not generate an analogous direct productivity-based informational term.

Using Corollary 1, the planner's education condition under private information can be written as

$$\overline{Y}_{h'} = (1+r)g_{h'}(h',h) + \Delta_{h'}, \tag{25}$$

where $\Delta_{h'}$ is the informational wedge derived in Section 4.3.

Under the baseline labor-requirement channel of Section 4, productive education reduces the labor required to implement a given output assignment. With the sign convention adopted in (17), this implies $\Delta_{h'} \leq 0$ whenever incentive constraints bind and $Y_{h'} > 0$. Section 6 shows that once labor supply and the slope of information rents are endogenous, additional productivity-rent terms may offset or reverse this baseline effect.

5.4 Decomposition

Combining the child's ex post benchmark, the full-information condition, and the private-information condition yields the education wedge decomposition.

Theorem 2 (Education wedge decomposition)**.** *Under private information, the realized education wedge satisfies*

$$Y_{h'}(\theta')-(1+r) = \underbrace{\Big(Y_{h'}(\theta')-\overline{Y}_{h'}\Big)}_{\textit{risk wedge}} + \underbrace{(1+r)\big(g_{h'}(h',h)-1\big)}_{\textit{technological wedge}} + \underbrace{\Delta_{h'}}_{\textit{informational wedge}}. \tag{26}$$

Moreover:

(i) *The risk wedge is type-specific. Taking expectations across types, its average satisfies*

$$\mathbb{E}\big[Y_{h'}(\theta') - \overline{Y}_{h'}\big] = -\frac{\operatorname{Cov}\big(u'\big(c'(\theta')\big), Y_{h'}(\theta')\big)}{\mathbb{E}\big[u'\big(c'(\theta')\big)\big]}. \tag{27}$$

Since $\overline{Y}_{h'}$ *is constant across types,*

$$\mathbb{E}\big[Y_{h'}(\theta') - \overline{Y}_{h'}\big] = \mathbb{E}[Y_{h'}(\theta')] - \overline{Y}_{h'}.$$

Thus, when high-return states are associated with low marginal utility, the average risk wedge is positive. Under the standard case in which higher-ability children receive higher consumption and $u'' < 0$*, marginal utility is lower in high-return states, so this covariance is negative.*

(ii) *The informational wedge is instrument-specific. It is given by* $\Delta_{h'}$ *from Corollary 1. Under the baseline labor-requirement channel of Section 4, productive education implies* $\Delta_{h'} \leq 0$*. With endogenous labor supply, Section 6 shows that the additional productivity-rent component is governed by*

$$\int_{\Theta} \omega(\theta') u'\big(c'(\theta')\big) Y_{\theta' h'}(\theta') f(\theta')\, d\theta',$$

where $\omega(\theta')$ *is the rent-weighting term defined there. If this term is positive and sufficiently large relative to the baseline labor-requirement component, the sign of the total informational wedge is reversed.*

Proof. The decomposition follows by adding and subtracting $\overline{Y}_{h'}$ from $Y_{h'}(\theta') - (1+r)$ and using the private-information education condition (25). The covariance representation in part (i) and the sign characterization in part (ii) are derived in Appendix D. □

The central implication is that education and financial bequests are not mechanism-equivalent instruments of intergenerational redistribution. Education carries risk, technological, and informational components. Financial bequests, by contrast, do not alter the production technology and therefore do not generate the direct productivity-based informational component identified in Theorem 1.

Although (26) is obtained by adding and subtracting $\overline{Y}_{h'}$, the decomposition is not arbitrary. Each term corresponds to a distinct comparison. The risk wedge compares realized and ex ante expected returns. The technological wedge compares a return-based benchmark with a resource-cost benchmark. The informational wedge compares the full-information allocation with the private-information allocation.

5.5 Interpretation

The decomposition in Theorem 2 separates the education wedge into three conceptually distinct components.

The risk wedge arises because education is chosen before child ability is realized. The planner evaluates education using the marginal-utility-weighted expected return, whereas the realized return depends on the ex post type. Equation (27) shows that the average risk wedge is governed by the covariance between marginal utility and the marginal return to education. Under standard monotone allocations, higher-ability children both have higher returns to education and receive higher consumption, so the covariance between marginal utility and the return to education is negative and the average risk wedge is positive.

The technological wedge reflects the fact that education is produced through a costly human-capital technology. A unit of financial wealth can be transferred at the gross return $1 + r$, while an additional unit of human capital requires $g_{h'}(h', h)$ units of resources. This component therefore depends on the benchmark used to evaluate education. If education is measured net of its resource cost, the technological wedge disappears.

Remark 2 (Benchmark dependence of the technological wedge)**.** *The technological wedge is benchmark-dependent. It reflects the difference between a return-based evaluation of education and a resource-cost evaluation. If education is evaluated net of its marginal resource cost, so that the benchmark is*

$$Y_{h'}(\theta') = (1 + r)g_{h'}(h', h),$$

then the technological wedge disappears. By contrast, the risk wedge and the informational wedge arise from uncertainty and incentive compatibility and therefore do not disappear under this change of benchmark.

The informational wedge is the direct incentive component derived in Section 4. It reflects the fact that education changes the labor requirement mapping and therefore affects incentive compatibility. As shown in Theorem 1, financial bequests have no analogous direct component because they do not enter the production technology.

The sign of the informational wedge should be interpreted together with Section 4. In the baseline labor-requirement representation, productive education relaxes one dimension of incentive provision, yielding $\Delta_{h'} \leq 0$ under the sign convention adopted there. This does not imply that education is always distorted upward: Section 6 shows that when labor supply and the slope of information rents are endogenous, the productivity-rent channel may offset or reverse this baseline effect.

Remark 3 (Relation to Bovenberg–Jacobs)**.** *Bovenberg and Jacobs show that, in a static Mirrlees environment, education subsidies and income taxation are complementary policy instruments. In the baseline labor-requirement representation considered here, productive education relaxes one dimension of incentive provision, which is consistent with the standard subsidy logic.*

The present framework adds two features that are absent from that benchmark: financial bequests as an alternative intergenerational transfer instrument, and an explicit dynastic incentive margin. Once labor supply and the slope of information rents are endogenous, Section 6 shows that the productivity-rent component governed by $Y_{\theta' h'}$ may offset or reverse the baseline subsidy logic.

In particular, when $Y_{\theta' h'} > 0$ and the productivity-rent component is sufficiently large relative to the baseline labor-requirement component, the optimal policy may tax rather than subsidize education—a reversal of the Bovenberg–Jacobs conclusion that arises precisely because financial bequests provide a purely budgetary alternative transfer margin.

The decomposition also organizes the comparative-statics implications discussed in Section 7.2. Changes in the dispersion of ability primarily affect the risk component, whereas changes in the return on financial wealth and in the weight placed on future generations affect the resource and incentive margins. These comparative statics can therefore be read as changes in the relative importance of the three components of (26).

Thus, the education wedge is not a single distortion. It is the sum of three mechanisms that operate through different margins: uncertainty, resource costs, and incentive provision.

6 Endogenous Labor Supply

This section extends the baseline environment by allowing the child to choose labor supply optimally. The purpose is to test the robustness of the direct informational asymmetry established in Sections 4 and 5, and to characterize how endogenous labor supply affects the sign of the education wedge.

In the baseline formulation, labor is pinned down by the output assignment through the labor requirement mapping. This isolates the effect of education on the cost of implementing assigned outputs. Once labor supply is endogenous, both educational investment and financial bequests affect behavior and therefore influence incentive provision through marginal utility and labor supply responses.

The central question is whether this additional behavioral channel eliminates the asymmetry between educational investment and financial transfers. We show that it does not. Endogenous labor supply makes financial bequests informationally relevant through indirect behavioral channels, but only education generates a direct productivity-rent component. This component is governed by $Y_{\theta' h'}$, and may offset or reverse the baseline labor-requirement effect identified in Section 4.

6.1 Environment

We maintain the environment of Section 4, but allow the child to choose labor supply. Preferences are

$$u\big(c'(\theta')\big) - v\big(l'(\theta')\big),$$

and output is

$$Y\big(l'(\theta'), h', \theta'\big).$$

Educational investment h' enters the production function directly, whereas financial bequests b' affect the child through the budget constraint. Given (b', h', θ'), the child chooses labor supply to solve

$$l'(\theta'; b', h') \in \arg\max_{l} \{u((1+r)b' + Y(l, h', \theta')) - v(l)\}.$$

The associated consumption level is

$$c'(\theta') = (1+r)b' + Y\big(l'(\theta'; b', h'), h', \theta'\big).$$

Under the maintained regularity conditions, the labor supply policy is well-defined, unique, and differentiable. Its first-order condition is

$$u'\big(c'(\theta')\big)Y_l\big(l'(\theta'), h', \theta'\big) = v'\big(l'(\theta')\big), \tag{28}$$

where, to simplify notation, we write $l'(\theta')$ for $l'(\theta'; b', h')$.

Lemma 2 (Envelope property with endogenous labor supply)**.** *For each type θ', let indirect utility be*

$$U(\theta') = u\big(c'(\theta')\big) - v\big(l'(\theta')\big),$$

where

$$c'(\theta') = (1+r)b' + Y\big(l'(\theta'), h', \theta'\big).$$

Then the marginal effect of educational investment on indirect utility satisfies

$$\frac{\partial U(\theta')}{\partial h'} = u'\big(c'(\theta')\big)Y_{h'}\big(l'(\theta'), h', \theta'\big).$$

Proof. The result follows from the child's labor-supply first-order condition and an envelope argument. The full derivation is in Appendix E. □

Lemma 2 shows that endogenous labor supply does not alter the direct marginal utility effect of education. Instead, it changes the informational wedge through the slope of information rents.

6.2 Informational wedges with endogenous labor supply

With endogenous labor supply, information rents are characterized by the envelope condition

$$U_{\theta'}(\theta') = u'\big(c'(\theta')\big)Y_{\theta'}\big(l'(\theta'), h', \theta'\big). \tag{29}$$

Thus, information rents depend on the sensitivity of output to type, evaluated at the equilibrium labor supply.

We use a tilde to distinguish the endogenous-labor informational effect from the baseline wedge in Sections 4 and 5. Let

$$\widetilde{R}_x$$

denote the unnormalised informational effect of an instrument $x \in \{h', b'\}$. The corresponding normalised wedge is

$$\widetilde{\Delta}_x = \frac{\widetilde{R}_x}{\beta \int_\Theta u'\big(c'(\theta')\big) f(\theta')\, d\theta'}. \tag{30}$$

Thus, $\widetilde{\Delta}_{h'}$ is the endogenous-labor counterpart of the informational wedge $\Delta_{h'}$ from Corollary 1.

The following theorem characterizes the unnormalised effects $\widetilde{R}_x$. Normalising them through (30) gives the corresponding wedges in the planner's first-order conditions.

Theorem 3 (Instrument-specific informational effects with endogenous labor)**.** *Suppose the assumptions of Lemma 2 hold and that the first-order approach is valid. Let*

$$\omega(\theta') = \frac{1 - F(\theta')}{f(\theta')}.$$

Then, for any instrument $x \in \{h', b'\}$, the unnormalised informational effect can be written as

$$\widetilde{R}_x = \int_\Theta \omega(\theta')\, U_{\theta' x}(\theta')\, f(\theta')\, d\theta'. \tag{31}$$

Moreover, the derivative of the rent slope with respect to x satisfies

$$U_{\theta' x}(\theta') = u''\big(c'(\theta')\big) c'_x(\theta') Y_{\theta'}\big(l'(\theta'), h', \theta'\big) + u'\big(c'(\theta')\big) \frac{\partial}{\partial x} Y_{\theta'}\big(l'(\theta'), h', \theta'\big), \tag{32}$$

where $c'_x(\theta')$ denotes the total derivative of consumption with respect to x.

For educational investment, the informational effect decomposes as

$$\widetilde{R}_{h'} = R^{\text{MU}}_{h'} + R^{\text{prod}}_{h'} + R^{\text{labor}}_{h'}, \tag{33}$$

where

$$R^{\text{MU}}_{h'} = \int_\Theta \omega(\theta') u''\big(c'(\theta')\big) c'_{h'}(\theta') Y_{\theta'}\big(l'(\theta'), h', \theta'\big) f(\theta')\, d\theta',$$

$$R^{\text{prod}}_{h'} = \int_\Theta \omega(\theta') u'\big(c'(\theta')\big) Y_{\theta' h'}\big(l'(\theta'), h', \theta'\big) f(\theta')\, d\theta',$$

and

$$R^{\text{labor}}_{h'} = \int_\Theta \omega(\theta') u'\big(c'(\theta')\big) Y_{\theta' l}\big(l'(\theta'), h', \theta'\big) l'_{h'}(\theta') f(\theta')\, d\theta'.$$

For financial bequests, the informational effect is

$$\widetilde{R}_{b'} = R_{b'}^{\mathrm{MU}} + R_{b'}^{\mathrm{labor}}, \tag{34}$$

where

$$R_{b'}^{\mathrm{MU}} = \int_{\Theta} \omega(\theta')u''\big(c'(\theta')\big)c'_{b'}(\theta')Y_{\theta'}\big(l'(\theta'), h', \theta'\big)f(\theta')\,d\theta',$$

and

$$R_{b'}^{\mathrm{labor}} = \int_{\Theta} \omega(\theta')u'\big(c'(\theta')\big)Y_{\theta' l}\big(l'(\theta'), h', \theta'\big)l'_{b'}(\theta')f(\theta')\,d\theta'.$$

Hence both instruments generate indirect informational effects through marginal utility and labor-supply responses, but only educational investment generates the direct productivity-rent component $R_{h'}^{\mathrm{prod}}$*.*

Proof. The result follows by differentiating the rent-slope equation (29) with respect to each instrument and using the reduced-form representation of informational rents. The full derivation is in Appendix E. □

Theorem 3 delivers the central robustness result of this section. Endogenous labor supply introduces marginal-utility and labor-supply channels for both instruments. However, the direct productivity-rent component $R_{h'}^{\mathrm{prod}}$ remains unique to education.

Corollary 2 (Persistence of the direct productivity asymmetry)**.** *Under the assumptions of Theorem 3, financial bequests affect incentive provision only through the indirect components* $R_{b'}^{\mathrm{MU}}$ *and* $R_{b'}^{\mathrm{labor}}$*. Educational investment affects incentive provision through these channels and, additionally, through the direct productivity-rent component* $R_{h'}^{\mathrm{prod}}$*.*

Thus, endogenous labor supply weakens the strict neutrality of financial bequests from the baseline model, but it does not eliminate the direct productivity asymmetry between education and bequests.

Remark 4 (Financial bequests with endogenous labor supply)**.** *Unlike in the baseline labor-requirement representation, financial bequests are not informationally neutral once labor supply is endogenous. In general,*

$$\widetilde{R}_{b'} = R_{b'}^{\mathrm{MU}} + R_{b'}^{\mathrm{labor}} \neq 0.$$

However, the effect of bequests operates only through marginal utility and labor-supply responses. Bequests do not generate a direct productivity-rent component because they do not enter the production function. This preserves the qualitative distinction from educational investment.

Proposition 4 (Sign of the direct productivity-rent component)**.** *Suppose that $u'(c'(\theta')) > 0$ and that $Y_{\theta' h'}(l'(\theta'), h', \theta')$ has a constant sign on Θ. Then the sign of $R_{h'}^{\text{prod}}$ is the sign of $Y_{\theta' h'}$. In particular, if human capital and ability are complements in production, so that*

$$Y_{\theta' h'}(l'(\theta'), h', \theta') > 0$$

on a set of positive measure, then

$$R_{h'}^{\text{prod}} > 0.$$

Proof. The result follows from the non-negativity of $\omega(\theta')$, $u'\big(c'(\theta')\big)$, and $f(\theta')$ in the integrand of $R_{h'}^{\text{prod}}$. These terms therefore pin down the sign of $R_{h'}^{\text{prod}}$ from the sign of $Y_{\theta' h'}$. The full derivation is in Appendix E. □

Proposition 4 is the key sign result of this extension. It shows that complementarity between human capital and ability generates a positive productivity-rent component. This is the channel that can offset or reverse the baseline labor-requirement effect from Section 4.

6.3 Relation to the baseline education wedge

The planner's first-order condition for education retains the same structure as in Section 5. With endogenous labor supply,

$$\overline{Y}_{h'} = (1 + r) g_{h'}(h', h) + \widetilde{\Delta}_{h'}, \tag{35}$$

where $\overline{Y}_{h'}$ denotes the expected marginal product of education weighted by marginal utility, as in (14), and

$$\widetilde{\Delta}_{h'} = \frac{\widetilde{R}_{h'}}{\beta \int_{\Theta} u'\big(c'(\theta')\big) f(\theta')\, d\theta'}.$$

The normalized endogenous-labor wedge decomposes as

$$\widetilde{\Delta}_{h'} = \widetilde{\Delta}_{h'}^{\text{MU}} + \widetilde{\Delta}_{h'}^{\text{prod}} + \widetilde{\Delta}_{h'}^{\text{labor}}, \tag{36}$$

where each component is obtained by normalizing the corresponding component of $\widetilde{R}_{h'}$ by

$$\beta \int_{\Theta} u'\big(c'(\theta')\big) f(\theta')\, d\theta'.$$

In the baseline labor-requirement representation, productive education generates a non-positive informational wedge under the sign convention of Section 4. With endogenous labor supply, the productivity-rent component may offset or reverse this effect.

Denote by h'^{FI} and h'^{PI} the optimal education levels under full information and private information, respectively.

Proposition 5 (Productivity rents and the Bovenberg–Jacobs reversal)**.** *Suppose that human capital and ability are complements in production, so that*

$$Y_{\theta' h'}(l'(\theta'), h', \theta') > 0$$

on a set of positive measure. Suppose further that the direct productivity-rent component dominates the remaining informational components:

$$R_{h'}^{\text{prod}} > -\left(R_{h'}^{\text{MU}} + R_{h'}^{\text{labor}}\right).$$

Then the normalized endogenous-labor informational wedge satisfies

$$\widetilde{\Delta}_{h'} > 0.$$

If, in addition, $\overline{Y}_{h'}$ is decreasing in h', then the private-information allocation features lower educational investment than the full-information allocation:

$$h'^{PI} < h'^{FI}.$$

Hence, relative to the financial bequest margin, optimal policy may tax rather than subsidize education. This reverses the standard Bovenberg–Jacobs subsidy logic when education amplifies informational rents through the productivity-rent channel.

Proof. The dominance condition implies $\widetilde{\Delta}_{h'} > 0$. Comparing the private-information and full-information education conditions then implies $h'^{PI} < h'^{FI}$ whenever $\overline{Y}_{h'}$ is decreasing in h'. The full derivation is in Appendix E. □

Remark 5 (Interpretation of the reversal condition)**.** *Proposition 5 is a sufficient-condition result. It does not imply that education is generally taxed under private information. The sign of the total education wedge depends on the relative magnitude of the productivity-rent component, the marginal-utility component, and the labor-supply component. The contribution is to identify the education-specific component and to show that it can, under complementarity, work against the standard subsidy force.*

6.4 Discussion

Endogenous labor supply refines the baseline results rather than overturning them. Both instruments now affect incentive provision through marginal utility and labor supply responses. However, educational investment remains distinct because it also changes the productivity mapping from ability to output.

This distinction is central for policy. Human capital investment is not merely another vehicle for transferring resources across generations. It is also a technological choice that determines how private information about ability is translated into output. Financial bequests shift resources without directly altering this mapping. Thus, even with endogenous labor supply, education and financial bequests remain non-equivalent instruments of intergenerational redistribution.

7 Discussion and Interpretation

The preceding sections establish that education and financial bequests are not mechanism-equivalent instruments of intergenerational redistribution. This section discusses the main implications of the decomposition and organizes the comparative statics of the model.

7.1 Mechanism equivalence and policy interpretation

The central implication of the analysis is that education is not merely a different way of transferring resources across generations. Financial bequests are budgetary transfers: they shift resources without directly altering the production mapping. Educational investment, by contrast, changes how private ability is translated into output.

This distinction matters for incentive provision. In the baseline labor-requirement representation, productive education reduces the labor required to implement assigned outputs and therefore relaxes one dimension of incentive compatibility. Under the sign convention adopted in Section 4, this corresponds to $\Delta_{h'} \leq 0$ whenever incentive constraints bind and education raises productivity.

With endogenous labor supply, however, education also affects the slope of information rents through the productivity-rent component identified in Section 6. When human capital and ability are complements, $Y_{\theta' h'} > 0$, this component is positive and may dominate the baseline labor-requirement effect. In that case the total informational wedge becomes positive and the planner may distort education downward relative to the full-information benchmark.

Thus, the model does not imply that education should always be subsidized or always be taxed. The sign of the optimal education distortion depends on which component of the wedge dominates. The key result is instead that the education margin is intrinsically different from the bequest margin: education directly changes the productivity and incentive structure of the economy, whereas financial bequests operate through the budget constraint.

7.2 Comparative-statics implications

The decomposition in Theorem 2 provides a useful organizing framework for comparative statics. Changes in fundamentals affect optimal education by shifting the risk, technological, and informational components of the education wedge.

First, changes in the dispersion of child ability primarily affect the risk component and the productivity-rent component. Greater ability dispersion increases the dispersion of realized returns to education. Through the covariance term in (27), this changes the gap between the realized return and the marginal-utility-weighted expected return. At the same time, if human capital and ability are complements, a more dispersed ability distribution may also increase the value of the productivity-rent component in Section 6. The net effect on education is therefore generally ambiguous without additional restrictions.

Second, changes in the return on financial wealth affect the resource-cost benchmark. A higher r raises the opportunity cost of investing in education rather than transferring financial wealth. This tends to make financial bequests more attractive relative to education. However, the total effect on optimal education also depends on how the change in r affects consumption, marginal utilities, and the informational wedge.

Third, changes in parental altruism affect the weight placed on the child's allocation. A higher β increases the importance of child utility in the parent's objective and therefore raises the value of future resources. Whether this leads to more or less education depends on how the increased weight on the child interacts with the incentive component of the education wedge.

These comparative statics are intentionally stated as implications rather than unconditional monotone predictions. The model shows that the direction of the response depends on the relative strength of the components in (26). This is useful empirically: changes in optimal education can be interpreted by asking whether the risk, technological, or informational component is moving most strongly.

7.3 Policy implications

The policy implication is not that education is socially undesirable. Education raises productivity and may relax incentive constraints in the baseline labor-requirement channel. The point is that education is not a neutral redistributive instrument. Because it changes the mapping from private ability to output, it also changes the incentive structure faced by the planner.

When the baseline labor-requirement effect dominates, education may be expanded relative to the full-information benchmark. When the productivity-rent component dominates, education may instead be reduced or taxed relative to the financial bequest margin. This sign dependence is the central policy lesson of the paper.

The availability of financial bequests is crucial for this comparison. Bequests provide a purely budgetary transfer margin against which the planner can evaluate education. Once this alternative instrument is present, the planner's problem is not simply whether to transfer resources to the next generation, but how to divide intergenerational transfers between a budgetary instrument and a productivity-changing instrument.

The analysis abstracts from borrowing constraints, general-equilibrium effects, peer effects, and external social returns to education. These forces may create additional reasons to subsidize education. The contribution of the paper is to isolate a distinct informational mechanism: even absent such forces, education differs from financial transfers because it affects how private information about ability is translated into output.

8 Conclusion

This paper studies intergenerational redistribution when parents can transfer resources through both financial bequests and educational investment. The main result is that these instruments are not mechanism-equivalent under private information. Financial bequests are budgetary transfers, while education changes the mapping from child ability to output.

The education wedge decomposes into risk, technological, and informational components. The risk component reflects the gap between ex ante and ex post returns to education. The technological component reflects the resource cost of producing human capital. The informational component is instrument-specific: it arises because education affects incentive compatibility through the production technology.

With endogenous labor supply, financial bequests also affect incentive provision through marginal utility and labor-supply responses. However, only education generates a direct productivity-rent component. When human capital and ability are complements, this component can dominate the baseline labor-requirement effect and reverse the standard education-subsidy logic.

A further extension would allow parents to possess private information about the distribution of child ability. In that case, education would also serve as a vehicle for using parental information about expected returns, whereas financial bequests would remain purely budgetary.

The analysis therefore highlights a central trade-off in intergenerational policy. Human capital investment is valuable because it raises productivity, but it also changes the incentive structure of the economy. Financial bequests transfer resources without directly changing this mapping. This distinction helps explain why optimal policy may treat education and financial transfers asymmetrically.

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A Full-Information Benchmark

This appendix derives the benchmark conditions used in the main text and establishes that, under full information, the parent and the child evaluate education according to the same marginal resource trade-off. The section serves two purposes. First, it characterizes the parent's optimal allocation between financial bequests and human-capital investment when there are no informational frictions. Second, it formulates a resource-consistent benchmark for intergenerational allocation and shows that the two conditions coincide.

A.1 Parent's Problem under Full Information

Consider a dynastic household consisting of a parent and a child. The parent chooses own consumption c, labor supply l, financial bequests b', and investment in the child's human capital h'. Preferences are given by

$$U^P = u(c) - v(l) + \beta\big[u(c') - v(l')\big], \tag{37}$$

where u is strictly increasing and concave, v is strictly increasing and convex, and $\beta \in (0,1)$ measures parental altruism toward the child.

The intergenerational budget constraints are

$$c + b' + g(h', h) = (1+r)b + Y(l, h, \theta), \tag{38}$$

$$c' = (1+r)b' + Y(l', h', \theta'). \tag{39}$$

Here b denotes inherited wealth, r is the interest rate, h is the parent's stock of human capital, and θ is parental ability. Education is produced according to the cost function $g(h', h)$, which specifies the resources required to attain the child's human capital level h' given parental background h. We assume $g_{h'}(h', h) > 0$.

Under full information, all relevant variables are observable and there are no incentive constraints. To isolate the intergenerational investment trade-off, the child's labor input l' is held fixed in the benchmark analysis. The parent's labor choice is separable from the comparison between financial bequests and educational investment and is included only for completeness.

We focus on an interior solution for the bequest and education margins. If either margin is at a corner, the corresponding first-order condition should be interpreted as a Kuhn–Tucker condition.

Substituting (39) into (37), the parent solves

$$\max_{c,l,b',h'} u(c) - v(l) + \beta\left[u((1+r)b' + Y(l', h', \theta')) - v(l')\right]$$

subject to

$$c + b' + g(h', h) = (1+r)b + Y(l, h, \theta).$$

The associated Lagrangian is

$$\begin{aligned}\mathcal{L} = u(c) - v(l) + \beta\left[u((1+r)b' + Y(l', h', \theta')) - v(l')\right] \\ + \lambda\Big[(1+r)b + Y(l, h, \theta) - c - b' - g(h', h)\Big].\end{aligned} \tag{40}$$

The first-order condition with respect to parental consumption is

$$u'(c) = \lambda. \tag{41}$$

The first-order condition with respect to parental labor supply is

$$v'(l) = \lambda Y_l(l, h, \theta). \tag{42}$$

The first-order condition with respect to financial bequests is

$$\lambda = \beta(1+r)u'(c'). \tag{43}$$

The first-order condition with respect to education is

$$\beta u'(c')Y_{h'}(l', h', \theta') = \lambda g_{h'}(h', h). \tag{44}$$

Combining (43) and (44) yields

$$Y_{h'}(l', h', \theta') = (1+r)g_{h'}(h', h). \tag{45}$$

Equation (45) is the parent's optimality condition for education under full information. The left-hand side is the marginal increase in the child's future income generated by higher human capital. The right-hand side is the return from allocating the marginal resources required for education to financial wealth instead. Since one additional unit of education requires $g_{h'}(h', h)$ units of resources, the relevant opportunity cost is $(1+r)g_{h'}(h', h)$.

A.2 Efficient Resource Benchmark

To compare the parent's education choice with a resource-consistent allocation rule, it is useful to construct a resource-based benchmark. This benchmark is not meant to describe the actual timing of the model. Rather, it is a hypothetical allocation problem that isolates the marginal trade-off between education and financial wealth.

Let R denote the total resources available for intergenerational transfer. The child chooses financial wealth b' and education h' to maximize future utility

$$U^C = u(c') - v(l'), \tag{46}$$

subject to

$$c' = (1+r)b' + Y(l', h', \theta'), \tag{47}$$

$$b' + g(h', h) = R. \tag{48}$$

As above, l' is held fixed in order to isolate the education-bequest trade-off.

Substituting (47) into the objective, the child solves

$$\max_{b',h'} \; u((1+r)b' + Y(l', h', \theta')) - v(l')$$

subject to $b' + g(h', h) = R$.

The associated Lagrangian is

$$\mathcal{L}^C = u((1+r)b' + Y(l', h', \theta')) - v(l') + \mu\big(R - b' - g(h', h)\big), \tag{49}$$

where μ is the multiplier on the child's resource constraint.

The first-order condition with respect to financial wealth is

$$\mu = u'(c')(1+r). \tag{50}$$

The first-order condition with respect to education is

$$u'(c')Y_{h'}(l', h', \theta') = \mu g_{h'}(h', h). \tag{51}$$

Substituting (50) into (51) gives

$$Y_{h'}(l', h', \theta') = (1+r)g_{h'}(h', h). \tag{52}$$

Equation (52) defines the efficient resource benchmark for education. It requires that reallocating resources from financial wealth to human capital yields no first-order gain. Since one additional unit of human capital absorbs $g_{h'}(h', h)$ units of resources, the marginal return to education must equal the return on financial wealth evaluated at the same resource cost.

A.3 Proof of Proposition 1

We compare the parent's optimality condition with the efficient resource benchmark under full information.

From the parent's problem, the first-order condition for bequests is

$$\lambda = \beta(1+r)u'(c'),$$

and the first-order condition for education is

$$\beta u'(c')Y_{h'}(l',h',\theta') = \lambda g_{h'}(h',h).$$

Substituting the bequest condition into the education condition gives

$$\beta u'(c')Y_{h'}(l',h',\theta') = \beta(1+r)u'(c')g_{h'}(h',h).$$

Since $u'(c') > 0$ and $\beta > 0$, this reduces to

$$Y_{h'}(l',h',\theta') = (1+r)g_{h'}(h',h). \tag{53}$$

Now consider the efficient resource benchmark. The first-order condition for financial wealth is

$$\mu = (1+r)u'(c'),$$

and the first-order condition for education is

$$u'(c')Y_{h'}(l',h',\theta') = \mu g_{h'}(h',h).$$

Substituting the wealth condition into the education condition gives

$$u'(c')Y_{h'}(l',h',\theta') = (1+r)u'(c')g_{h'}(h',h).$$

Since $u'(c') > 0$, this reduces to

$$Y_{h'}(l',h',\theta') = (1+r)g_{h'}(h',h). \tag{54}$$

Equations (53) and (54) are identical. Thus, under full information, the parent's education choice and the efficient resource benchmark are governed by the same marginal resource trade-off. In both cases, the marginal productivity gain from human capital is equated to the return from allocating the marginal resources required for education to financial wealth.

The coincidence follows from the fact that the altruistic parent fully internalizes the child's welfare. The altruism parameter β scales the child's utility in the parent's objective, but it also scales the marginal benefit of financial bequests and the marginal benefit of education in the same way. Once the education margin is compared to the bequest margin, β cancels out.

It follows that the parent chooses the efficient level of education whenever the solution is unique. A sufficient condition for uniqueness is that $Y_{h'}(l', h', \theta')$ is strictly decreasing in h', while $g_{h'}(h', h)$ is weakly increasing in h', as would hold under diminishing returns to human capital and convex education costs. In that case, the two sides of (53)–(54) cross at most once, implying a unique education choice.

The continuum version used in the subsequent analysis follows the same logic: the realized marginal product $Y_{h'}(l', h', \theta')$ is replaced by the marginal-utility-weighted expected marginal product $\overline{Y}_{h'}$.

This proves Proposition 1.

B Ability Risk

This appendix derives the parent's education condition under ability risk and proves the risk-wedge representation used in the main text. The key object is the marginal-utility-weighted expected return to education. Because education is chosen before the child's ability is realized, this ex ante object generally differs from the realized marginal return.

As in the benchmark analysis, the child's labor input is treated as fixed in this section in order to isolate the effect of ability risk. Throughout this appendix, $l'(\theta')$ denotes the child's labor input, which is treated as fixed when differentiating with respect to educational investment. In the private-information appendix, the same role is played by the labor requirement mapping $\ell(y; h', \theta')$, which is induced by an assigned output level.

The analysis is based on a resource-based benchmark that compares marginal returns to the marginal resource cost of education.

B.1 Independent Ability

Suppose the child's ability is drawn independently of parental ability:

$$\theta' \sim F.$$

Parents choose financial bequests b' and education h' before θ' is realized. Future consumption is given by

$$c'(\theta') = (1+r)b' + Y(l'(\theta'), h', \theta').$$

The parent's expected objective is

$$u(c) - v(l) + \beta\mathbb{E}[u(c'(\theta')) - v(l'(\theta'))] .$$

The Lagrangian is

$$\mathcal{L} = u(c) - v(l) + \beta\mathbb{E}[u(c'(\theta')) - v(l'(\theta'))] + \lambda\Big[(1+r)b + Y(l, h, \theta) - c - b' - g(h', h)\Big]. \tag{55}$$

We focus on an interior solution for the bequest and education margins. If either margin is at a corner, the corresponding first-order condition should be interpreted as a Kuhn–Tucker condition.

Define

$$\bar{u}' \equiv \mathbb{E}[u'(c'(\theta'))] . \tag{56}$$

Since $u' > 0$, we have $\bar{u}' > 0$.

The first-order condition for financial bequests is

$$\lambda = \beta(1+r)\bar{u}'. \tag{57}$$

The first-order condition for education is

$$\beta\mathbb{E}[u'(c'(\theta'))Y_{h'}(l'(\theta'), h', \theta')] = \lambda g_{h'}(h', h). \tag{58}$$

Substituting (57) into (58) gives

$$\mathbb{E}[u'(c'(\theta'))Y_{h'}(l'(\theta'), h', \theta')] = (1+r)\bar{u}' g_{h'}(h', h).$$

Dividing by $\bar{u}' > 0$ yields

$$\overline{Y}_{h'} = (1+r)g_{h'}(h', h), \tag{59}$$

where

$$\overline{Y}_{h'} \equiv \frac{\mathbb{E}[u'(c'(\theta'))Y_{h'}(l'(\theta'), h', \theta')]}{\mathbb{E}[u'(c'(\theta'))]}. \tag{60}$$

The object $\overline{Y}_{h'}$ is the ex ante marginal return to education weighted by marginal utility. In contrast to financial bequests, which yield the risk-free intertemporal return, the return to education depends on realized ability and is therefore uncertain. Equation (59) shows that parents choose education by equating this marginal-utility-weighted expected return to the return on financial wealth, evaluated at the marginal resource cost of education.

B.2 Proof of Proposition 2

Let

$$Y_{h'}(\theta') \equiv Y_{h'}(l'(\theta'), h', \theta')$$

to simplify notation. The type-specific risk wedge is

$$Y_{h'}(\theta') - \overline{Y}_{h'}. \tag{61}$$

It measures the difference between the realized marginal return to education for type θ' and the ex ante marginal-utility-weighted return that governs the parent's choice.

Using (59), the same object can be written at the optimum as

$$Y_{h'}(\theta') - (1+r)g_{h'}(h', h),$$

because $\overline{Y}_{h'} = (1+r)g_{h'}(h', h)$. Thus, the risk wedge can be interpreted either as the gap between realized and ex ante marginal returns or, at the optimum, as the gap between the realized marginal return and the resource cost of education evaluated at the return on financial wealth.

Taking expectations across types gives

$$\mathbb{E}\big[Y_{h'}(\theta') - \overline{Y}_{h'}\big] = \mathbb{E}[Y_{h'}(\theta')] - \overline{Y}_{h'}. \tag{62}$$

By definition,

$$\overline{Y}_{h'} = \frac{\mathbb{E}[u'(c'(\theta'))Y_{h'}(\theta')]}{\mathbb{E}[u'(c'(\theta'))]}.$$

Using the identity

$$\mathbb{E}[u'(c'(\theta'))Y_{h'}(\theta')] = \mathbb{E}[u'(c'(\theta'))]\,\mathbb{E}[Y_{h'}(\theta')] + \mathrm{Cov}(u'(c'(\theta')), Y_{h'}(\theta'))\,,$$

we obtain

$$\overline{Y}_{h'} = \mathbb{E}[Y_{h'}(\theta')] + \frac{\mathrm{Cov}(u'(c'(\theta')), Y_{h'}(\theta'))}{\mathbb{E}[u'(c'(\theta'))]}.$$

Substituting this expression into (62) yields

$$\mathbb{E}\big[Y_{h'}(\theta') - \overline{Y}_{h'}\big] = -\frac{\mathrm{Cov}(u'(c'(\theta')), Y_{h'}(\theta'))}{\mathbb{E}[u'(c'(\theta'))]}. \tag{63}$$

Equation (63) is the covariance representation of the average risk wedge. To sign this expression, suppose that $Y_{h'\theta'} > 0$, so that higher ability raises the marginal return to education, and that the allocation is monotone, so that $c'(\theta')$ is increasing in θ'. Since u is concave, higher consumption implies lower marginal utility. Hence

$$\mathrm{Cov}(u'(c'(\theta')), Y_{h'}(\theta')) < 0,$$

and the average risk wedge is positive.

This proves Proposition 2.

B.3 Intergenerational Ability Persistence

Suppose ability evolves according to

$$\theta' \sim F(\theta' \mid \theta),$$

so that parental ability is informative about the distribution of child ability. The parent conditions choices on θ. Define

$$\bar{u}'(\theta) \equiv \mathbb{E}[u'(c'(\theta')) \mid \theta]\,. \tag{64}$$

The same argument as in Section B.1 gives the conditional education condition

$$\overline{Y}_{h'}(\theta) = (1+r)g_{h'}(h', h), \tag{65}$$

where

$$\overline{Y}_{h'}(\theta) \equiv \frac{\mathbb{E}[u'(c'(\theta'))Y_{h'}(l'(\theta'), h', \theta') \mid \theta]}{\mathbb{E}[u'(c'(\theta')) \mid \theta]}. \tag{66}$$

Thus, ability persistence makes the ex ante return to education dynasty-specific. Differences in parental ability affect education through the conditional distribution $F(\theta' \mid \theta)$ and through the marginal-utility weights induced by the allocation. This heterogeneity reflects differences in expected returns and does not arise from incentive considerations.

B.4 Proof of Proposition 3

This result is conditional on the marginal-utility-weighted expected return to education being increasing in parental ability. To compare types at a common education level, write $\overline{Y}_{h'}(\theta, h')$ for the conditional weighted return at education level h', so that

$$\overline{Y}_{h'}(\theta) = \overline{Y}_{h'}(\theta, h'(\theta))$$

at the optimum.

Formally, suppose that $\overline{Y}_{h'}(\theta, h')$ is increasing in θ. A sufficient condition is that $F(\theta' \mid \theta)$ shifts to the right with θ, $Y_{h'}(l', h', \theta')$ is increasing in θ', and the induced marginal-utility weights do not overturn this monotonicity.

Let $h'(\theta)$ denote the education level chosen by a parent of type θ. It is characterized by

$$\overline{Y}_{h'}(\theta, h'(\theta)) = (1 + r)g_{h'}(h'(\theta), h). \tag{67}$$

Suppose that $\overline{Y}_{h'}(\theta, h')$ is strictly decreasing in h', while $g_{h'}(h', h)$ is weakly increasing in h'. Let $\theta_2 > \theta_1$. Since $\overline{Y}_{h'}(\theta, h')$ is increasing in θ, we have

$$\overline{Y}_{h'}(\theta_2, h'(\theta_1)) > \overline{Y}_{h'}(\theta_1, h'(\theta_1)) = (1 + r)g_{h'}(h'(\theta_1), h).$$

At the education level chosen by type θ_1, the marginal expected return for type θ_2 exceeds the marginal resource cost. Since $\overline{Y}_{h'}(\theta, h')$ is strictly decreasing in h' and $g_{h'}(h', h)$ is weakly increasing in h', restoring equality in (67) requires a weakly higher education level:

$$h'(\theta_2) \geq h'(\theta_1).$$

If the monotonicity in θ is strict and the solution is interior and unique, then

$$h'(\theta_2) > h'(\theta_1).$$

This proves Proposition 3.

C Private Information and Direct Informational Asymmetry

This appendix provides technical details for Section 4. It proceeds in four steps. First, it states the regularity and implementability assumptions used in the private-information characterization. Second, it presents a two-type benchmark that illustrates how educational investment enters incentive constraints through the labor requirement mapping. Third, it proves Theorem 1. Finally, it proves Corollary 1.

C.1 Regularity and Implementability Assumptions

We maintain the following assumptions throughout this appendix.

Assumption 1 (Preferences and technology)**.** *The utility function u is twice continuously differentiable, strictly increasing, and strictly concave. The disutility of labor function v is twice continuously differentiable, strictly increasing, and strictly convex. The production function $Y(l, h, \theta)$ is twice continuously differentiable, satisfies $Y_l(l, h, \theta) > 0$, and is weakly increasing in human capital h. The education cost function $g(h', h)$ is twice continuously differentiable, increasing, and convex in h'.*

Assumption 2 (Labor requirement mapping)**.** *For every feasible tuple (y, h', θ'), there exists a unique labor requirement $\ell(y; h', \theta')$ satisfying*

$$Y\big(\ell(y; h', \theta'), h', \theta'\big) = y.$$

Moreover, $\ell(y; h', \theta')$ is continuously differentiable in all arguments, with

$$\ell_y(y; h', \theta') > 0, \qquad \ell_{\theta'}(y; h', \theta') < 0.$$

Thus, higher ability reduces the labor required to produce a given output level.

Assumption 3 (Implementability)**.** *The planner's problem admits an interior solution for the bequest and education margins. Under private information, the Spence–Mirrlees single-crossing condition holds, and the optimal allocation is monotone. Hence the first-order approach is valid, so that global incentive compatibility can be represented by the associated local incentive constraints.*

Assumption 2 follows from the implicit function theorem whenever $Y_l(l, h, \theta) > 0$. Indeed, differentiating

$$Y\big(\ell(y; h', \theta'), h', \theta'\big) = y$$

with respect to h', holding y fixed, gives

$$\ell_{h'}(y; h', \theta') = -\frac{Y_{h'}\big(\ell(y; h', \theta'), h', \theta'\big)}{Y_l\big(\ell(y; h', \theta'), h', \theta'\big)}. \tag{68}$$

Thus, if education raises productivity, $Y_{h'} > 0$, then $\ell_{h'} < 0$: higher education lowers the labor required to implement any given output assignment.

C.2 A Two-Type Illustration

This subsection presents a simple two-type example that illustrates the mechanism behind the direct informational asymmetry. The example is not used to establish the sign of the continuum wedge. Its purpose is only to show why education enters incentive constraints through the productivity channel, whereas financial bequests do not.

Suppose child ability takes two values $i \in \{L, H\}$, with $\theta_H > \theta_L > 0$, and probabilities π_H and $\pi_L = 1 - \pi_H$. Let second-period output for type i be y'_i. For this illustration, suppose output is produced according to

$$Y'_i(l'_i, h') = \theta_i^{\xi}(h')^{1-\xi} l'_i, \qquad \xi \in (0, 1).$$

The labor requirement associated with output y'_i is therefore

$$\ell_i(y'_i; h') = \frac{y'_i}{\theta_i^{\xi}(h')^{1-\xi}}. \tag{69}$$

Differentiating gives

$$\ell_{i,h'}(y'_i; h') = -\frac{1-\xi}{h'}\ell_i(y'_i; h') < 0. \tag{70}$$

Thus, in this example, higher education reduces the labor required to produce any assigned output level.

Under private information, the planner offers bundles (c'_L, y'_L) and (c'_H, y'_H). The high-type incentive constraint is

$$u(c'_H) - v\big(\ell_H(y'_H; h')\big) \geq u(c'_L) - v\big(\ell_H(y'_L; h')\big). \tag{71}$$

The labor required for the high type to mimic the low-type allocation is

$$\widehat{\ell}_L^H = \ell_H(y'_L; h') = \frac{y'_L}{\theta_H^{\xi}(h')^{1-\xi}}.$$

Define the high-type incentive slack by

$$\Phi(h') = u(c'_H) - v\big(\ell_H(y'_H; h')\big) - u(c'_L) + v\big(\widehat{\ell}_L^H\big).$$

Differentiating the slack with respect to h', holding the assigned bundles (c'_L, y'_L) and (c'_H, y'_H) fixed, gives

$$\frac{\partial \Phi}{\partial h'} = -v'\big(\ell_H(y'_H; h')\big)\ell_{H,h'}(y'_H; h') + v'\big(\widehat{\ell}_L^H\big)\frac{\partial \widehat{\ell}_L^H}{\partial h'}. \tag{72}$$

Since both labor derivatives are negative, the first term in (72) is positive and the second term is negative. The first term reflects the fact that education reduces the labor cost of truthful implementation for the high type. The second reflects the fact that education also reduces the labor cost of mimicking the low-type allocation. Hence the net effect of education on a particular two-type incentive slack is generally ambiguous and depends on the relative strength of these two channels.

The important point for the main text is not the sign of $\partial\Phi/\partial h'$ in this two-type illustration. The important point is that education enters the incentive constraint through the labor requirement mapping. Financial bequests, by contrast, shift resources through the budget constraint but do not alter $\ell_i(y; h')$. Thus, financial bequests generate no direct productivity-based term analogous to the one generated by education.

C.3 Proof of Theorem 1

Proof. By the revelation principle, it is without loss of generality to restrict attention to direct mechanisms

$$\{c'(\theta'), y'(\theta')\}_{\theta' \in [\underline{\theta}, \bar{\theta}]}.$$

For a child of true ability θ' who reports $\hat{\theta}'$, utility is

$$V(\theta', \hat{\theta}') = u\big(c'(\hat{\theta}')\big) - v\big(\ell(y'(\hat{\theta}'); h', \theta')\big),$$

where $\ell(y; h', \theta')$ is implicitly defined by

$$Y\big(\ell(y; h', \theta'), h', \theta'\big) = y.$$

Truthful implementation requires

$$V(\theta', \theta') \geq V(\theta', \hat{\theta}') \qquad \text{for all } \theta', \hat{\theta}'.$$

Define truthful utility by

$$U(\theta') = V(\theta', \theta') = u\big(c'(\theta')\big) - v\big(\ell(y'(\theta'); h', \theta')\big).$$

Under Assumption 3, the first-order approach is valid. The local incentive constraints can therefore be represented by the associated envelope condition and monotonicity restriction. The envelope theorem implies

$$U'(\theta') = -v'\big(\ell(y'(\theta'); h', \theta')\big)\ell_{\theta'}\big(y'(\theta'); h', \theta'\big) \geq 0, \tag{73}$$

since $\ell_{\theta'} < 0$ and $v' > 0$.

Let $\mu(\theta') \geq 0$ denote the multiplier on the local incentive constraint for type θ', and let λ denote the multiplier on the resource constraint.

The parent budget constraint is

$$c + b' + g(h', h) = (1 + r)b + Y(l, h, \theta),$$

and child consumption satisfies

$$c'(\theta') = (1 + r)b' + y'(\theta').$$

We now derive the first-order conditions for b' and h'. A marginal increase in b' raises child consumption uniformly through

$$\frac{\partial c'(\theta')}{\partial b'} = 1 + r.$$

Financial bequests do not enter the production function and do not affect the labor requirement mapping

$$\ell(y; h', \theta').$$

Therefore differentiating the Lagrangian with respect to b' generates no direct productivity-based informational term. The bequest condition is

$$\lambda = \beta(1 + r) \int_{\underline{\theta}}^{\overline{\theta}} u'\big(c'(\theta')\big) f(\theta')\, d\theta'. \tag{74}$$

Educational investment differs because it changes the labor requirement associated with assigned output levels. The direct incentive channel is

$$h' \longrightarrow \ell(y'(\theta'); h', \theta') \longrightarrow v\big(\ell(y'(\theta'); h', \theta')\big) \longrightarrow \text{local incentive constraints.}$$

Differentiating the Lagrangian with respect to h', treating $(c'(\theta'), y'(\theta'))$ as separate control variables in the standard envelope step, yields

$$\beta \int_{\underline{\theta}}^{\overline{\theta}} u'\big(c'(\theta')\big) Y_{h'}\big(\ell(y'(\theta'); h', \theta'), h', \theta'\big) f(\theta')\, d\theta' = \lambda g_{h'}(h', h) + R_{h'}, \tag{75}$$

where

$$R_{h'} = -\int_{\underline{\theta}}^{\overline{\theta}} \mu(\theta') \frac{\partial}{\partial h'} \left[u\big(c'(\theta')\big) - v\big(\ell(y'(\theta'); h', \theta')\big)\right] f(\theta')\, d\theta'. \tag{76}$$

The sign of $R_{h'}$ follows from the labor requirement derivative. Since $c'(\theta')$ and $y'(\theta')$ are treated as separate controls in the envelope step,

$$\frac{\partial}{\partial h'}\left[u\big(c'(\theta')\big) - v\big(\ell(y'(\theta'); h', \theta')\big)\right] = -v'\big(\ell(y'(\theta'); h', \theta')\big)\ell_{h'}(y'(\theta'); h', \theta').$$

Define

$$M_{h'}(\theta') \equiv -\frac{\partial}{\partial h'}\left[u\big(c'(\theta')\big) - v\big(\ell(y'(\theta'); h', \theta')\big)\right].$$

Then

$$M_{h'}(\theta') = v'\big(\ell(y'(\theta'); h', \theta')\big)\ell_{h'}(y'(\theta'); h', \theta'), \tag{77}$$

and hence

$$R_{h'} = \int_{\underline{\theta}}^{\overline{\theta}} \mu(\theta') M_{h'}(\theta') f(\theta')\, d\theta'. \tag{78}$$

By the implicit function theorem,

$$\ell_{h'}(y; h', \theta') = -\frac{Y_{h'}\big(\ell(y; h', \theta'), h', \theta'\big)}{Y_l\big(\ell(y; h', \theta'), h', \theta'\big)}.$$

If $Y_{h'} > 0$ and $Y_l > 0$, then

$$\ell_{h'}(y; h', \theta') < 0.$$

Since $v' > 0$, it follows from (77) that

$$M_{h'}(\theta') < 0.$$

Therefore, because $\mu(\theta') \geq 0$,

$$R_{h'} \leq 0,$$

with strict inequality whenever incentive constraints bind on a set of positive measure. Thus, in the baseline labor-requirement representation, productive education relaxes this dimension of incentive provision under the sign convention adopted in the main text.

Equations (74)–(78) establish the direct informational asymmetry. Financial bequests enter the planner's problem through the resource constraint and generate no direct productivity-based informational term. Educational investment enters the labor requirement mapping and generates the term $R_{h'}$. This proves Theorem 1. □

C.4 Proof of Corollary 1

Proof. Starting from the education condition (75), divide both sides by

$$\beta \int_{\underline{\theta}}^{\overline{\theta}} u'\big(c'(\theta')\big) f(\theta')\, d\theta'.$$

The denominator is strictly positive because $\beta > 0$, $u' > 0$, and f is a density.

Using the bequest condition (74), we have

$$\frac{\lambda g_{h'}(h', h)}{\beta \int_{\underline{\theta}}^{\overline{\theta}} u'\big(c'(\theta')\big) f(\theta')\, d\theta'} = (1+r) g_{h'}(h', h).$$

Define

$$\overline{Y}_{h'} = \frac{\int_{\underline{\theta}}^{\overline{\theta}} u'\big(c'(\theta')\big) Y_{h'}\big(\ell(y'(\theta'); h', \theta'), h', \theta'\big) f(\theta')\, d\theta'}{\int_{\underline{\theta}}^{\overline{\theta}} u'\big(c'(\theta')\big) f(\theta')\, d\theta'}.$$

Then the education condition becomes

$$\overline{Y}_{h'} = (1+r) g_{h'}(h', h) + \Delta_{h'}, \tag{79}$$

where

$$\Delta_{h'} = \frac{R_{h'}}{\beta \int_{\underline{\theta}}^{\overline{\theta}} u'\big(c'(\theta')\big) f(\theta')\, d\theta'}. \tag{80}$$

This substitution uses the bequest optimality condition to express the resource multiplier in terms of the marginal utility value of financial transfers. The wedge $\Delta_{h'}$ is therefore measured relative to the bequest margin.

Under full information, $\mu(\theta') \equiv 0$, so $R_{h'} = 0$ and $\Delta_{h'} = 0$. The condition then reduces to the full-information benchmark

$$\overline{Y}_{h'} = (1+r) g_{h'}(h', h).$$

This proves Corollary 1. □

D Education Wedge Decomposition

This appendix provides the formal derivation of Theorem 2. The decomposition separates the wedge between the child's ex post return-based valuation of education and the return on financial wealth into three components: a risk component, a technological component, and an informational component.

Throughout this appendix, write

$$Y_{h'}(\theta') \equiv Y_{h'}\big(\ell(y'(\theta'); h', \theta'), h', \theta'\big),$$

where $\ell(y'(\theta'); h', \theta')$ is the labor input associated with the assigned output $y'(\theta')$. Equivalently, this labor input may be denoted by $l'(\theta')$ at the implemented allocation.

D.1 Proof of Theorem 2

Proof. The child's return-based benchmark compares the realized marginal return to education with the return on financial wealth:

$$Y_{h'}(\theta') = 1 + r. \tag{81}$$

This benchmark is not an actual choice condition. It is a return-based reference point used to measure the wedge between education and financial wealth after ability has been realized.

Under private information, the planner's education condition from Corollary 1 is

$$\overline{Y}_{h'} = (1 + r)g_{h'}(h', h) + \Delta_{h'}, \tag{82}$$

where

$$\overline{Y}_{h'} = \frac{\int_{\underline{\theta}}^{\overline{\theta}} u'\big(c'(\theta')\big) Y_{h'}(\theta') f(\theta')\, d\theta'}{\int_{\underline{\theta}}^{\overline{\theta}} u'\big(c'(\theta')\big) f(\theta')\, d\theta'}$$

is the marginal-utility-weighted expected marginal return to education, and $\Delta_{h'}$ is the informational wedge derived in Corollary 1.

Starting from the realized education wedge, add and subtract $\overline{Y}_{h'}$:

$$Y_{h'}(\theta') - (1 + r) = \Big(Y_{h'}(\theta') - \overline{Y}_{h'}\Big) + \Big(\overline{Y}_{h'} - (1 + r)\Big). \tag{83}$$

Using (82),

$$\overline{Y}_{h'} - (1 + r) = (1 + r)g_{h'}(h', h) + \Delta_{h'} - (1 + r),$$

and therefore

$$\overline{Y}_{h'} - (1 + r) = (1 + r)\big(g_{h'}(h', h) - 1\big) + \Delta_{h'}.$$

Substituting this expression into (83) yields

$$Y_{h'}(\theta') - (1+r) = \underbrace{\Big(Y_{h'}(\theta') - \overline{Y}_{h'}\Big)}_{\text{risk wedge}} + \underbrace{(1+r)\big(g_{h'}(h',h) - 1\big)}_{\text{technological wedge}} + \underbrace{\Delta_{h'}}_{\text{informational wedge}}. \tag{84}$$

This proves the decomposition in (26).

We now prove part (i). The risk wedge is type-specific:

$$Y_{h'}(\theta') - \overline{Y}_{h'}.$$

Taking expectations across types gives

$$\mathbb{E}\big[Y_{h'}(\theta') - \overline{Y}_{h'}\big] = \mathbb{E}[Y_{h'}(\theta')] - \overline{Y}_{h'}. \tag{85}$$

The equality follows because $\overline{Y}_{h'}$ is constant across types.

Let

$$m(\theta') \equiv u'\big(c'(\theta')\big).$$

By definition,

$$\overline{Y}_{h'} = \frac{\mathbb{E}[m(\theta')Y_{h'}(\theta')]}{\mathbb{E}[m(\theta')]}.$$

Using the covariance identity

$$\mathbb{E}[m(\theta')Y_{h'}(\theta')] = \mathbb{E}[m(\theta')]\mathbb{E}[Y_{h'}(\theta')] + \mathrm{Cov}(m(\theta'), Y_{h'}(\theta')),$$

we obtain

$$\overline{Y}_{h'} = \mathbb{E}[Y_{h'}(\theta')] + \frac{\mathrm{Cov}(m(\theta'), Y_{h'}(\theta'))}{\mathbb{E}[m(\theta')]}.$$

Substituting this expression into (85) gives

$$\mathbb{E}\big[Y_{h'}(\theta') - \overline{Y}_{h'}\big] = -\frac{\mathrm{Cov}\big(u'\big(c'(\theta')\big), Y_{h'}(\theta')\big)}{\mathbb{E}\big[u'\big(c'(\theta')\big)\big]}. \tag{86}$$

This proves the covariance representation in part (i). Under monotone allocations with $Y_{h'\theta'} > 0$, higher ability raises the marginal return to education. If higher-ability children also receive higher consumption, then concavity of

u implies that marginal utility is lower in high-return states. The covariance term is then negative, so the average risk wedge is positive.

Part (ii) follows from the definition of $\Delta_{h'}$ in Corollary 1, the sign characterization in Theorem 1, and the endogenous-labor decomposition in Section 6. In the baseline labor-requirement representation, Appendix C shows that productive education implies $R_{h'} \leq 0$. Since

$$\Delta_{h'} = \frac{R_{h'}}{\beta \int_{\underline{\theta}}^{\overline{\theta}} u'\big(c'(\theta')\big) f(\theta')\, d\theta'},$$

and the denominator is strictly positive, it follows that $\Delta_{h'} \leq 0$ in the baseline environment.

With endogenous labor supply, the total informational wedge includes an additional productivity-rent component governed by

$$\int_{\Theta} \omega(\theta') u'\big(c'(\theta')\big) Y_{\theta' h'}(\theta') f(\theta')\, d\theta',$$

where $\omega(\theta')$ is the rent-weighting term defined in Section 6. The formal derivation of this component and the conditions under which it dominates the baseline labor-requirement component are provided in Appendix E. If this component is positive and sufficiently large, the sign of the total informational wedge is reversed.

This proves Theorem 2. □

D.2 Benchmark Dependence of the Technological Wedge

The technological wedge in (84) is

$$(1+r)\big(g_{h'}(h', h) - 1\big).$$

It appears because the child's benchmark in (81) compares the realized marginal return to education with the return on financial wealth on a return basis, whereas the planner evaluates education net of its marginal resource cost $g_{h'}(h', h)$.

If instead the benchmark is defined on a resource-cost basis as

$$Y_{h'}(\theta') = (1+r) g_{h'}(h', h),$$

then the education wedge becomes

$$Y_{h'}(\theta') - (1+r)g_{h'}(h', h).$$

Adding and subtracting $\overline{Y}_{h'}$ gives

$$Y_{h'}(\theta') - (1+r)g_{h'}(h', h) = \Big(Y_{h'}(\theta') - \overline{Y}_{h'}\Big) + \Big(\overline{Y}_{h'} - (1+r)g_{h'}(h', h)\Big). \tag{87}$$

Using the private-information condition (82),

$$\overline{Y}_{h'} - (1+r)g_{h'}(h', h) = \Delta_{h'}.$$

Therefore, under the resource-cost benchmark,

$$Y_{h'}(\theta') - (1+r)g_{h'}(h', h) = \underbrace{\Big(Y_{h'}(\theta') - \overline{Y}_{h'}\Big)}_{\text{risk wedge}} + \underbrace{\Delta_{h'}}_{\text{informational wedge}}. \tag{88}$$

The technological wedge therefore reflects the difference between a return-based benchmark and a resource-cost benchmark. It is not an additional informational distortion.

D.3 Informational Wedge and Relation to Endogenous Labor

The informational wedge $\Delta_{h'}$ is derived in Corollary 1. In the baseline labor-requirement representation of Section 4, it is given by

$$\Delta_{h'} = \frac{R_{h'}}{\beta \int_{\underline{\theta}}^{\overline{\theta}} u'\big(c'(\theta')\big) f(\theta')\, d\theta'}.$$

Since the denominator is strictly positive, the sign of $\Delta_{h'}$ is the sign of $R_{h'}$. Appendix C shows that, under $Y_{h'} > 0$, $Y_l > 0$, and the sign convention adopted in the main text,

$$R_{h'} \leq 0, \qquad \Delta_{h'} \leq 0.$$

Thus, in the baseline labor-requirement representation, productive education relaxes one dimension of incentive provision.

This conclusion is specific to the baseline environment. With endogenous labor supply, Section 6 shows that education also affects the slope of information rents through the productivity-rent component governed by $Y_{\theta' h'}$. The formal derivation of this component and the conditions under which it dominates the baseline labor-requirement component are provided in Appendix E. When this productivity-rent component is positive and sufficiently large, it may offset or reverse the baseline sign of the informational wedge.

E Endogenous Labor Supply

This appendix provides the formal derivations underlying Section 6. It first proves the envelope property with endogenous labor supply. It then derives the reduced-form informational term using the envelope representation of information rents and shows how this term decomposes differently for educational investment and financial bequests. Finally, it proves the sign characterization of the direct productivity-rent component and the conditional reversal result.

E.1 Proof of Lemma 2

Given (b', h', θ'), the child chooses labor supply l', with consumption determined by

$$c' = (1 + r)b' + Y(l', h', \theta'). \tag{89}$$

Equivalently, the child solves

$$\max_{c', l'} \; u(c') - v(l')$$

subject to (89). The Lagrangian is

$$\mathcal{L} = u(c') - v(l') + \lambda\Big[(1 + r)b' + Y(l', h', \theta') - c'\Big].$$

The first-order conditions are

$$u'(c') = \lambda, \qquad v'(l') = \lambda Y_l(l', h', \theta').$$

Eliminating λ gives the labor-supply condition

$$u'(c')Y_l(l', h', \theta') = v'(l'). \tag{90}$$

Let indirect utility for type θ' be

$$U(\theta') = u(c'(\theta')) - v(l'(\theta')).$$

Differentiating indirect utility with respect to educational investment h' gives

$$\frac{\partial U(\theta')}{\partial h'} = u'\big(c'(\theta')\big)\frac{\partial c'(\theta')}{\partial h'} - v'\big(l'(\theta')\big)\frac{\partial l'(\theta')}{\partial h'}.$$

From the budget constraint,

$$\frac{\partial c'(\theta')}{\partial h'} = Y_{h'}\big(l'(\theta'), h', \theta'\big) + Y_l\big(l'(\theta'), h', \theta'\big)\frac{\partial l'(\theta')}{\partial h'}.$$

Substituting this expression into the derivative of indirect utility yields

$$\begin{aligned}\frac{\partial U(\theta')}{\partial h'} = u'\big(c'(\theta')\big)&\left[Y_{h'}\big(l'(\theta'), h', \theta'\big) + Y_l\big(l'(\theta'), h', \theta'\big)\frac{\partial l'(\theta')}{\partial h'}\right]\\ &- v'\big(l'(\theta')\big)\frac{\partial l'(\theta')}{\partial h'}.\end{aligned}$$

Using the labor-supply condition (90), the terms involving $\partial l'(\theta')/\partial h'$ cancel. Hence

$$\frac{\partial U(\theta')}{\partial h'} = u'\big(c'(\theta')\big)Y_{h'}\big(l'(\theta'), h', \theta'\big).$$

This proves Lemma 2.

E.2 Envelope Condition for Information Rents

Let $U(\theta')$ denote indirect utility under truthful reporting. Under the first-order approach, incentive compatibility implies an envelope condition for information rents. We derive it explicitly.

For a truthful type θ', indirect utility is

$$U(\theta') = u\big(c'(\theta')\big) - v\big(l'(\theta')\big),$$

where

$$c'(\theta') = (1+r)b' + Y\big(l'(\theta'), h', \theta'\big).$$

Differentiating $U(\theta')$ with respect to θ' gives

$$U'(\theta') = u'\big(c'(\theta')\big)\frac{dc'(\theta')}{d\theta'} - v'\big(l'(\theta')\big)\frac{dl'(\theta')}{d\theta'}.$$

Using the budget constraint,

$$\frac{dc'(\theta')}{d\theta'} = Y_l\big(l'(\theta'), h', \theta'\big)\frac{dl'(\theta')}{d\theta'} + Y_{\theta'}\big(l'(\theta'), h', \theta'\big).$$

Therefore,

$$\begin{aligned} U'(\theta') = u'\big(c'(\theta')\big) &\left[Y_l\big(l'(\theta'), h', \theta'\big)\frac{dl'(\theta')}{d\theta'} + Y_{\theta'}\big(l'(\theta'), h', \theta'\big)\right] \\ &- v'\big(l'(\theta')\big)\frac{dl'(\theta')}{d\theta'}. \end{aligned}$$

By the labor-supply condition (90),

$$v'\big(l'(\theta')\big) = u'\big(c'(\theta')\big)Y_l\big(l'(\theta'), h', \theta'\big).$$

Thus the terms involving $dl'(\theta')/d\theta'$ cancel, yielding

$$U'(\theta') = u'\big(c'(\theta')\big)Y_{\theta'}\big(l'(\theta'), h', \theta'\big). \tag{91}$$

This is the rent-slope equation used in Section 6.

E.3 Reduced-Form Informational Term

Let $x \in \{h', b'\}$ denote an intergenerational transfer instrument. The effect of x on incentive provision can be expressed using the envelope representation of information rents.

From (91),

$$U'(\theta') = u'\big(c'(\theta')\big)Y_{\theta'}\big(l'(\theta'), h', \theta'\big).$$

Integrating the rent-slope equation gives

$$U(\theta') = U(\underline{\theta}) + \int_{\underline{\theta}}^{\theta'} U'(s)\, ds.$$

Substituting this representation into expected utility gives

$$\int_{\underline{\theta}}^{\overline{\theta}} U(\theta')f(\theta')\, d\theta' = U(\underline{\theta}) + \int_{\underline{\theta}}^{\overline{\theta}} \left[\int_{\underline{\theta}}^{\theta'} U'(s)\, ds\right] f(\theta')\, d\theta'.$$

Changing the order of integration yields

$$\int_{\underline{\theta}}^{\overline{\theta}} \left[\int_{\underline{\theta}}^{\theta'} U'(s)\, ds \right] f(\theta')\, d\theta' = \int_{\underline{\theta}}^{\overline{\theta}} \big(1 - F(s)\big) U'(s)\, ds.$$

Renaming s as θ', this can be written as

$$\int_{\underline{\theta}}^{\overline{\theta}} \big(1 - F(\theta')\big) U'(\theta')\, d\theta' = \int_{\underline{\theta}}^{\overline{\theta}} \omega(\theta') U'(\theta') f(\theta')\, d\theta',$$

where

$$\omega(\theta') = \frac{1 - F(\theta')}{f(\theta')}. \tag{92}$$

Since $F(\theta') \leq 1$ and $f(\theta') > 0$ on the support,

$$\omega(\theta') \geq 0.$$

Differentiating the rent term with respect to the instrument x gives the reduced-form informational effect

$$\widetilde{R}_x = \int_{\underline{\theta}}^{\overline{\theta}} \omega(\theta') U_{\theta' x}(\theta') f(\theta')\, d\theta'. \tag{93}$$

Using (91), this becomes

$$\widetilde{R}_x = \int_{\underline{\theta}}^{\overline{\theta}} \omega(\theta') \frac{\partial}{\partial x} \left[u'\big(c'(\theta')\big) Y_{\theta'}\big(l'(\theta'), h', \theta'\big) \right] f(\theta')\, d\theta'.$$

E.4 Proof of Theorem 3

We now decompose $U_{\theta' x}(\theta')$ for each instrument $x \in \{h', b'\}$. From the rent-slope equation (91),

$$U_{\theta'}(\theta') = u'\big(c'(\theta')\big) Y_{\theta'}\big(l'(\theta'), h', \theta'\big).$$

Differentiating with respect to x gives

$$\begin{aligned} U_{\theta' x}(\theta') &= u''\big(c'(\theta')\big) c'_x(\theta') Y_{\theta'}\big(l'(\theta'), h', \theta'\big) \\ &\quad + u'\big(c'(\theta')\big) \frac{\partial}{\partial x} Y_{\theta'}\big(l'(\theta'), h', \theta'\big). \end{aligned} \tag{94}$$

Substituting (94) into (93) yields the general representation of the informational effect.

For educational investment $x = h'$, the production function depends directly on h'. Hence

$$\frac{\partial}{\partial h'} Y_{\theta'}\big(l'(\theta'), h', \theta'\big) = Y_{\theta' h'}\big(l'(\theta'), h', \theta'\big) + Y_{\theta' l}\big(l'(\theta'), h', \theta'\big) l'_{h'}(\theta').$$

Therefore,

$$\begin{aligned} U_{\theta' h'}(\theta') = {} & u''\big(c'(\theta')\big) c'_{h'}(\theta') Y_{\theta'}\big(l'(\theta'), h', \theta'\big) \\ & + u'\big(c'(\theta')\big) Y_{\theta' h'}\big(l'(\theta'), h', \theta'\big) \\ & + u'\big(c'(\theta')\big) Y_{\theta' l}\big(l'(\theta'), h', \theta'\big) l'_{h'}(\theta'). \end{aligned} \tag{95}$$

Substituting (95) into (93) gives

$$\widetilde{R}_{h'} = R^{\mathrm{MU}}_{h'} + R^{\mathrm{prod}}_{h'} + R^{\mathrm{labor}}_{h'}, \tag{96}$$

where

$$R^{\mathrm{MU}}_{h'} = \int_{\underline{\theta}}^{\overline{\theta}} \omega(\theta') u''\big(c'(\theta')\big) c'_{h'}(\theta') Y_{\theta'}\big(l'(\theta'), h', \theta'\big) f(\theta')\, d\theta', \tag{97}$$

$$R^{\mathrm{prod}}_{h'} = \int_{\underline{\theta}}^{\overline{\theta}} \omega(\theta') u'\big(c'(\theta')\big) Y_{\theta' h'}\big(l'(\theta'), h', \theta'\big) f(\theta')\, d\theta', \tag{98}$$

$$R^{\mathrm{labor}}_{h'} = \int_{\underline{\theta}}^{\overline{\theta}} \omega(\theta') u'\big(c'(\theta')\big) Y_{\theta' l}\big(l'(\theta'), h', \theta'\big) l'_{h'}(\theta') f(\theta')\, d\theta'. \tag{99}$$

For financial bequests $x = b'$, the production function does not depend directly on b'. Thus

$$\frac{\partial}{\partial b'} Y_{\theta'}\big(l'(\theta'), h', \theta'\big) = Y_{\theta' l}\big(l'(\theta'), h', \theta'\big) l'_{b'}(\theta').$$

Therefore,

$$\begin{aligned} U_{\theta' b'}(\theta') = {} & u''\big(c'(\theta')\big) c'_{b'}(\theta') Y_{\theta'}\big(l'(\theta'), h', \theta'\big) \\ & + u'\big(c'(\theta')\big) Y_{\theta' l}\big(l'(\theta'), h', \theta'\big) l'_{b'}(\theta'). \end{aligned} \tag{100}$$

Substituting (100) into (93) gives

$$\widetilde{R}_{b'} = R^{\mathrm{MU}}_{b'} + R^{\mathrm{labor}}_{b'}, \tag{101}$$

where

$$R_{b'}^{\mathrm{MU}} = \int_{\underline{\theta}}^{\overline{\theta}} \omega(\theta') u''\big(c'(\theta')\big) c'_{b'}(\theta') Y_{\theta'}\big(l'(\theta'), h', \theta'\big) f(\theta')\, d\theta', \tag{102}$$

$$R_{b'}^{\mathrm{labor}} = \int_{\underline{\theta}}^{\overline{\theta}} \omega(\theta') u'\big(c'(\theta')\big) Y_{\theta' l}\big(l'(\theta'), h', \theta'\big) l'_{b'}(\theta') f(\theta')\, d\theta'. \tag{103}$$

Equations (96) and (101) show that both instruments generate informational effects through marginal utility and labor-supply responses. However, only educational investment generates the direct productivity-rent component $R_{h'}^{\mathrm{prod}}$. This proves Theorem 3.

E.5 Planner's First-Order Condition and Normalization

The unnormalised informational effect $\widetilde{R}_{h'}$ enters the planner's first-order condition for education:

$$\beta \int_{\underline{\theta}}^{\overline{\theta}} u'\big(c'(\theta')\big) Y_{h'}\big(l'(\theta'), h', \theta'\big) f(\theta')\, d\theta' = \lambda g_{h'}(h', h) + \widetilde{R}_{h'}. \tag{104}$$

The bequest first-order condition is

$$\lambda = \beta(1+r) \int_{\underline{\theta}}^{\overline{\theta}} u'\big(c'(\theta')\big) f(\theta')\, d\theta'. \tag{105}$$

Substituting (105) into (104) and dividing by

$$\beta \int_{\underline{\theta}}^{\overline{\theta}} u'\big(c'(\theta')\big) f(\theta')\, d\theta'$$

gives

$$\overline{Y}_{h'} = (1+r) g_{h'}(h', h) + \widetilde{\Delta}_{h'}, \tag{106}$$

where

$$\overline{Y}_{h'} = \frac{\int_{\underline{\theta}}^{\overline{\theta}} u'\big(c'(\theta')\big) Y_{h'}\big(l'(\theta'), h', \theta'\big) f(\theta')\, d\theta'}{\int_{\underline{\theta}}^{\overline{\theta}} u'\big(c'(\theta')\big) f(\theta')\, d\theta'}$$

and

$$\widetilde{\Delta}_{h'} = \frac{\widetilde{R}_{h'}}{\beta \int_{\underline{\theta}}^{\overline{\theta}} u'\big(c'(\theta')\big) f(\theta')\, d\theta'}. \tag{107}$$

Since the denominator in (107) is strictly positive, the sign of the normalized wedge $\widetilde{\Delta}_{h'}$ is the sign of the unnormalised effect $\widetilde{R}_{h'}$.

E.6 Proof of Corollary 2

Proof. The result follows immediately from the decompositions (96) and (101). Educational investment generates three informational components:

$$R_{h'}^{\text{MU}}, \qquad R_{h'}^{\text{prod}}, \qquad R_{h'}^{\text{labor}}.$$

Financial bequests generate only two:

$$R_{b'}^{\text{MU}}, \qquad R_{b'}^{\text{labor}}.$$

The missing term for bequests is the direct productivity-rent component

$$R_{h'}^{\text{prod}} = \int_{\underline{\theta}}^{\overline{\theta}} \omega(\theta') u'\big(c'(\theta')\big) Y_{\theta' h'}\big(l'(\theta'), h', \theta'\big) f(\theta')\, d\theta'.$$

This term exists only because education enters the production technology directly. Financial bequests do not enter $Y(l', h', \theta')$, and therefore have no analogous component. Hence endogenous labor supply makes financial bequests informationally relevant through behavioral responses, but does not eliminate the direct productivity asymmetry between education and bequests. □

E.7 Proof of Proposition 4

Proof. From (98), the direct productivity-rent component is

$$R_{h'}^{\text{prod}} = \int_{\underline{\theta}}^{\overline{\theta}} \omega(\theta') u'\big(c'(\theta')\big) Y_{\theta' h'}\big(l'(\theta'), h', \theta'\big) f(\theta')\, d\theta'.$$

By definition,

$$\omega(\theta') = \frac{1 - F(\theta')}{f(\theta')}.$$

Since $F(\theta') \le 1$ and $f(\theta') > 0$ on the support, we have

$$\omega(\theta') \ge 0.$$

Moreover, $u'(c'(\theta')) > 0$ and $f(\theta') > 0$. Hence all terms multiplying $Y_{\theta' h'}\big(l'(\theta'), h', \theta'\big)$ inside the integral are non-negative, and strictly positive wherever $\omega(\theta') > 0$.

Therefore, if

$$Y_{\theta' h'}\big(l'(\theta'), h', \theta'\big) \ge 0$$

for all θ', then

$$R_{h'}^{\text{prod}} \ge 0.$$

If the inequality is strict on a set of positive measure with $\omega(\theta') > 0$, then

$$R_{h'}^{\text{prod}} > 0.$$

Analogously, if $Y_{\theta' h'} \le 0$ for all θ', then $R_{h'}^{\text{prod}} \le 0$, with strict inequality under the corresponding strictness condition. Thus, when $Y_{\theta' h'}$ has a constant sign on the relevant support, the sign of $R_{h'}^{\text{prod}}$ is the sign of $Y_{\theta' h'}$. □

E.8 Proof of Proposition 5

Proof. Let h'^{FI} denote the full-information education level and h'^{PI} the private-information education level in the endogenous-labor environment. Under full information, the education condition is

$$\overline{Y}_{h'}^{FI} = (1 + r) g_{h'}(h', h). \tag{108}$$

Under private information with endogenous labor supply, the corresponding condition can be written as

$$\overline{Y}_{h'}^{PI} = (1 + r) g_{h'}(h', h) + \widetilde{\Delta}_{h'}, \tag{109}$$

where $\widetilde{\Delta}_{h'}$ denotes the normalized total informational wedge in the endogenous-labor environment.

From (96), the unnormalised informational effect is

$$\widetilde{R}_{h'} = R_{h'}^{\text{MU}} + R_{h'}^{\text{prod}} + R_{h'}^{\text{labor}}.$$

By Proposition 4, if human capital and ability are complements in production, so that $Y_{\theta' h'} > 0$, then $R_{h'}^{\text{prod}} > 0$ whenever the strict inequality holds on a set of positive measure.

Suppose that this productivity-rent component is sufficiently large relative to the remaining components, so that

$$R_{h'}^{\text{prod}} > -\left(R_{h'}^{\text{MU}} + R_{h'}^{\text{labor}}\right).$$

Then

$$\widetilde{R}_{h'} = R_{h'}^{\text{MU}} + R_{h'}^{\text{prod}} + R_{h'}^{\text{labor}} > 0.$$

Since

$$\widetilde{\Delta}_{h'} = \frac{\widetilde{R}_{h'}}{\beta \int_{\underline{\theta}}^{\overline{\theta}} u'\big(c'(\theta')\big) f(\theta')\, d\theta'},$$

and the denominator is strictly positive, we have

$$\widetilde{\Delta}_{h'} > 0.$$

Comparing (108) and (109), private information with $\widetilde{\Delta}_{h'} > 0$ requires a higher expected marginal return to education than full information:

$$\overline{Y}_{h'}^{PI} > \overline{Y}_{h'}^{FI}.$$

If the marginal-utility-weighted expected marginal return to education $\overline{Y}_{h'}$ is decreasing in h', this higher required return is achieved by choosing a lower level of education. Hence

$$h'^{PI} < h'^{FI}.$$

In this case, the private-information allocation features less educational investment than the full-information allocation. Equivalently, optimal policy taxes rather than subsidizes education relative to the financial bequest margin. This is the conditional reversal of the standard Bovenberg–Jacobs subsidy logic. □

F Regularity and First-Order Approach

Throughout the analysis with private information, we impose standard regularity conditions ensuring that the first-order approach can be used. The utility functions u and v are twice continuously differentiable, with $u' > 0$, $u'' < 0$, $v' > 0$, and $v'' > 0$. The production function $Y(l, h, \theta)$ is continuously differentiable and satisfies $Y_l > 0$. The education cost function $g(h', h)$

is increasing and convex in h'. The type distribution has compact support $[\underline{\theta}, \overline{\theta}]$ with density $f > 0$.

The labor requirement mapping is defined by

$$Y\big(\ell(y; h, \theta), h, \theta\big) = y.$$

By the implicit function theorem,

$$\ell_y(y; h, \theta) = \frac{1}{Y_l\big(\ell(y; h, \theta), h, \theta\big)} > 0.$$

If higher ability raises output at a given labor input, so that $Y_\theta(l, h, \theta) > 0$, then

$$\ell_\theta(y; h, \theta) = -\frac{Y_\theta\big(\ell(y; h, \theta), h, \theta\big)}{Y_l\big(\ell(y; h, \theta), h, \theta\big)} < 0.$$

Thus, higher ability reduces the labor required to produce any given output level.

Under the Spence–Mirrlees single-crossing condition, the optimal allocation is monotone and local incentive constraints are sufficient for global incentive compatibility. Under these maintained assumptions, the planner's problem can be analyzed using the local incentive constraints and the associated envelope representation.

The first-order conditions derived in the main text should therefore be interpreted as characterizing an interior optimum under the validity of the first-order approach.