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Abstract

Less liquid markets for government bonds (LLMs) are characterized by many well recognized challenges which reduce the reliability of the classic Nelson-Siegel-Svensson (NSS) parsimonious approach. We document key stylized facts about government bond markets concerning liquidity, diversity of maturities available, bid-ask spread in price quotes, as well as price distortion in the very short end of the curve due to switch auctions. Based on these facts, we augment the NSS approach with model- and data-driven endogenous system of weights which permits reliable estimation of yield curves in LLMs. We apply our approach to the data for one of the largest European emerging markets: Poland. Through a battery of sensitivity analyses we show that there exists a class of weights that systematically gives better results than the classic NSS approach. The best fit weights have at least the same weight for the short end of the curve as a sum for all other tenors of bonds. It proves that inferring from the liquidity in particular maturities raises the information content and quality of yield curve estimation, which links our results to the expectation hypotheses. Unlike findings for most mature markets (e.g. US), for Poland there is a limited domain where pure expectations hypothesis (PEH) cannot be ruled out. Moreover, expectations hypothesis (EH) holds in Poland for almost all horizons. The existence of term premia structure explains the differences between compounded rates of returns from shorter investments and longer term zero-coupon yields of corresponding maturity.

Keywords:
yield curve estimation, parsimonious form, (pure) expectations hypothesis

JEL Classification
G12, G15, G17

Corresponding author
Marcin Dec (m.dec@grape.org.pl)

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Foundation of Admirers and Mavens of Economics
ul. Koszykowa 59/7
00-660 Warsaw
Poland

W | grape.org.pl
E | grape@grape.org.pl
TT | GRAPE_ORG
FB | GRAPE.ORG
PH | +48 799 012 202

# Introduction

The general objective of this paper is to offer novel insights on possible quantitative solutions to typical of less liquid government bond markets (such as Polish one) challenges when estimating yield curve, namely: (i) generally shorter recorded history compared to advanced economies, (ii) extreme sensitivity of risk premium inference due to the issues with estimation of the short end of the yield curve, (iii) insufficient diversity of maturities of available bonds, (iv) insufficiently precise price quotes for many of the off-the-run securities. These issues imply that the estimates of the yield curve run a high risk of being spurious, due to over-fitting and are characteristic for many less liquid markets (LLM). For the estimation purposes, we will use data from Poland as an example of such markets. The notion of LLMs is not precise in the literature, it is defined by comparison to liquid bond markets such as the major advanced economies. We define *less liquid government bond markets* indirectly via measure of market size as the ones with at least 80 bln USD equivalent outstanding amounts of general government debt securities in local currency, but excluding sovereign issuers from the United States, the United Kingdom, Japan, the Euro Area, Switzerland, Canada and Australia. Using Bank for International Statistics data as of the end of December 2019 the following countries would fall into this group: Brazil, Chile, Czechia, Hungary, India, Indonesia, Israel, Malaysia, Mexico, Korea, Poland, Singapore, South Africa, Thailand and Turkey.

In this article we develop a class of weighting schemes which improves fit relative to conventionally used methods. We operationalise fit as mean absolute error and smoothness of estimated forward curves. We propose to combine the outstanding amounts and market liquidity data with market level information on prices of government bonds in the form of weighting schemes. Prior literature (i.e. Dziwok (2004, 2013)) for LLMs and Gurkaynak et al. (2011) for developed markets), offered approaches that lack a reference to liquidity measures such as: equal weights of yield differences or weights derived from modified duration on price differences. We postulate that inferring from the liquidity in particular maturities raises the information content of the estimation and, unlike earlier approaches, is model-consistent.

Moreover, we posit here that Pure Expectations Hypothesis (PEH) does not hold universally in LLMs. Empirical research demonstrated that in liquid markets (US) Pure Expectations Hypothesis, claiming that that future rate is an unbiased predictor of the future spot rate, holds for short horizons and durations, but not universally (Fama & Bliss (1987), Campbell & Shiller (1991), Cochrane & Piazzesi (2005)). We will obtain verification of PEH across horizons and durations analogous to the literature for the US and compare the results between the less liquid (Poland) markets and liquid market (US).

These two findings have straight forward implications for the general practice in the financial less liquid markets to utilize IBOR type rates to construct the short end of the yield curves. The use of IBOR type rates is motivated by availability, but they are not strictly speaking risk free instruments. Consequently, incorporating them in the process of obtaining the risk premia is internally inconsistent. Our proposed method replaces IBOR type rates with internally consistent method of blending transactional data of government bonds and some implied money market instruments without turning to indicative prices. This is particularly relevant, because in many markets, as Polish one, low liquidity in government bonds is accompanied by structural over-liquidity of the banking sector.

Our method is innovative along two main lines. First, our decision to jointly study yield curve fit and term structure estimation is a unique proposal, to our best knowledge, not found in the literature covering less liquid government bond markets. This vertical integration in the modelling of term structure of interest rates is crucial for precision and meaningful inference mainly due to the challenges of blending information from relevant money market sub-segment into the curve on one hand and the sensitivity of term premia extraction to the behaviour of short-end of the yield curve, on the other. This modelling complexity is even more pronounced in the light of interest rate benchmarks reform (henceforth: BMR), because after the Financial Crisis of 2007-8 (henceforth: FC) and LIBOR manipulation scandal, the regulatory pressure towards transaction-based rather than survey-based indices has increased greatly. In principle, in our modelling and estimations we hold to this rule indeed that the raw data we use shall be solely based on transactional information (from trading platforms and central banks) but not the indicative prices from the general financial information distributors. Our approach, in contrast to the earlier literature, exploits to the maximum the available micro-structural data of bond market: turnover (monthly and daily, where available), outstanding amounts, bid-ask spreads, number of transactions traded daily, bid and ask yields. For less liquid markets, so far separate (disintegrated) research threads were conducted: theory of parsimonious fitting, yield curve estimation, expectations hypothesis testing and the extraction of risk premia structure. Studies on risk premia in less liquid markets are rare as the literature is dominated by research conducted on developed markets. The very exceptions (i.e. Jabłecki et al. (2016), Kucera et al. (2017)) are limited to raw term premia presentation and comparisons based on unaltered methodology used for US markets which in turn is supplied with zero coupon yields generically prepared by third party providers (Bloomberg or Refinitiv). There are examples of extensive usage of estimated term premia in calibration and design of macroeconomic models (as in Kolasa & Wesołowski (2020)), but yet again these calculations of term premia are based on US markets originated algorithm and rely on zero-coupon bond data taken as given from external sources. We acknowledge that there are numerous studies of less liquid markets as Czech and Polish ones, but they are concentrated on yield curve estimation challenges  fitting and smoothing - and almost never extend to term premia extraction ( Hladíková & Radová (2012), Kladívko (2010), Slavík (2001), Świętoń (2002), Cięciwa (2003), Marciniak (2006), Kliber (2009), Dziwok (2004, 2013)). To the best of our knowledge no in-depth study of vertical robustness of yield curve and term-premia estimation exists for markets other than in the US, the UK, Australia, Canada and the Euro Area. Our study draws heavily on the idiosyncrasy of a particular micro-structure of a given bond market and hence fills that gap, at least in part.

Second, we intend to deliver more meaningful, realistic and interpretable time-series of yield curves than the ones produced by methods overly concentrated on their smoothness and perfect fit to all observed data without market practise view on weights of different bond series. We believe, that in the existent literature focuses too much on the graphic side of the curve without proper attention to the economical sense of the results. Yield curve is an imaginary object that spans from a discrete set of points; hence the major problem is how we define the continuous function of interest rates in the first place, rather than running an exercise in smoothing. This is why, out of a relatively diversified catalogue of potential tools of choice we would focus on merits of parsimonious modelling. The yield curve used in further decomposition should be a representation of market expectations of the future interest rates behaviour and being such should allow for some series of bonds to be dear or cheap to the curve. The methodology developed should also identify structural mispricing due to tax reasons and supply related information policies of a particular sovereign issuer. These desired characteristics are absent in the results of implementations of such methods as B-spline models stabilized with a variable roughness penalty seen in Marciniak (2006).

## Characteristics of Polish government bonds market

Debt securities are instrumental in financing current budget deficit and in refinancing the existing (rolling) debt, amounting to approximately 86.1% of 1.088,19 bln PLN public debt as of the end of June 2020[1]. There are retail, wholesale domestic and wholesale foreign (denominated in other currencies than PLN and placed in the international markets) government bonds available for investing and trading in Poland. Despite playing a prominent role in socially beneficial saving behaviour promotion, retail segment is relatively small (3.3% of public debt) and usually linked to CPI, whereas

[1]cf. *Public Debt 06/2020. Monthly newsletter. Ministry of Finance. Republic of Poland*

the internationally placed bonds (17.2% of public debt) are denominated in foreign currencies, hence their information potential is connected directly with foreign term structure and expectations with regards to its evolution as well as an evaluation of Poland's default risk for external investor, but not much with Polish interest rates term structure.

Therefore and of because the nature of this enquiry we narrow the scope of bond types of our interest to wholesale domestic ones with fixed coupon, without any additional features like call/put options (51.3% of public debt). These kind of bonds are, primarily, 2-year zero coupon bonds, 5-, 10- and 20-year fixed coupon bearing bonds without special features, denominated in PLN (4.7%, 18.1%, 20.3% and 6.6% of public debt, respectively). Polish government securities (T-bonds and T-bills, henceforth: TS) recent history (after the communist regime fallen in 1989) is relatively young as first auction of T-bills took place in May 1991 (in material form during that time) and first actions of fixed coupon bonds, namely: 2-year - OS0696 and 5-year - OS0699 were held on 17th February 1994. Since then the development of the secondary market was strongly correlated with the State Budget borrowing requirements but the quality of the market in terms of diversified numbers of investors in different segments of the curve (i.e. banks, investment funds, pension funds, insurance companies - including the ones offering life insurance, and foreign investors as a separate, and itself heterogeneous, group) has been growing with the pension funds reform, the development of interbank OTC market and Polish capital market in a broader sense.

There are two other segments in wholesale domestic group, that we leave as out of scope for estimation purposes: CPI-linkers and floaters (0.4% and 16.9% of public debt, respectively), as any inference of future interest paths that may be derived and extracted from the price history of bonds of these types is dependent on the proper estimation of the *base* yield curve and term premia in the first place.

In Poland the system of **Treasury Securities Dealers** (henceforth: TSD, authorised Primary Dealers and Bank Gospodarstwa Krajowego (state owned bank)) was implemented in 2002, in order to regulate the primary and secondary bond market in consultation with *buy-side* participants with the ultimate goal of better transparency and lower public debt financing costs. It involved defining types of participants in auctions, conditions and obligations of both: investors and the Ministry of Finance. Treasury BondSpot Poland (henceforth: BondSpot or BS)[2] - as electronic wholesale trading platform and one of the key elements in the TSD system, the successor of the Electronic Treasury Securities Market (2002 - 2004) plays a key role in both creating cheap, transparent trading environment and in price discovery and market information dissemination to the public and for debt management purposes. Since the beginning the platform offered at least one - so called *fixing* - a time window in a day in which Primary Dealers are obliged to place two-way quotes (both bid and ask prices) for the majority of existing securities with certain maximum (regulated) *bid-ask-spread* (BAS) depending on the current maturity of a particular series. On the other hand, Primary Dealers have been granted an exclusive right to purchase T-bills and bonds on the primary market with an intention of distributing them among others via BondSpot. The platform distinguishes between three types of participants[3]: **Market Makers** (Primary Dealers) who commit themselves to constantly providing two-way quotes to the general market, **Market Takers** - who basically may place an order in the system against Market Maker offer and **Institutional Investors** with an access to a *request for quote* feature used to communicate directly with a Market Maker of their choice in order to trade. In the moment of writing this Chapter a minimum amounts of 5 mln PLN and their multiples thereof can be traded. Fixing prices of securities being reference prices for the domestic

[2]On the official BondSpot's page we read: "Established on November 25, 2004, BondSpot Poland is an effect of cooperation with an Italian company MTS S.p.A., the first European electronic market for government bonds and the founding member of MTS Group. Today, the group of MTS companies - MTS Galaxy is the leading market in Europe for the trading of fixed income securities. It has over a thousand participants throughout Europe, with average transaction volumes exceeding 85 bln EUR a day (single-counted)".

[3]In September 2020 there were, 28 participants, including 13 banks holding the status of Primary Dealer

debt market are set twice a day during two fixing sessions each trading day.

There are three types of **primary market operations** in Poland: *sale auction*, *switch auction* and *buy-back auction*. Since 2012 the *sale auctions* are carried out in uniform price formula which means that the successful bidding investors buy bonds at minimum accepted price (*cut-off*) from the order book[4]. Once the price is known, additional non-competitive bids may be placed and TSD buy bonds at the minimum accepted price (up to 15% of total sales on a given action). Usually there are 2-4 auctions a month with some 1-4 securities on offer. The bonds are tapped - which means that for several tenders the same series is auctioned - leading to increase of outstanding amount and in consequence the liquidity on the secondary market and lower servicing costs for the future issues. Less common but still very important as a tool in debt management are *switch auctions* in which the Minister of Finance buys back bonds with near redemption dates before their originally planned maturity date, while selling longer term bonds in exchange without any cash flows. The least common auctions are the *buy-backs* in which the Ministry targets a few series of bonds (usually with very short maturity) and buys them back for cash in multiple-price styled auction. The latter two types of operations reduce refinancing risk, help to build large outstanding amounts in benchmark issues and reduce number of illiquid series.

## Data selection

In order to properly estimate yield curves and propose new wieghting system we would need the following data on government bonds market and their characteristics and some auxiliary variables:

1. **on government bonds secondary market (dynamic)**: daily[5] data by bonds series of preferably *firm*[6] prices or yields to maturity (ideally: bid, ask, mid), daily data on volume traded by series, number of trades, volume weighted trading prices at least for some widely used electronic platform and monthly data for the whole secondary market

2. **on government bonds primary market (static)**: databases with characteristics of all bonds with regard to: (1) coupon values, payment and *ex-dividend* dates, (2) primary auction results (regular and switch auctions) and other important operations on securities which may influence the proper calculation of outstanding amounts on particular settlement dates

3. **ultra short risk free interest rates** i.e NBP official interest rates, short term money market rate POLONIA time series and volume of NBP bills auctioned regularly week by week as well as, less frequent, so called *tuning* operations.

Our research showed that there are the following possible choices of sources of necessary data:

1. **Polish Ministry of Finance** offers a general access information on public debt on its website, where it provides extensive historical and current data on T-bonds and T-bills including, but not limited to: (1) schedules of coupon payments and interest periods dates for every issued bond since 1994, (2) securities operations (primary auctions and their results, switch operations, early redemptions, special operations etc.) since 1994, (3) monthly turnover on the secondary market by particular series of bonds since April 2014[7], (4) legal supporting materials (Auction procedures, Issuance procedures, Letters of issue, Rules and Regulations

[4]However, in the years of 1994-2011 sale auctions were held in the multiple-price auction system - meaning that bidders bought bonds at a price submitted in their order (potentially different price for each investor

[5]end-of-day or other fixed time that is uniform across the whole time series

[6]as opposed to *indicative*, which are prone to contribution mistakes without any economic motivation to correct it. *Firm* prices are those on which it is possible to trade

[7]In May 2019 we were granted a permission to use additional data sent by the Secretariat of Public Debt Department in Polish Ministry of Finance for the period staring from July 2004

Governing the Activities of the Treasury Securities Dealer, The Public Finance Sector Debt Management Strategy)

2. **BondSpot** via its webpage [8] makes available all fixings information since 26th November 2004, by bond series bid/ask/mid prices and *ytm*-s are available in one or two fixings a day regime. On the top of it BS publishes data about trades conducted on the platform per bond series i.e number of trades, volume, min and max prices and best bid and ask prices. All of the prices published by BondSpot are *firm* in that a market taker could have always used them to trade, which makes them very valuable for our inference and estimation of term structure of Polish interest rates.

3. **Refinitiv** [9] collects and privately (access is subject to monthly fee and long term contracts for majority of interested parties[10]) makes available probably broader information than Bond Spot in some areas, but in the case of Polish government bonds no volume data on the *over-the-counter* markets nor electronic platforms is available and what is more important for our inference is that the bid ask prices and *ytm*-s provided for each series even *tick-by-tick* are indicative only and no one is guaranteed to trade on these. Prices on the secondary market are collected by Reuters since 1997, but the quality of data, judging by the frequency of change is rather poor until 2002-2004 where the prices seem to be updated regularly.

4. **Bloomberg** offers Polish bond data for a slightly shorter period but of very similar depth and breadth and character as Refinitiv. The company produces their own estimations of zero coupon curves derived from government bonds price data[11]. These curves may be potentially used to compare with our estimations.

5. **National Bank of Poland** provides all the history of the monetary policy decisions, time series of benchmark *overnight* interest rate POLONIA (since January 24th, 2005), historical and current information on open market operations including amounts tendered. Unfortunately it does not publish any time series of yield curve estimated parameters, in style of Fed or ECB.

6. **Warsaw Stock Exchange** is an important source of *firm* prices and volume data of all types of Polish government bonds, but this platform is dedicated to serve retail investors and usual turnover and the size of bid and ask orders are very tiny (a few pieces of bonds in some series) and the total turnover in TS on WSE is minuscule as compared even with BS, let alone the total secondary market. Therefore, BS and MinFin data are more valuable given our aim.

Reconciling the list of data we need with what is available we have decided on the optimal maximum lifespan of homogeneously good quality data (both on prices/*ytm*-s and trading activity/volumes) to be the period between 1st January 2005 and 30th June 2020 (daily and monthly). Moreover our main sources would be: Polish Ministry of Finance, BondSpot and National Bank of Poland, whereas Bloomberg and Refinitiv would be used in robustness and comparison studies. As we will show later in the Figure 9, amongst other characteristics, the number of bonds under fixing on BondSpot (second tile) and the *cover ratio* as a proportion of this number and the total fixed coupon outstanding bonds (last tile). The number of bonds subject to fixing decreased after the FC to 13 from 15-17 in the beginning of 2005 but then it steadily rose to 18-21 in 3 years to stay on that level till the time of writing this thesis and giving an average of 17.2 bonds being fixed everyday in

[8] *https://www.bondspot.pl*, although it is worth noting an elaborate and at some points cumbersome process of collecting all of the data using *web scrapping* techniques and aggregating information from more than 8.000 html tables in one Matlab database covering the period of 2005:01-2020:06

[9] formerly Thomson Reuters, and Reuters

[10] The author has been granted an access to data and the right to use it in academic publications, including dissertation by Thomson Reuters on 11 January 2018 - with side letter dated June 20th, 2018

[11] via a service called BVAL GSAC (Government, supranational, agency, corporates issuer & sector curves). Calculation details are not publicly available and the algorithm is patent pending

2005:01-2020:06 period. Not all of the fixed coupon bonds are subject to fixing procedures on BS. There is a certain number of very short bonds (under 6-8 months to maturity) or currently being issued for the first time. Average cover ratio in the period was 84%, which means that on average 3.2 bonds which had non zero outstanding amounts were not quoted on the fixing, which we assume is a sufficient cover of the bonds traded in the market.

Table 1: Dates with less then 10 eligible bonds on BondSpot fixing

| Date | Number of eligible bonds |
|---|---|
| 14-Mar-2005 | 4 |
| 19-Jul-2005 | 8 |
| 10-Oct-2008 | 6 |
| 15-Oct-2008 | 2 |
| 23-Oct-2008 | 7 |

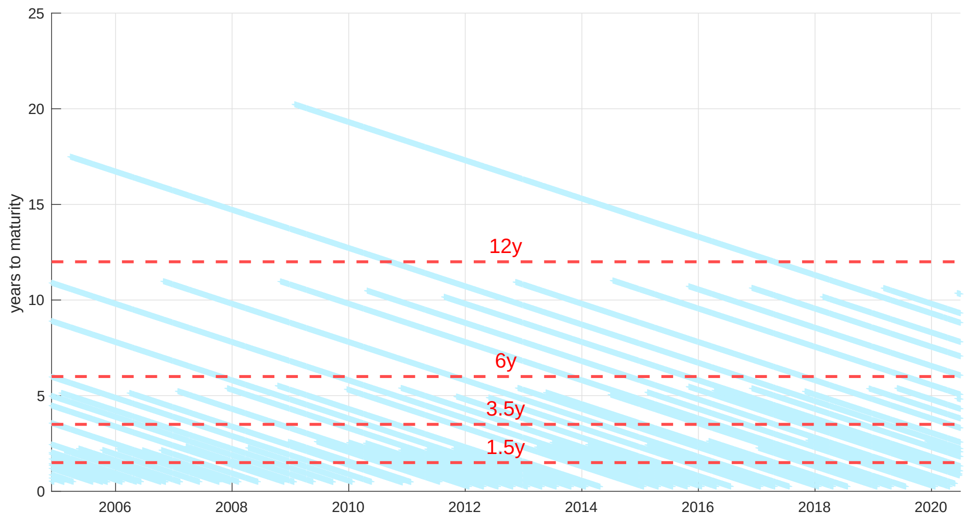
Notes: *eligible* means here: with no less than 0.85 years to maturity.

There are only 5 days of data from BS that we have decided not to include for estimation purposes because of too few bonds under fixing (we took 10 as a minimum number of bonds), which is depicted in the Table 1. The main reason for Primary Dealers' temporal decrease in scrutiny and care in quoting fixing prices on the platform is a very turbulent time in the Financial Crisis mayhem. Finally, such defined a collection of data we will call a *dataset* (DS).

## Stylised facts on Polish bonds (and their impact on filtering)

In this section we introduce a division of the yield curve into segments (buckets) by current years to maturity into [0, 1.5], (1.5, 3.5], (3.5, 6], (6, 12] and (12, 30] years. This proposal is based on the historical analysis depicted in the Figure 2. It will serve as an additional dimension in our enquiry.

Figure 2: Proposed segmentation of bonds on BondSpot in the period 2005:01-2020:06




Notes: (1) every light blue line represents one series of bond subject to fixing on BS (2) Red dashed lines are the proposed segments' division

Thorough analysis of the *dataset* has led to the following stylised facts:

1. **Monthly secondary TS market's turnover is of the same magnitude as monthly sum of NBP bills auctioned at reference rate**. We observed that the monthly average TS turnover (outright) in the whole period stood at 196.2 bln PLN with highs around 400-450 bln PLN and lows around 100 bln PLN, where as the average monthly sum of NBP bills auctioned is 309 bln PLN with greater volatility between 50 and 650 bln PLN until the beginning of COVID19 Pandemic and around 850 bln PLN in 2020:06.

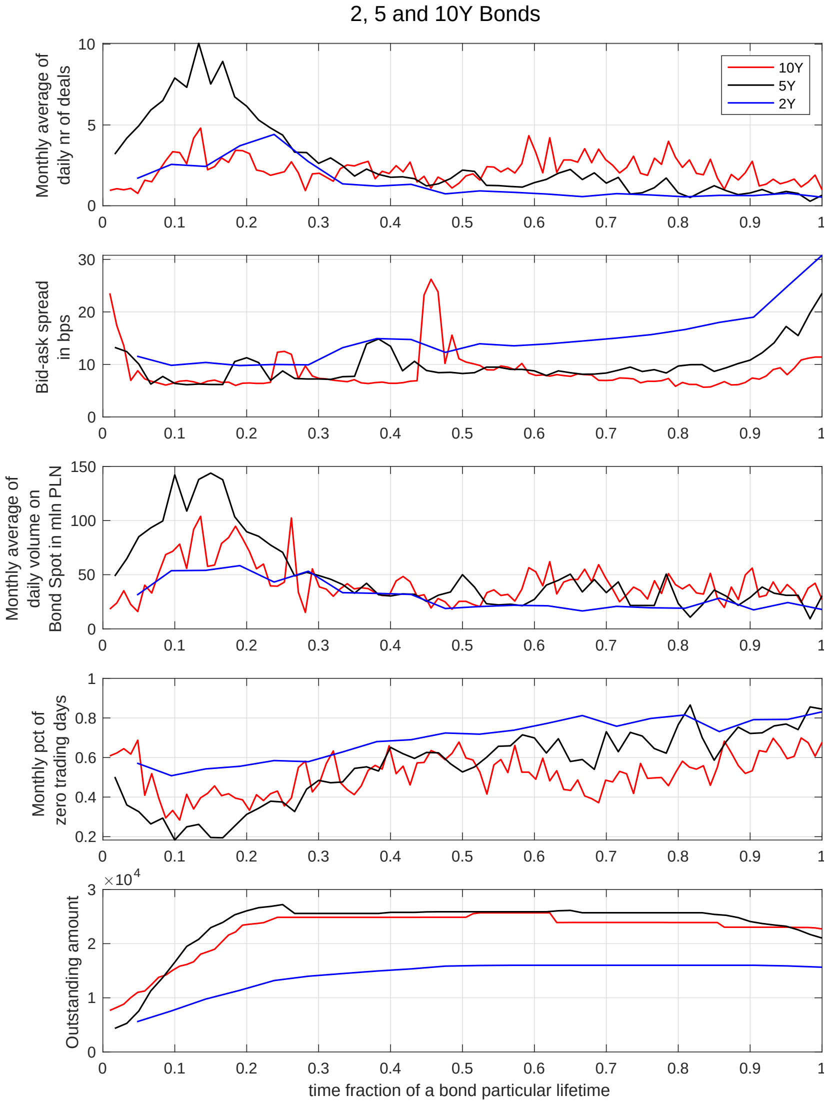
Figure 1: Selected characteristics of Polish fixed coupon government bonds traded on BondSpot in 2005:01-2020:06 **by segments**

| | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 | 2011 | 2012 | 2013 | 2014 | 2015 | 2016 | 2017 | 2018 | 2019 | 2020$H1$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $OA([0,1.5])$ | 48590 | 44230 | 51631 | 52948 | 51408 | 66328 | 99858 | 106604 | 81082 | 72560 | 91508 | 66385 | 79065 | 76322 | 67685 | 86692 |
| $OA((1.5,3.5])$ | 59992 | 78764 | 87634 | 84715 | 89947 | 119549 | 106790 | 98955 | 112911 | 108352 | 96154 | 104826 | 109970 | 132429 | 144443 | 122702 |
| $OA((3.5,6])$ | 60872 | 58962 | 52706 | 59775 | 62865 | 78647 | 72567 | 83818 | 91178 | 78282 | 84041 | 113466 | 126009 | 104692 | 98118 | 102449 |
| $OA((6,12])$ | 29235 | 44288 | 54152 | 46564 | 61643 | 65496 | 96719 | 93326 | 108537 | 83944 | 86701 | 98821 | 115127 | 118313 | 134309 | 138281 |
| $OA((12,30])$ | 4134 | 8262 | 15247 | 21261 | 26470 | 23287 | 6573 | 7774 | 10353 | 8713 | 8529 | 8529 | 2593 | 0 | 0 | 0 |
| $OA([0,30])$ | 202824 | 234506 | 261371 | 265264 | 292333 | 353307 | 382507 | 390476 | 404061 | 351851 | 366932 | 392027 | 432763 | 431756 | 444556 | 450125 |
| $NB([0,1.5])$ | 5.12 | 4.26 | 3.89 | 3.29 | 2.86 | 2.96 | 3.89 | 5.04 | 3.92 | 3.64 | 4.59 | 3.76 | 3.76 | 3.76 | 3.86 | 4.72 |
| $NB((1.5,3.5])$ | 4.35 | 4.90 | 4.94 | 4.10 | 4.07 | 5.12 | 4.77 | 4.04 | 5.00 | 5.51 | 5.00 | 4.82 | 5.14 | 6.00 | 6.26 | 5.11 |
| $NB((3.5,6])$ | 3.80 | 2.86 | 2.47 | 3.02 | 3.02 | 3.21 | 3.15 | 3.81 | 3.93 | 3.82 | 4.28 | 5.25 | 5.48 | 4.24 | 3.82 | 3.73 |
| $NB((6,12])$ | 2.00 | 2.17 | 2.82 | 2.16 | 2.81 | 2.97 | 4.18 | 4.13 | 4.82 | 4.29 | 3.97 | 3.80 | 4.51 | 4.87 | 5.37 | 5.07 |
| $NB((12,30])$ | 0.76 | 1.00 | 1.00 | 1.00 | 1.91 | 1.72 | 1.00 | 1.00 | 0.99 | 1.00 | 1.00 | 1.00 | 0.30 | 0.00 | 0.00 | 0.00 |
| $NB([0,30])$ | 16.03 | 15.20 | 15.12 | 13.57 | 14.68 | 15.98 | 16.99 | 18.02 | 18.67 | 18.26 | 18.83 | 18.65 | 19.19 | 18.87 | 19.31 | 18.63 |
| $V([0,1.5])$ | 66.37 | 69.13 | 59.35 | 29.89 | 45.91 | 77.56 | 129.03 | 184.25 | 91.88 | 116.78 | 103.86 | 68.64 | 54.34 | 58.54 | 38.84 | 34.92 |
| $V((1.5,3.5])$ | 136.31 | 137.09 | 72.14 | 46.72 | 74.92 | 209.60 | 210.38 | 270.84 | 240.61 | 240.06 | 119.52 | 103.98 | 123.42 | 134.79 | 60.20 | 21.48 |
| $V((3.5,6])$ | 327.95 | 362.75 | 246.63 | 125.65 | 79.22 | 260.58 | 194.31 | 409.21 | 313.33 | 331.63 | 259.87 | 349.33 | 231.67 | 115.87 | 71.32 | 36.21 |
| $V((6,12])$ | 84.85 | 138.63 | 170.94 | 59.22 | 65.85 | 271.42 | 288.48 | 494.13 | 448.58 | 379.60 | 333.14 | 283.47 | 198.57 | 150.43 | 113.67 | 63.28 |
| $V((12,30])$ | 1.49 | 4.91 | 27.64 | 28.40 | 17.17 | 41.94 | 5.09 | 40.05 | 21.23 | 15.66 | 8.34 | 6.33 | 1.04 | 0.00 | 0.00 | 0.00 |
| $V([0,30])$ | 616.98 | 712.50 | 576.71 | 289.87 | 283.07 | 861.11 | 827.28 | 1398.48 | 1115.63 | 1083.74 | 824.74 | 811.76 | 609.03 | 459.63 | 284.03 | 155.90 |
| $YTM([0,1.5])$ | 4.96 | 4.23 | 4.80 | 6.29 | 4.53 | 4.02 | 4.55 | 4.27 | 2.76 | 2.28 | 1.58 | 1.44 | 1.46 | 1.25 | 1.24 | 0.71 |
| $YTM((1.5,1.5])$ | 5.01 | 4.64 | 5.11 | 6.27 | 5.13 | 4.70 | 4.84 | 4.32 | 3.07 | 2.51 | 1.78 | 1.73 | 2.00 | 1.73 | 1.58 | 0.95 |
| $YTM((3.5,6])$ | 5.17 | 4.99 | 5.34 | 6.18 | 5.57 | 5.24 | 5.34 | 4.52 | 3.40 | 2.89 | 2.16 | 2.25 | 2.64 | 2.34 | 1.90 | 1.22 |
| $YTM((6,12])$ | 5.21 | 5.20 | 5.46 | 6.10 | 5.96 | 5.68 | 5.89 | 4.94 | 3.87 | 3.36 | 2.56 | 2.84 | 3.27 | 3.09 | 2.28 | 1.63 |
| $YTM((12,30])$ | 5.11 | 5.38 | 5.53 | 6.06 | 6.16 | 5.92 | 6.14 | 5.23 | 4.27 | 3.80 | 2.87 | 3.26 | 3.70 | - | - | - |
| $YTM([0,30])$ | 5.07 | 4.72 | 5.16 | 6.22 | 5.40 | 5.00 | 5.21 | 4.54 | 3.35 | 2.82 | 2.04 | 2.13 | 2.40 | 2.12 | 1.77 | 1.13 |
| $YRFRA([0,1.5])$ | 1.00 | 0.96 | 0.98 | 1.01 | 0.98 | 1.00 | 1.01 | 0.87 | 0.76 | 0.98 | 0.83 | 0.87 | 0.87 | 0.87 | 0.88 | 0.98 |
| $YRFRA((1.5,3.5])$ | 2.14 | 2.33 | 2.35 | 2.30 | 2.28 | 2.36 | 2.20 | 2.36 | 2.46 | 2.34 | 2.36 | 2.43 | 2.41 | 2.50 | 2.38 | 2.38 |
| $YRFRA((3.5,6])$ | 4.61 | 4.26 | 4.38 | 4.77 | 4.47 | 4.64 | 4.47 | 4.51 | 4.46 | 4.51 | 4.68 | 4.67 | 4.49 | 4.45 | 4.60 | 4.49 |
| $YRFRA((6,12])$ | 9.30 | 8.48 | 8.50 | 8.48 | 8.49 | 8.99 | 9.03 | 8.86 | 8.38 | 8.19 | 8.18 | 8.38 | 8.98 | 8.93 | 8.48 | 8.09 |
| $YRFRA((12,30])$ | 17.10 | 16.22 | 15.23 | 14.23 | 16.23 | 16.43 | 17.81 | 16.81 | 15.81 | 14.81 | 13.81 | 12.80 | 12.16 | - | - | - |
| $YRFRA([0,30])$ | 3.96 | 4.11 | 4.33 | 4.40 | 5.49 | 5.30 | 4.95 | 4.68 | 4.76 | 4.58 | 4.35 | 4.52 | 4.40 | 4.27 | 4.22 | 4.01 |
| $BAS([0,1.5])$ | 20.04 | 20.48 | 20.44 | 45.49 | 40.03 | 31.29 | 17.74 | 13.54 | 19.50 | 10.40 | 12.39 | 15.19 | 13.30 | 12.76 | 17.61 | 25.02 |
| $BAS((1.5,3.5])$ | 10.37 | 10.51 | 11.64 | 25.53 | 20.46 | 16.31 | 9.86 | 6.47 | 7.42 | 5.72 | 6.28 | 6.40 | 5.57 | 5.67 | 7.24 | 10.28 |
| $BAS((3.5,6])$ | 8.93 | 7.14 | 5.48 | 14.81 | 12.96 | 9.81 | 6.72 | 4.76 | 5.87 | 4.69 | 4.64 | 4.58 | 4.21 | 4.47 | 4.62 | 7.51 |
| $BAS((6,12])$ | 6.58 | 6.22 | 5.02 | 12.55 | 11.63 | 9.28 | 6.18 | 3.66 | 5.08 | 4.07 | 4.20 | 4.26 | 4.34 | 4.27 | 4.31 | 8.10 |
| $BAS((12,30])$ | 10.16 | 9.37 | 5.40 | 10.44 | 22.49 | 19.21 | 16.50 | 3.98 | 14.67 | 7.59 | 18.82 | 18.00 | 14.46 | - | - | - |
| $BAS([0,30])$ | 12.65 | 12.04 | 11.25 | 24.66 | 21.38 | 16.75 | 10.53 | 7.30 | 9.48 | 6.19 | 7.63 | 7.85 | 6.58 | 6.48 | 7.99 | 12.93 |
| $ZTD([0,1.5])$ | 0.83 | 0.81 | 0.83 | 0.85 | 0.88 | 0.73 | 0.62 | 0.70 | 0.79 | 0.68 | 0.77 | 0.84 | 0.87 | 0.88 | 0.90 | 0.92 |
| $ZTD((1.5,3.5])$ | 0.64 | 0.68 | 0.75 | 0.81 | 0.80 | 0.64 | 0.51 | 0.48 | 0.52 | 0.54 | 0.64 | 0.73 | 0.77 | 0.78 | 0.83 | 0.91 |
| $ZTD((3.5,6])$ | 0.52 | 0.35 | 0.41 | 0.56 | 0.71 | 0.42 | 0.31 | 0.21 | 0.28 | 0.27 | 0.42 | 0.46 | 0.56 | 0.67 | 0.63 | 0.78 |
| $ZTD((6,12])$ | 0.47 | 0.42 | 0.48 | 0.55 | 0.64 | 0.40 | 0.26 | 0.14 | 0.19 | 0.25 | 0.34 | 0.35 | 0.47 | 0.52 | 0.60 | 0.76 |
| $ZTD((12,30])$ | 0.88 | 0.80 | 0.57 | 0.52 | 0.81 | 0.62 | 0.66 | 0.25 | 0.49 | 0.52 | 0.77 | 0.72 | 0.83 | - | - | - |
| $ZTD([0,30])$ | 0.66 | 0.63 | 0.65 | 0.70 | 0.77 | 0.57 | 0.45 | 0.39 | 0.44 | 0.44 | 0.57 | 0.60 | 0.67 | 0.71 | 0.74 | 0.84 |

**Note**: OA - outstanding amount in mln PLN , NB - number of bonds under fixing, V - volume traded on Bond Spot in mln PLN, YTM - yield to maturity in pct points, YRFRA - time to maturity in years, BAS - bid-ask spread in bps, ZTD - zero trading days share in total number of trading days, $(x, y]$ means a segment of bonds with current time to maturity between $x$ and $y$ years (excluding $x$ and including $y$).

2. **The volume traded (both on BS platform and on the market as a whole), as well as daily number of transactions follow common pattern during the lifetime of a certain bond type**. In the Figure 3 (and Figures 26, 27, 28 in Appendix), especially for 2Y and 5Y bond we observe that the volume on average rises steadily to peak after approximately $1/6th$ of bond's lifetime (for 2Y: after 4 months, 5Y: after 8 months , 10Y: after 15 months). The maximum values are on average 2-4 times higher than the corresponding mean. Soon after the peak volume and number of deals pull to the mean for more or less the same duration - $1/6th$ of bond's lifetime. This *pyramid* shape is clearly recognizable on all of these figures. During the remaining $2/3rds$ of bond's lifetime these statistics are eroding slowly with values slightly lower than the corresponding mean. This process starts approximately when the outstanding amount stops to climb (because there were no further auction).
These patters are observed not only in BondSpot volume data but also in MinFin data for the total secondary market turnover, which is depicted at the Figure 4.

Figure 3: Selected averaged liquidity measures of 2Y, 5Y and 10Y fixed coupon government bonds traded on BondSpot platform

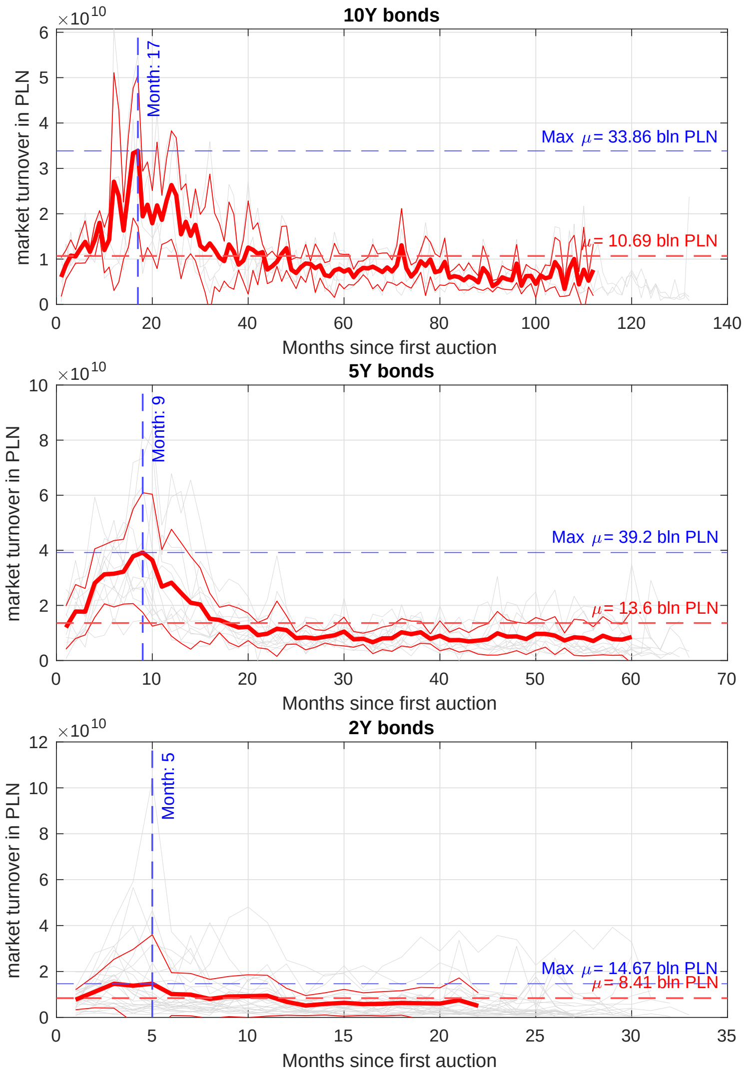



Notes: (1) Two year bonds included: OK0113, OK0114, OK0116, OK0406, OK0407, OK0408, OK0419, OK0709, OK0710, OK0711, OK0712, OK0713, OK0714, OK0715, OK0716, OK0717, OK0720, OK0806, OK0807, OK0808, OK1012, OK1018, OK1206, OK1207, OK1208, OK0419 (2) Five year bonds included: PS0310, PS0412, PS0413, PS0414, PS0415, PS0416, PS0418, PS0420, PS0511, PS0718, PS0719, PS1016 (3) Ten year bonds included: DS1013, DS1015, DS1017, DS1019

3. **The bid-ask spread (on BS) rises approximately two- or threefold above bond's lifetime mean in the last year**. Again the pattern is pronounced in the Figure 3 (and

Figures 26, 27, and a bit less so in Figure 28 in the Appendix). The rise is accompanied by fall in the outstanding amount due to switch operations in the last year of the bonds lifespan.

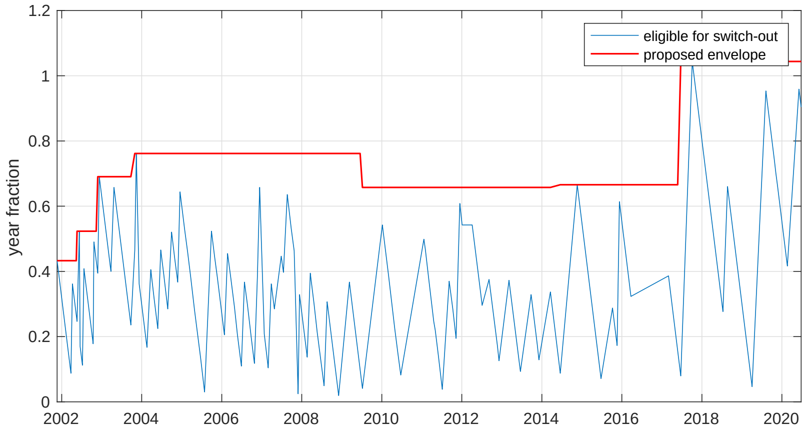
Figure 4: Averaged turnover of fixed coupon 2, 5, 10Y government bonds during their lifespan in months




Notes: (1) thinner red lines indicate one standard deviation up and down from the mean on a certain month, (2) (2) Only bonds with full life history contained in the period 2005:Q1 - 2020:Q3 were taken into consideration (2) Two year bonds included: OK0113, OK0114, OK0116, OK0406, OK0407, OK0408, OK0419, OK0709, OK0710, OK0711, OK0712, OK0713, OK0714, OK0715, OK0716, OK0717, OK0720, OK0806, OK0807, OK0808, OK1012, OK1018, OK1206, OK1207, OK1208, OK0419 (2) Five year bonds included: PS0310, PS0412, PS0413, PS0414, PS0415, PS0416, PS0418, PS0420, PS0511, PS0718, PS0719, PS1016 (3) Ten year bonds included: DS1013, DS1015, DS1017, DS1019

4. **Zero trading days patterns resemble mirror reflection of the ones observed for volume and number of trades**. The minimum values of ZTD share in total trading days in a given month are detected in the first $1/6th$ of bond's lifetime (for 2Y: after 2 months, 5Y: after 10 months , 10Y: after 3 months). On average ZTD shares are lower for longer bonds, i.e. 10Y - 0.5, 5Y - 0.6, 2Y - 0.7, while the 2Y bonds experience big swing in values from 0 to above 0.8 and 10y bonds ZTD shares mildly vary between 0.3 and 0.7.

Figure 5: Maximum time to maturity of bonds eligible to switch




Notes: (1) the red line represents an envelope based on moving averages (2) datascope: 2002- 2020:H1

5. **Historically the lowest bid-ask spreads were observed in the segment of (6, 12] years to maturity and the highest in the short-end of the curve [0, 1.5] years**. Table 1 reports the yearly average BAS for each segment for the whole DS (more than fifteen years).

6. **Ultra long end of Polish yield curve (12, 30] is very erratically inhabited with only one or two series quoted on fixing, and no representation since 2018 till now**. The BAS observed in this segment is comparable with less liquid segment of (1.5, 3.5] years.

7. **Switch auctions influence prices by increasing BAS due to very limited motivation on both sides: potential buyers' side who have alternative strategy of rolling NBP bills and potential sellers' who maybe better off using these bonds to buy longer and more liquid ones**. In the Figure 5 we show the history of maximum years to maturity of the bonds an investor wishing to buy longer bonds may switch from. It is clear that at the end of 2017 qualitative change occurred and this maximum level increased from 0.6-0.8 to 1-1.1. In the light of other above mentioned stylised facts it seems wise to exclude bonds with less than 0.85 years until mid 2017 and less than 1.20 from that point of time onwards.

Table 2: Statistics of liquidity measures of Polish fixed coupon government bonds traded on BondSpot in 2005:01-2020:06 - by segments

| | Amihud | | Roll | | Gamma | |
|---|---|---|---|---|---|---|
| Segment | mean | std | mean | std | mean | std |
| $[0, 1.5]$ | 0.0446 | 0.0323 | -184.20 | 115.29 | 0.0058 | 0.0491 |
| $(1.5, 3.5]$ | 0.0382 | 0.0305 | -124.94 | 128.41 | -0.0047 | 0.0358 |
| $(3.5, 6]$ | 0.0327 | 0.0290 | -135.29 | 146.39 | -0.0126 | 0.0468 |
| $(6, 12]$ | 0.0289 | 0.0242 | -145.05 | 161.00 | -0.0102 | 0.0483 |
| $(12, 30]$ | 0.0447 | 0.0326 | -197.76 | 258.92 | 0.0068 | 0.1079 |
| $[0, 30]$ | 0.0375 | 0.0280 | -138.02 | 110.34 | -0.0047 | 0.0323 |

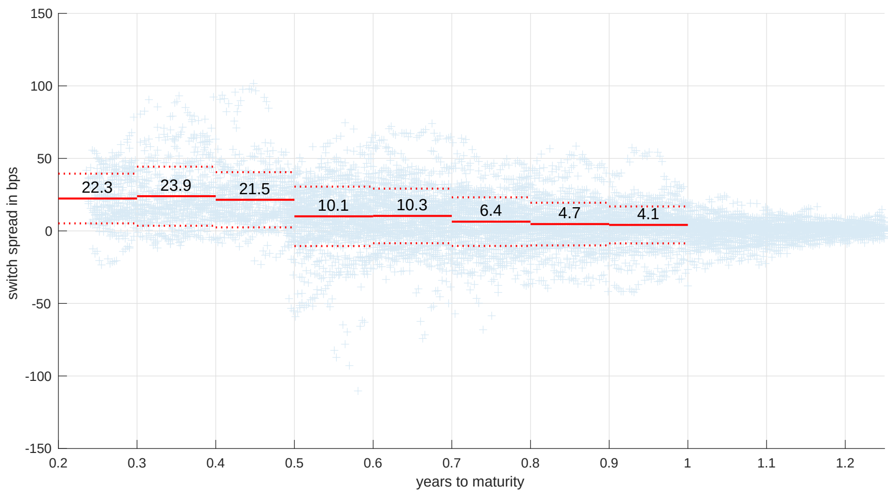
**Notes**: Amihud's illiquidity measure (yield change as a proxy of return, volume taken from BS, when ZTD: volume of 0.1 was imputed to avoid division by zero) , Roll's effective spread measure (the averages reported in this table for Roll's measure are premultiplied by 1e6 solely for the clearer presentation), $\gamma$ a measure proposed by Bao et al. (2011) (the averages reported in this table for G measure are premultiplied by 1e6 as well), Roll's and Gamma measures use 22 days window for covariance $Cov(\Delta ytm_t, \Delta ytm_{t+1})$, where $ytm_t$ is a $ytm$ at the fixing at $t$ date of a certain bond we measure the liquidity.

8. **Historically the least liquid segments were [0, 1.5] and (12,30] years to maturity**. Table 2 presents the mean and standard deviation of three main liquidity measures we characterised in the previous chapter, namely: Amihud's, Roll's and BPW's $\gamma$. It is important to note that the higher Amihud's and BPW's and the lower Roll's measures the less liquid segment appears. Hence, data give a mixed picture as to which segment is the most liquid.

Since Amihud's measure combines two types of information: total return and the volume that follows, we are inclined to say that the most liquid segment is (6, 12] with 0.0289 average and standard deviation of only 0.0242 for the whole 15-year period. Table 8 shows yearly averages of these measures by segment, which offers yet another argument for that claim, as in 8 out of 15 years the segment of (6, 12] has the lowest Amihud's read.

9. **Switch operations make short bonds (eligible to switch from) richer than the interpolated interest rate between NBP bills and** $[1.0, 1.5]$ **segment of bonds**. Key concept we use here is *switch spread* as a difference between linearly interpolated rate between NBP rate and the average *ytm* in the segment $[1.0, 1.5]$ and the *ytm* of a particular shorter than 1.25 years bonds. The rationale is based on the observation that NBP rate is a strong alternative for short term investors in *risk-free* instruments and it is rare (or not recorded) that the market expects the whole monetary policy easing or tightening cycle to last only a year. Table 12 presents descriptive statistics by subsegment of $[0.2, 1.0]$ and Figure 6 ilustrates all the pairs: years to maturity and switch spread for the whole sample period. Clearly, bonds up to 0.5 years to maturity were on average overvalued by $22.5 - 23.9$ bps, the bonds with current tenors in the range of $(0.5, 0.7]$ by $10.1 - 10.3$ bps and the ones falling into time bracket of $(0.7, 1.0]$ by some 4 bps.

Figure 6: *Switch spread* of Polish fixed coupon government bonds in 2005:01-2020:06




Notes: (1) Switch spread is a difference between linearly interpolated rate between NBP rate and the average *ytm* in the segment $[1.0, 1.5]$ and the *ytm* of a particular shorter than 1.25 years bonds (2) light blue crosses represent single observations (3) red solid lines present averages in a given segment and dotted red lines indicate one standard deviation above and below this mean

10. **All segment-wise average yield time series are *trend stationary* when corrected for long term variance a mode de Newley-West for lags of at least 18-months**. Table 3 presents the results of different tests (Kwiatkowski et al. (1992)) for trend stationarity (stationarity around a deterministic trend) for various monthly lags used in long term variance calculations. [12]

[12] We decided to use KPSS approach as the other frameworks like Augmented Dickey-Fuller work with different null hypothesis (that the times series has unit root) and the way in which classical hypothesis testing is carried out ensures that the null hypothesis is accepted unless there is strong evidence against it. It has been shown that these tests have low power against stable autoregressive alternatives with roots near unity and against fractionally integrated alternatives

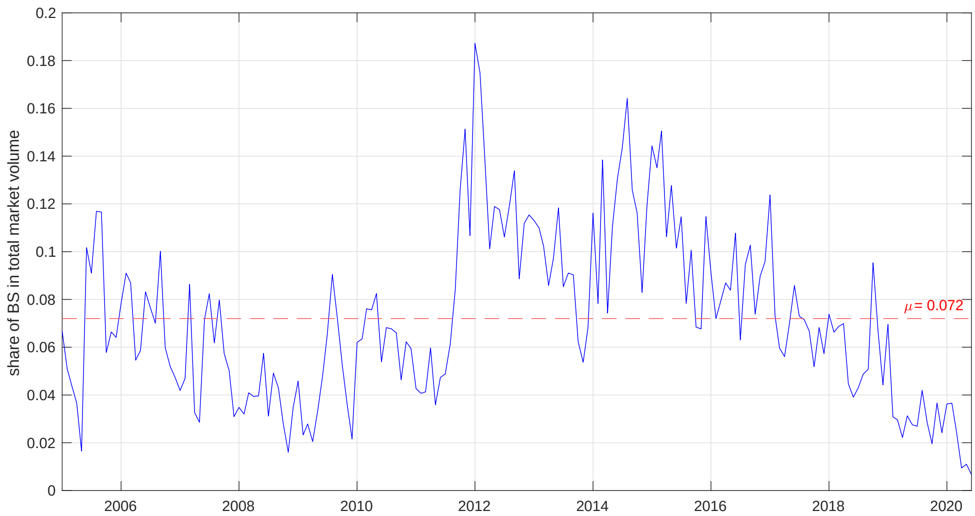
Table 3: KPSS tests with different lags in 2005:01-2020:06 - by segments

| | $[0, 1.5]$ | $(1.5, 3.5]$ | $(3.5, 6.0]$ | $(6.0, 12.0]$ |
|---|---|---|---|---|
| $H(6m)$ | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| $H(8m)$ | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| $H(10m)$ | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| $H(12m)$ | 0 | 1.0000 | 1.0000 | 1.0000 |
| $H(14m)$ | 0 | 1.0000 | 1.0000 | 1.0000 |
| $H(16m)$ | 0 | 0 | 1.0000 | 1.0000 |
| $H(18m)$ | 0 | 0 | 0 | 0 |
| $pV(6m)$ | 0.0100 | 0.0100 | 0.0100 | 0.0100 |
| $pV(8m)$ | 0.0194 | 0.0100 | 0.0100 | 0.0100 |
| $pV(10m)$ | 0.0381 | 0.0178 | 0.0118 | 0.0100 |
| $pV(12m)$ | 0.0596 | 0.0303 | 0.0223 | 0.0192 |
| $pV(14m)$ | 0.0836 | 0.0452 | 0.0356 | 0.0292 |
| $pV(16m)$ | 0.1000 | 0.0633 | 0.0479 | 0.0420 |
| $pV(18m)$ | 0.1000 | 0.0814 | 0.0665 | 0.0541 |
| $stat(6m)$ | 0.2438 | 0.3012 | 0.3255 | 0.3392 |
| $stat(8m)$ | 0.1909 | 0.2346 | 0.2538 | 0.2647 |
| $stat(10m)$ | 0.1603 | 0.1952 | 0.2112 | 0.2205 |
| $stat(12m)$ | 0.1408 | 0.1696 | 0.1831 | 0.1914 |
| $stat(14m)$ | 0.1279 | 0.1518 | 0.1633 | 0.1709 |
| $stat(16m)$ | 0.1189 | 0.1388 | 0.1485 | 0.1556 |
| $stat(18m)$ | 0.1127 | 0.1291 | 0.1371 | 0.1438 |

**Notes**: (1) Null hypothesis: $H_0$ average yields in a given segment are *trend stationary* (2) $H(lag)$ is a result of KPSS test evaluation and: if it reads 0 it means that the test fails to reject the null hypothesis that the times series is trend stationary, and when it is 1 - the test rejects this null hypothesis. All tests are done witg 5% nominal significance level (3) $pV(lag)$ indicate p-value of test statistics, (4) $stat(lag)$ reports the KPSS statistics values (5) lags mean *autocovariance lags* to include in the Newey-West estimator of the long-run variance

11. **Share of BondSpot in total secondary market turnover is erratic and the list of bonds traded is periodically shallow**. Figure 7 shows the history of this ratio over the last 15 years. Since 2012 the share is falling steadily from $0.15 - 0.18$ to below 0.01 recently. The process transpires also from Figure 9 (6th tile) with average *zero trading days* share rising from 0.35 in 2012 to above 0.80 in June 2020. With such a low representation the volume data from BondSpot cannot be reliably taken as a basis for weighting system in our yield curve estimations.

Figure 7: Share of BondSpot in total market turnover 2005:01-2020:06



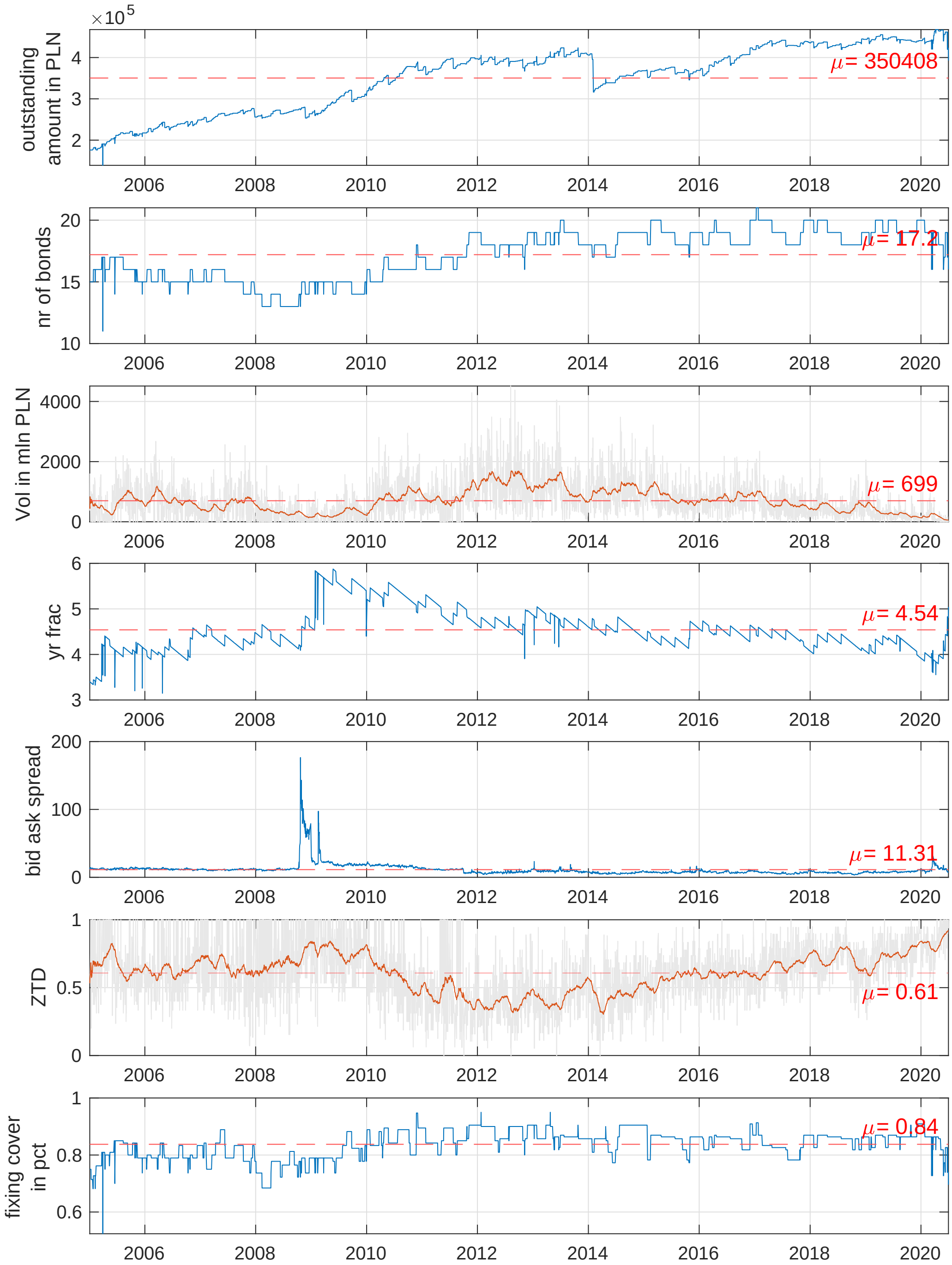

Notes: (1) outright transactions were considered only, as relevant in our enquiry.

Figure 8: Selected liquidity measures of Polish fixed coupon government bonds traded on BondSpot in 2005:01-2020:06 **by segments**

| | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 | 2011 | 2012 | 2013 | 2014 | 2015 | 2016 | 2017 | 2018 | 2019 | 2020$H1$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $A([0,1.5])$ | 0.0589 | 0.0463 | 0.0429 | 0.0967 | 0.0787 | 0.0386 | 0.0269 | 0.0381 | 0.0484 | 0.0272 | 0.0298 | 0.0330 | 0.0275 | 0.0252 | 0.0367 | 0.0712 |
| $A((1.5,3.5])$ | 0.0538 | 0.0429 | 0.0346 | 0.0934 | 0.0700 | 0.0357 | 0.0218 | 0.0210 | 0.0299 | 0.0280 | 0.0334 | 0.0279 | 0.0228 | 0.0219 | 0.0239 | 0.0611 |
| $A((3.5,6])$ | 0.0662 | 0.0290 | 0.0184 | 0.0720 | 0.0564 | 0.0237 | 0.0145 | 0.0110 | 0.0215 | 0.0194 | 0.0329 | 0.0269 | 0.0225 | 0.0270 | 0.0320 | 0.0679 |
| $A((6,12])$ | 0.0377 | 0.0294 | 0.0197 | 0.0567 | 0.0532 | 0.0226 | 0.0119 | 0.0069 | 0.0189 | 0.0197 | 0.0221 | 0.0190 | 0.0230 | 0.0279 | 0.0359 | 0.0854 |
| $A((12,30])$ | - | - | - | - | 0.0495 | 0.0314 | 0.0302 | 0.0114 | 0.0488 | 0.0398 | 0.0791 | 0.0576 | 0.0175 | - | - | - |
| $A([0,30])$ | 0.0559 | 0.0392 | 0.0310 | 0.0819 | 0.0651 | 0.0309 | 0.0199 | 0.0207 | 0.0313 | 0.0252 | 0.0335 | 0.0288 | 0.0245 | 0.0251 | 0.0320 | 0.0721 |
| $R([0,1.5])$ | -268.16 | -211.36 | -215.77 | -216.35 | -251.18 | -172.26 | -154.04 | -178.98 | -211.32 | -108.97 | -183.37 | -151.48 | -136.34 | -110.72 | -167.67 | -234.28 |
| $R((1.5,3.5])$ | -244.65 | -140.63 | -86.70 | -228.39 | -112.68 | -99.08 | -101.95 | -112.52 | -139.26 | -158.20 | -99.25 | -94.31 | -51.42 | -76.14 | -81.01 | -75.05 |
| $R((3.5,6])$ | -309.35 | -157.19 | -74.20 | -184.09 | -77.58 | -63.08 | -107.23 | -114.38 | -141.94 | -199.43 | -86.44 | -79.76 | -68.17 | -110.41 | -86.06 | -46.86 |
| $R((6,12])$ | -238.83 | -154.57 | -52.30 | -246.89 | -105.82 | -56.27 | -120.19 | -84.41 | -187.66 | -213.81 | -63.23 | -111.50 | -91.36 | -77.55 | -48.90 | -137.38 |
| $R((12,30])$ | - | - | - | - | -166.01 | -133.05 | -68.10 | -51.79 | -213.25 | -165.35 | -296.15 | -145.29 | -13.51 | - | - | - |
| $R([0,30])$ | -266.75 | -173.15 | -115.19 | -212.22 | -137.28 | -98.03 | -112.62 | -124.11 | -170.50 | -167.15 | -125.35 | -108.55 | -81.72 | -90.01 | -91.16 | -130.96 |
| $G([0,1.5])$ | 0.0233 | 0.0072 | 0.0182 | -0.0314 | -0.0154 | 0.0102 | 0.0093 | 0.0091 | 0.0090 | 0.0013 | 0.0138 | 0.0071 | 0.0073 | 0.0029 | 0.0087 | 0.0189 |
| $G((1.5,3.5])$ | 0.0108 | -0.0053 | -0.0029 | -0.0023 | -0.0385 | -0.0065 | -0.0022 | 0.0008 | -0.0057 | 0.0023 | -0.0046 | -0.0004 | -0.0032 | 0.0009 | 0.0006 | -0.0339 |
| $G((3.5,6])$ | 0.0212 | -0.0082 | -0.0077 | -0.0248 | -0.0412 | -0.0112 | -0.0027 | -0.0041 | -0.0277 | -0.0015 | -0.0315 | -0.0108 | -0.0063 | 0.0023 | -0.0046 | -0.0730 |
| $G((6,12])$ | -0.0011 | -0.0065 | -0.0120 | 0.0327 | -0.0323 | -0.0132 | 0.0002 | -0.0146 | -0.0161 | 0.0123 | -0.0433 | -0.0164 | -0.0079 | -0.0040 | -0.0160 | -0.0394 |
| $G((12,30])$ | - | - | - | - | 0.0106 | 0.0031 | -0.0029 | -0.0147 | -0.0083 | 0.0075 | 0.0723 | -0.0048 | -0.0071 | - | - | - |
| $G((0,30])$ | 0.0166 | -0.0014 | 0.0007 | -0.0108 | -0.0306 | -0.0050 | 0.0002 | -0.0016 | -0.0099 | 0.0030 | -0.0080 | -0.0045 | -0.0032 | 0.0003 | -0.0033 | -0.0295 |

**Notes**: A - Amihud's illiquidity measure (yield change as a proxy of return, volume taken from BS, when ZTD: volume of 0.1 was imputed to avoid division by zero) , R - Roll's effective spread measure (the averages reported in this table for Roll's measure are pre-multiplied by $1e6$ solely for the clearer presentation), G - $\gamma$ a measure proposed by Bao et al. (2011) (the averages reported in this table for G measure are pre-multiplied by $1e6$ as well), "-" indicates that there are no bonds in a certain segment to calculate the liquidity measures properly. Roll's and Gamma measures use 22 days window for covariance $Cov(\Delta ytm_t, \Delta ytm_{t+1})$, where $ytm_t$ is a $ytm$ at the fixing at $t$ date of a certain bond we measure the liquidity.

Figure 9: Selected characteristics of Polish fixed coupon government bonds traded on BondSpot in 2005:01-2020:06 - all segments




# Filtering rules and weight system framework

The yield curve estimation design in the less liquid markets (as Polish one) has to overcome some qualitative imperfections as compared with, for example US or UK markets. When deciding on filtering out some bonds and then augmenting the information pool with available non-price data, we had the following principles in mind:

- **Principle 1.** Minimise the share of arbitrary decisions.
- **Principle 2.** Do not exclude bonds from the sample entirely, but diminish their weight accordingly, unless the pricing is systemically distorted.

- **Principle 3.** Include as much reliable and useful information (static and dynamic data) as possible to reflect importance of a certain bond in the yield curve formation.
- **Principle 4.** Include only *risk free* rates.

To that end, and bearing in mind the data availability and the stylised facts drafted in the previous subsection we propose the following *filtering (out) rules*:

- **Rule 1.** from the broad set of Polish government bonds we exclude CPI-linkers, floaters, foreign denominated and retail bonds. This is a usual choice in yield curve estimation literature.
- **Rule 2.** we take all pricing information of fixed and zero coupon bonds which are subject to fixing on BondSpot, with an exception in the next bullet:
- **Rule 3.** we exclude bonds with less than 0.85 or 1.20 years to maturity (till mid 2017 and after that period) because their prices and therefore *ytm* are distorted significantly by the switch operations of Polish MinFin (as it was clearly shown).

It is worth underlying firstly, that we **do not exclude** bonds with current maturity greater than 12 years despite proven characteristics of this *ultra long* segment being under-represented (erratic presence and only 1-2 bonds in the segment if any) and the most illiquid of all the segments distinguished. We will let auxiliary data to speak for themselves (turnover and outstanding amounts) via the weight system. Secondly, we also **do not exclude** any bonds that may be described as *off-the-run* or *on-the-run* as it is common in the literature of developed, liquid markets (Gurkaynak et al. (2011)), mainly because of bonds scaristy common to less liquid markets (even if we take all of the bonds they will constitute only a 1/4 or 1/5 of the number of available bonds in US or UK markets, let alone excluding more *off-the-run* bonds). Yet again here, we will differentiate the bonds importance and quality of data through proper weights.

In order to reach our goal of using maximum depth and widest scope of information in the estimation of yield curves we propose the approach summarised below:

1. we include NBP bills rates (rebased to 365 days)[13] as a good ultra-short interest rate which influences expectations of market participants and offer a plausible alternative for any short-horizon investor and stable anchor for the beginning of yield curve. Other typical choices of the interest rate of short-end of the curve include: T-bills rates or xIBOR rates from money market. In Poland T-bills were absent from the financial market's history for a substantial period of time and their current share in both outstanding amounts of public debt as well as in total turnover is minuscule. The WIBOR rates are, of course available, but they are under going a methodology change. What is more, they intertwine *risk free rates* with some degree of *default risk*, which is against our approach to yield curve modelling here - we stick to the pure *default risk free* instruments. Additionally, the underlying market of, say WIBOR1M, WIBOR3M or WIBOR6M is very shallow and there are almost no transactions in the real market. Hence the informational reliability and data quality is questionable.
2. the weight of the ultra-short point on a curve would have to be in line with its significance for fixed income investors in Poland, hence: of the magnitude of the sum of weights of all other points.
3. we would use both outstanding amount and turnover based information to construct the weights, which constitutes a unique way of dealing with market data quality. Combining these two sources of dynamic information allows to flexibly treat *on-* and *off-the-run* issues as well as reflect bigger relative importance in yield curve estimations of huge issues with very high share in turnover and vice versa.

[13] recall that the bonds' *ytm* in Poland is measured as if a year has 365 days, whereas NBP's reference rate is in fact a yield to maturity in simple interest rate model calculated per year of 360 days, hence it should be rebased to 365 days for comparability with the others

Turning now to the possible **framework for weights system** we will test in our yield curve estimation, we distinguish the subsequent degrees of freedom:

1. using or not the Rule 3 (filtering out the eligible for switch bonds) - 2 variants
2. using the full domain of tenors of leaving the longest segment $(12, 30]$ out of estimation - 2 variants
3. for each $t$ use weights for outstanding amounts ($W_{t,i}^{oa}$) or for turnover share ($W_{t,i}^{vol}$) or both for all bonds $i$ under fixing - 3 variants
4. use a weight for the shortest point of the curve (NBP rate) of 1 or 2 times bigger than the sum of all other weights used for bonds - 2 variants

On the top of these 24 sets (as a simple multiplication of variants listed above $2 \times 2 \times 3 \times 2$) we investigate the naive classic system of all equal weights (points 3 and 4 above) in 4 settings as a result of choices made for point 1 and 2, ending up in total 28 systems under investigation.

## Polish yield curve estimation

We have decided to choose, estimate and experiment with a parsimonious yield curve environment, namely, in the popular **Nelson-Siegel-Svensson** form (Nelson & Siegel (1987) and its Svensson (1993, 1994) extension - henceforth: NSS). The reasons for such choice are:

1. we are interested in morphology, dynamics and forecasting power of Polish yield curve with is a much closer goal to the one of monetary and fiscal authorities and further away from trading and valuation of government securities domain in which *tightness* of the estimated curves is key. In our enquiry, we recognise the following attractive characteristics of NSS fitting (recall from the literature review):
   (a) the greatest flexibility at the short end of the curve, where it is needed the most also for the term premia structure estimation.
   (b) estimated curve is asymptotically flat for ultra long maturities by construction and definition.
   (c) ability to capture local lack of monotonicity of the term structure (spot and forward rates) thanks to two *humps*
   (d) easy decomposition of parsimonious curve to *level*, *slope* and *curvature* elements
   (e) the resultant curves are usually very smooth if compared with *spline*, which is a very desirable feature as a yield curve may be viewed by (macro)economists as a collection of inter-temporal marginal rates of substitution. With such an interpretation in mind it would be unreasonable to expect yield curve to be rough or zig-zacking.
2. in what follows, we propose to overcome almost all the numerical challenges (drawback of the NSS approach) listed in the previous chapter
   (a) *each market may have its own heuristics with regard to possible shapes on the yield curve* - we have acknowledged it by carefully examining Polish market's data and have ran multiple tests and visual data inspection in order to create a list of different starting vectors of parameters $\Theta$ implying various shapes of yield curves encountered.
   (b) *the constrained optimisation methods may become particularly slow* - we have tasted and used Matlab's routine of *internal-point* whereas key step in achieving efficiency is the preparation of bonds cash flow matrix and time to maturity vectors prior to the iteration *pre se*. The details will be given in this subsection.

(c) *different combinations of starting parameters may produce an equally good fit to observed data* - we have not experienced the precise same *goodness-of-fit* measures for different parameters, but we accept that such situation is probable, and therefore we have developed more composite measure of *goodness* including wider scope of statistics. Rules of such a ranking will be presented in subsequent subsections.

(d) *there is usually a set of starting values of* $\Theta$ *needed [...]* - as explained above, such a set was prepared with more than 30 staring vectors, based on historical contexts.

(e) *overly smooth specification of the yield curve fitting may disguise some important issue-characteristic or term structure related economic information in government bonds' prices (i.e. tax effects, supply effects)* - we have already proposed in filtering rules a treatment of such specific bonds in Poland either by excluding them (*switch bonds*) or limiting their informational value in line with the importance for the market, which is measured by the outstanding amounts and turnover on the secondary market.

3. we would avoid the drawbacks of *spline* methods, which are particularly undesired in the task ahead):

   (a) lack of underlying financial or economic theory

   (b) poor asymptotic behaviour of the long term rates

   (c) oscillations of the estimated forward rates

   (d) the estimations depend greatly on the location of the knot points between different segments of the curve, arbitrarily chosen in the procedure

The NSS parsimony accompanied by different weighting systems testing environment provide us with enough degrees of freedom in our pursue of better than traditional setups. We have obtained average errors of magnitude less than one basis point, whereas the bid ask spreads observed in the market in different segments ranged from 5-6 to 30-40 basis (cf. Table 1) in the cases of ultra short *switch bonds*. Therefore we believe that there are other qualities of the estimated yield curves we should concentrate (i.e. smoothness) rather than *spurious precision*.
In this subsection we implement and test 28 particular filtering and weight systems to estimate full time series of parameters of NSS yield curve $\Theta = (\beta_0, \beta_1, \beta_2, \beta_3, \tau_1, \tau_2)$ for every date $t$ in a dataset (DS):

$$y(x) = \beta_0 + (\beta_1 + \beta_2)\frac{1 - e^{\left(-\frac{x}{\tau_1}\right)}}{x/\tau_1} - \beta_2 e^{\left(-\frac{x}{\tau_1}\right)} + \beta_3 \left[\frac{1 - e^{\left(-\frac{x}{\tau_2}\right)}}{\tau/\tau_2} - e^{\left(-\frac{x}{\tau_2}\right)}\right] \tag{1}$$

At the first glance, the form of parsimonious curve may seem assuming continuous compounding of interest rate, but in reality the exponential function used has just a shape-forming role and we can easily estimate yield curve for spot rates that are continuously compounded ($\mathcal{C}_{yc}^{cont}$) and with annual capitalisation ($\mathcal{C}_{yc}^{ann}$) separately. Obviously, they will have slightly different parameters and should be used to produce discount factors using different formulae. The yields are used to discount cash-flows from bonds to produce their estimated dirty prices. In the final presentation of the chosen system that would be checked for robustness we would provide NSS parameters for both versions as well as a refitted NSS curve into *ytm** curve that is implied but previously fitted zero coupon yields.

The general problem we are facing at every date $t$ in our dataset is finding the vector of parameters $\Theta$ which solves with assumed, sufficient accuracy:

$$\min_{\Theta} \mathcal{O} = \min_{\Theta} \left\{ \sum_{i=1}^{N} W_i (P_i(\Theta) - p_i)^2 + W^{nbp}(R(\Theta) - r^{nbp})^2 \right\} \quad \text{s.t.} \quad \mathcal{C}(\Theta) \tag{2}$$

where $W_i$ is a combined weight of modified duration $W_i^{md}$ and either $W_i^{oa}$ or $W_i^{vol}$ or their sum:

$$W_i = \begin{cases} W_i^{md} W_i^{oa}, & \text{outstanding amounts weights only} \\ W_i^{md} W_i^{vol}, & \text{turnover weights only} \\ W_i^{md}\left(W_i^{oa} + W_i^{vol}\right), & \text{combined weights} \end{cases} \tag{3}$$

and the weight for the *ultra-short* end of the curve is defined as follows (with $\gamma$ being a scalar multiplier (1 or 2, in our simulations)):

$$W^{nbp} = \begin{cases} \gamma \sum_{i=1}^{N} W_i^{oa}, & \text{outstanding amounts weights only} \\ \gamma \sum_{i=1}^{N} W_i^{vol}, & \text{turnover weights only} \\ \gamma \sum_{i=1}^{N}\left(W_i^{oa} + W_i^{vol}\right), & \text{combined weights} \end{cases} \tag{4}$$

Additionally, in the objective function we have $R(\Theta)$ which stands for estimated short term interest rate using set of parameters $\Theta$ and $r^{nbp}$ is the prevailing on that day NBP rate (rebased to 365 days). It is worth underlying that the weights $W_i^{oa}$, $W_i^{vol}, W_i^{md}$ are calculated for every day $t$ in the dataset, based on static data.

**Input:** $\forall t$ prices $p_{i,t}$, weights $W_{i,t}$, rates $r_t^{nbp}$ and a list starting values $\hat{\Theta}$
**Output:** time series of optimal $\Theta^*$ for each $t$
**for** $t \in DS$ **do**
  **retrieve** from database for date $t$: $p_i$ and chosen set of weights $W_i$ for every $i$-bond and $W^{nbp}$, $r^{nbp}$
  **create** $\mathcal{B}^{cf}$ and vector $\mathbf{t}$ using static data at $t$
  **for** $s \in \hat{\Theta}$ **do**
    **while** *tolerance conditions not met* **do**
      **calculate** $P(\Theta_k) = \mathcal{B}^{cf} \times df(\Theta_k, \mathbf{t})$, $R(\Theta_k)$
      **calculate** objective function value in $k$-th iteration
    **end**
    **return** $\Theta_s^*$
  **end**
  **choose** $\Theta_s^*$ with the lowest objective function's value ($\mathcal{O}$)
  **store** $\Theta^* \equiv \min_s \mathcal{O}(\Theta_s^*)$ for the date $t$
**end**

**Algorithm 1:** Calculating time series of the optimal $\Theta^*$ (**faster** version)

We believe that it is reasonable in the prevailing negative interest rate environments that the typical constraint for NSS on the short term zero-coupon rates to be greater than zero ($\beta_0 + \beta_1 > 0$) should be modified and set at $-2\%$. On one hand, this will give sufficient space for the curve to be fitted in Polish case of mid 2020, and on the other will still act as a non-slack constraint of parameter space. The negative boundary for short term rates at this level is more and more common in yield curve estimation in the Switzerland or the Eurozone and we believe that is allows for some, although minimal these days, possibility of some negative rates, improving the fit in this segment of the curve. We reckon that the assumption that long term interest rates are positive is still valid ($\beta_0 > 0$). Hence the modified constraints are:

$$\mathcal{C}(\Theta): \begin{cases} \beta_0 > 0 \\ \beta_0 + \beta_1 > -2\% \\ \tau_1 > 0 \\ \tau_2 > \tau_1 \end{cases} \tag{5}$$

There are some technical niceties which are crucial for the algorithm to converge fast enough for the whole dates in DS scope. Firstly, we implement some version of *trusted region* for our parameters $\Theta$. We checked experimentally broad enough boundaries for parameters $\Theta$ when minimising: lower bound $\underline{\Theta} = [-0.10, -0.15, -2, -2, 0.15, 0]$ and upper bound $\overline{\Theta} = [0.15, 0.15, 2, 2, 10, 305]$. Secondly, in the minimisation problem 2 we need swift calculation of the objective function's value as we are to call it roughly $1e6$ times[14] in the algorithm. Hence if our basic call to objective function lasts just one second, the total calculation time for a DS would end up in the region of 2 weeks. Testing time of all 28 weighting sets of parameters would be prohibitively long. The solution to this issue is to create an aggregated matrix of cash flows for each bond $\mathcal{B}^{cf}$ (the same for each iteration) and multiply it by a vector of discount factors $\mathbf{df}_k = df(\Theta_k, \mathbf{t})$ different for each iteration $k$ in $\Theta$ space, instead of summing of weighted squared differences $P_i(\Theta_k) - p_i$ bond by bond (for $i \in [1, N_t]$, where $N_t$ is a particular number of bonds taken to estimation at date $t$).

In particular, we define $\mathcal{B}^{cf}$ for a given date (with dimensions: $[N_t \times \sum_{i=1}^{N_t} M_i]$, where $M_i$ is the number of future cash flows from $i$-th bond):

$$\mathcal{B}^{cf} = \begin{bmatrix} CF_{1,1} & ... & CF_{1,j} & ... & CF_{1,M_1} & 0 & 0 & 0 & 0 & 0 & ... & ... & ... & ... & ... & ... & 0 \\ 0 & 0 & 0 & 0 & 0 & CF_{2,1} & ... & CF_{2,j} & ... & CF_{2,M_2} & 0 & ... & ... & ... & ... & ... & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \ddots & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & ... & 0 & CF_{N_t,1} & ... & CF_{N_t,j} & ... & CF_{N_t,M_2} \end{bmatrix} \tag{6}$$

and the vector $\mathbf{t}$ consists of column-wise common year fractions of cashflows in $\mathcal{B}^{cf}$ matrix:

$$\mathbf{t} = \begin{bmatrix} t_{1,1} & ... & t_{1,i} & ... & t_{1,M1} & t_{2,1} & ... & t_{2,i} & ... & t_{2,M_2} & ... & t_{N_t,1} & ... & t_{N_t,i} & ... & t_{N_t,M_2} \end{bmatrix}' \tag{7}$$

Finally we allow function $df()$ to return a vector of discount factors (annually or continuously compounded - depending on a set-up) taking arguments of the above-defined $\mathbf{t}$ and the current set of parameters in $k$-th iteration $\Theta_k$, which in turn generate zero-coupon rates prevailing for those parameters. In such defined framework we have the column vector of modelled prices:

$$P(\Theta_k) = \mathcal{B}^{cf} \times df(\Theta_k, \mathbf{t}) \tag{8}$$

The speed advantage of this approach (Algorithm 1) as compared with calling cash flow schedule for each bond in each objective function call (Algorithm 2 in the Appendix on page 53 ) is precisely due to creation of re-usable big cash flow matrix of all bonds on a certain date and changing only the discount factor vector in every iteration and multiplication of a matrix by a vector is relatively cheap operation.

[14] we have approximately 4000 days and the experientially checked number of calls per yield curve estimation for a given day is in the range of 100-300

Table 4: List of starting vectors of parameters $\hat{\Theta}$

| $\beta_0$ | $\beta_1$ | $\beta_2$ | $\beta_3$ | $\tau_1$ | $\tau_2$ | $\beta_0$ | $\beta_1$ | $\beta_2$ | $\beta_3$ | $\tau_1$ | $\tau_2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $ytm_{N-1}$ | $ytm_1 - ytm_{N-1}$ | -0.5 | 0.1 | 1 | 15 | 0.05 | 0 | 0.5 | -1 | 2 | 15 |
| $ytm_{N-1}$ | $ytm_1 - ytm_{N-1}$ | -0.5 | -1.5 | 1 | 15 | $ytm_{N-1}$ | 0 | -1 | 1.1 | 2 | 22 |
| $ytm_{N-1}$ | $ytm_1 - ytm_{N-1}$ | 0.5 | 2 | 1 | 15 | $ytm_{N-1}$ | 0 | -1 | -2 | 2 | 15 |
| $ytm_{N-1}$ | $ytm_1 - ytm_{N-1}$ | 1 | 3 | 1 | 15 | $ytm_{N-1}$ | 0 | 0.5 | -1 | 2 | 15 |
| $ytm_{N-1}$ | $ytm_1 - ytm_{N-1}$ | -0.5 | 0.1 | 1 | 19 | 0.06 | -0.020 | -0.01 | -0.03 | 0.35 | 30 |
| $ytm_{N-1}$ | $ytm_1 - ytm_{N-1}$ | -1 | 0.1 | 2 | 15 | 0.04 | -0.015 | -0.01 | -0.03 | 0.35 | 30 |
| $ytm_{N-1}$ | $ytm_1 - ytm_{N-1}$ | -2 | 0.1 | 2 | 15 | 0.02 | -0.005 | -0.01 | -0.03 | 0.35 | 30 |
| $ytm_{N-1}$ | $ytm_1 - ytm_{N-1}$ | 1 | -0.5 | 1 | 15 | 0.06 | -0.020 | -0.01 | -0.03 | 0.55 | 20 |
| $ytm_{N-1}$ | $ytm_1 - ytm_{N-1}$ | 2 | 0.2 | 2 | 19 | 0.04 | -0.015 | -0.01 | -0.03 | 0.55 | 20 |
| $ytm_{N-1}$ | $ytm_1 - ytm_{N-1}$ | -0.1 | 0.5 | 1 | 8 | 0.02 | -0.005 | -0.01 | -0.03 | 0.55 | 20 |
| $ytm_{N-1}$ | $ytm_1 - ytm_{N-1}$ | -1 | 0.1 | 2 | 8 | 0.04 | 0.020 | -0.01 | 0.03 | 0.35 | 30 |
| $ytm_{N-1} \times 1.5$ | $ytm_1 - ytm_{N-1}$ | -1 | 0.1 | 2 | 19 | 0.02 | 0.015 | -0.01 | 0.03 | 0.35 | 30 |
| $ytm_{N-1} \times 1.5$ | $ytm_1 - ytm_{N-1}$ | -1 | 0.1 | 2 | 22 | 0.01 | 0.005 | -0.01 | 0.03 | 0.35 | 30 |
| $ytm_{N-1} \times 0.5$ | $ytm_1 - ytm_{N-1}$ | -1 | 0.1 | 2 | 19 | 0.04 | 0.020 | -0.01 | 0.03 | 0.55 | 20 |
| $ytm_{N-1} \times 0.5$ | $ytm_1 - ytm_{N-1}$ | -1 | 0.1 | 2 | 22 | 0.02 | 0.015 | -0.01 | 0.03 | 0.55 | 20 |
| 0.05 | 0 | -1 | 1.1 | 2 | 22 | 0.01 | 0.005 | -0.01 | 0.03 | 0.55 | 20 |
| 0.05 | 0 | -1 | 1.1 | 1 | 10 | | | | | | |

**Notes**: (1) additionally we use last fitted parameters (from $t-1$ date) (2) $ytm_N$ is the *ytm* of a bond that is the longest in a fixing table for a given date $t$, analogously: $ytm_{N-1}$ is the *ytm* of a bond that is second to the longest and $ytm_1$ is the *ytm* of the shortest bond

For a single optimisation run we used *fmincon* function in Matlab with its default set-up based on the algorithm of *interior point* described and developed by Byrd et al. (1999) and Waltz et al. (2006).
In what follows, we have analysed the results of Polish government yield curve estimations in 28 different weight/filtering systems in the ensuing groups of measures:

1. **statistics of estimated parameters** $\Theta_t$ i.e. mean, median, standard deviation, interquartile range, max-min range
2. **statistics of *goodness-of-fit*** MAE, WMAE, maximum absolute difference, hit ratio, cheap-/rich ratios,
3. **auxiliary characteristics of estimated interest rates**: smoothness (henceforth: SMO, as in Equation **??**), short rates fit[15] , volatility and level of synthetic interest rates in segments
4. **optimisation algorithm - related**: exit flags, number of iterations, number of calls to objective function, execution time (in seconds)

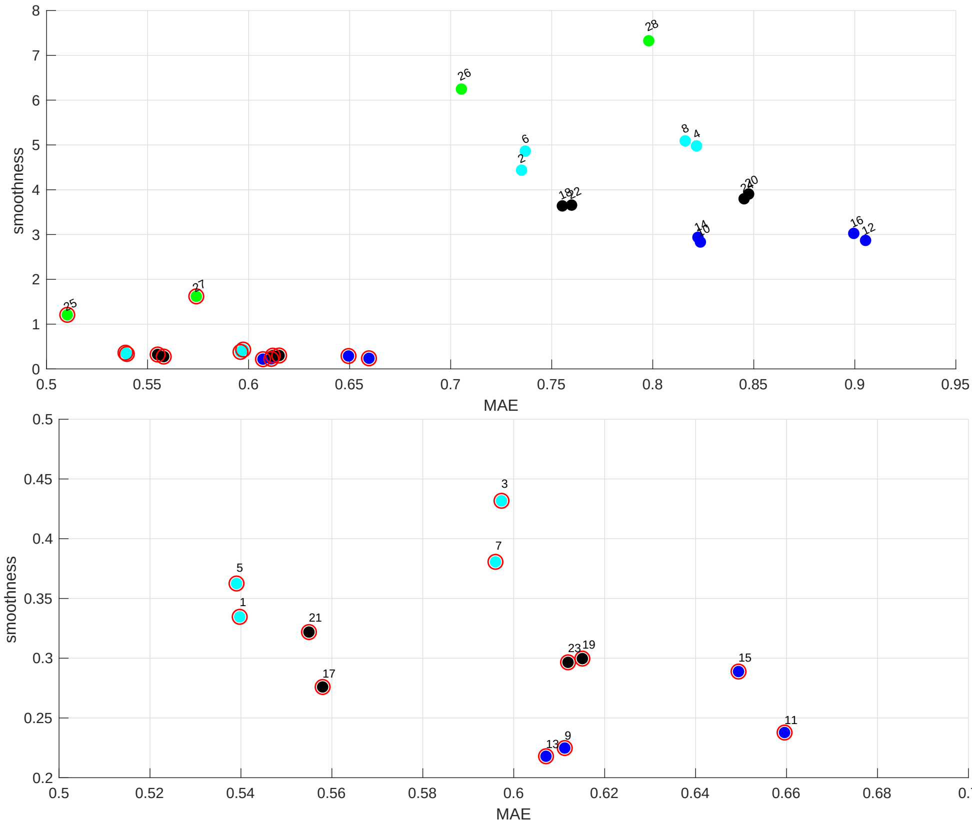
With the aim to verify a hypothesis that *there exist a class of weighting schemes which improves fit relative to conventionally used methods*, we propose two angles of analysis. First we select two dimensions: MAE for the *goodness-of-fit* and SME for the *smoothness/roughness* in which we plot the results for all the systems (Figure 10). Recall from the literature review, that the smoothness measure proposed is close to zero when the yield curve is very smooth and higher in the situations in which the yield curve is *rough* in the sense that the second derivative changes sign quite often. Having the results for all 28 sets we made the following observations:

1. conventional weighting (all equal weights in the yield space) results in the roughest and not well fitted yield curves
2. systems based on combined outstanding amounts and turnover give less rough yield curves then conventional with only slightly worse mean error
3. systems based on solely on outstanding amounts produce even smoother curves but with a trade-off with MAE
4. systems based on solely on turnover give the smoothest curves and yet again worse MAE

[15] a difference between fitted $y(1/52)$ and rebased for 365 days NBP reference rate - in basis points

5. excluding *eligible-for-switch* bonds improves smoothness (lowers the roughness) and error (lowers MAE) significantly (roughness by the factor of 5 and MAE by approx. 30%).

Figure 10: *Goodness-of-fit* and *smoothness* of all 28 tested weight systems




Notes: (1) Lower panel zooms in a part of the upper panel for MAE in the range of $[0.5, 0.7]$ and *smoothness* in the range of $[0.2, 0.5]$ (2) the labels are in line with number of weight systems in Table **??**. (3) green dots - are for system with $W = 4$ (equal weights), cyan dots - $W = 3$ (combined outstanding amount and turnover), blue dots - $W = 2$ (turnover), black dots - $W = 1$ (outstanding amounts) (4) ∘ indicates a system where *eligible for switch bonds* are excluded from the estimation.

From the set of pairs: (MAE, SMO) we may extract the ones which form a trade-off frontier (henceforth: TOF) - the subset of pairs for which there is no better MAE without worsening SMO and vice-versa. This subset consists of the weight systems with numbers: 1, 5, 13, 17 and 25. Common features of these systems are the decisions: to exclude *eligible-for-switch* bonds and **not** to truncate the domain to 12 years. With the exception of system labelled 25 (equal weights), the TOF's systems are based on the idea of heavy weight of the short end of the curve (with $\gamma = \{1, 2\}$. Solely this findings are enough to support the hypothesis of existence of a class of weights that improves fit relative to conventional methods of equal weights.

As a second angle we propose here a **composite measure** based on rankings of the full set of 28 systems in the following categories: average standard deviation of yields in selected tenors of the curve, average max-min range, average interquartile range, average MAE, average MAD (mean absolute difference) all over the time axis, short rate fit (an absolute difference between fitted $y(1/52)$ and rebased for 365 days NBP reference rate) and the smoothness/roughness measure introduced in the previous chapter. Therefore we have 3 volatility, 3 error and 1 smoothness measures fro which the less is better - less volatility, smaller errors and less rough curve. Upper panel of Table 5 presents readings of these measures whereas the lower panel consists of their rankings as well as the average rank across the seven categories. The best 5 systems as far as this composite measure is concerned are: 1, 17, 5, 21 and ex equo: 9, 19 and 23. Yet again these systems have similar characteristics to the ones in TOF and have a common part of systems with numbers: 1, 5 and 17.

Table 5: Volatility, error and smoothness statistics for all 28 tested weight systems

| | Weight system | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | exc. sw | $< 12y$ | W | $\gamma$ | STD | MMR | IQR | MAE | MAD | SRF | SMO |
| 1. | Y | N | 3 | 2 | 0.01549 | 0.06695 | 0.02887 | 0.53973 | 8.94 | 0.06911 | 0.33464 |
| 2. | N | N | 3 | 2 | 0.01559 | 0.06843 | 0.02879 | 0.73507 | 13.30 | 0.18038 | 4.43741 |
| 3. | Y | Y | 3 | 2 | 0.01551 | 0.06739 | 0.02888 | 0.59731 | 8.44 | 0.05495 | 0.43175 |
| 4. | N | Y | 3 | 2 | 0.01561 | 0.06906 | 0.02880 | 0.82167 | 12.64 | 0.16288 | 4.97872 |
| 5. | Y | N | 3 | 1 | 0.01549 | 0.06683 | 0.02887 | 0.53903 | 8.87 | 0.13172 | 0.36253 |
| 6. | N | N | 3 | 1 | 0.01560 | 0.06874 | 0.02880 | 0.73691 | 13.26 | 0.35590 | 4.86259 |
| 7. | Y | Y | 3 | 1 | 0.01551 | 0.06734 | 0.02888 | 0.59597 | 8.39 | 0.10495 | 0.38065 |
| 8. | N | Y | 3 | 1 | 0.01561 | 0.06908 | 0.02881 | 0.81605 | 12.60 | 0.31642 | 5.09149 |
| 9. | Y | N | 2 | 2 | 0.01549 | 0.06689 | 0.02889 | 0.61122 | 11.06 | 0.07343 | 0.22483 |
| 10. | N | N | 2 | 2 | 0.01558 | 0.06865 | 0.02882 | 0.82359 | 15.79 | 0.16588 | 2.83313 |
| 11. | Y | Y | 2 | 2 | 0.01551 | 0.06734 | 0.02891 | 0.65956 | 9.86 | 0.06069 | 0.23779 |
| 12. | N | Y | 2 | 2 | 0.01560 | 0.06897 | 0.02882 | 0.90533 | 14.80 | 0.15185 | 2.86892 |
| 13. | Y | N | 2 | 1 | 0.01549 | 0.06704 | 0.02889 | 0.60710 | 11.02 | 0.13718 | 0.21794 |
| 14. | N | N | 2 | 1 | 0.01558 | 0.06854 | 0.02883 | 0.82237 | 15.84 | 0.33084 | 2.93779 |
| 15. | Y | Y | 2 | 1 | 0.01551 | 0.06746 | 0.02891 | 0.64944 | 9.74 | 0.10915 | 0.28884 |
| 16. | N | Y | 2 | 1 | 0.01559 | 0.06889 | 0.02883 | 0.89948 | 14.74 | 0.28565 | 3.02518 |
| 17. | Y | N | 1 | 2 | 0.01548 | 0.06713 | 0.02886 | 0.55797 | 9.26 | 0.07709 | 0.27593 |
| 18. | N | N | 1 | 2 | 0.01558 | 0.06865 | 0.02878 | 0.75527 | 13.74 | 0.19543 | 3.63927 |
| 19. | Y | Y | 1 | 2 | 0.01550 | 0.06718 | 0.02888 | 0.61509 | 8.60 | 0.06311 | 0.29967 |
| 20. | N | Y | 1 | 2 | 0.01560 | 0.06896 | 0.02881 | 0.84746 | 13.21 | 0.18602 | 3.90386 |
| 21. | Y | N | 1 | 1 | 0.01548 | 0.06711 | 0.02886 | 0.55496 | 9.17 | 0.14036 | 0.32189 |
| 22. | N | N | 1 | 1 | 0.01558 | 0.06859 | 0.02879 | 0.75983 | 13.79 | 0.38757 | 3.65704 |
| 23. | Y | Y | 1 | 1 | 0.01550 | 0.06714 | 0.02887 | 0.61194 | 8.57 | 0.11622 | 0.29654 |
| 24. | N | Y | 1 | 1 | 0.01560 | 0.06895 | 0.02881 | 0.84522 | 13.24 | 0.35922 | 3.79850 |
| 25. | Y | N | 4 | - | 0.01551 | 0.06818 | 0.02887 | 0.51024 | 7.47 | 1.23639 | 1.20791 |
| 26. | N | N | 4 | - | 0.01564 | 0.06877 | 0.02877 | 0.70530 | 10.26 | 4.31371 | 6.24691 |
| 27. | Y | Y | 4 | - | 0.01553 | 0.06825 | 0.02888 | 0.57409 | 7.10 | 1.09017 | 1.62035 |
| 28. | N | Y | 4 | - | 0.01566 | 0.06906 | 0.02881 | 0.79804 | 9.86 | 3.93117 | 7.32579 |

| | Weight system | | | | | | ranks in the category | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | exc. sw | $< 12y$ | W | $\gamma$ | STD | MMR | IQR | MAE | MAD | SRF | SMO | Average mark |
| 1. | Y | N | 3 | 2 | 4.00 | 3.00 | 17.00 | 3.00 | 8.00 | 4.00 | 9.00 | **6.86** |
| 2. | N | N | 3 | 2 | 19.00 | 15.00 | 4.00 | 16.00 | 22.00 | 16.00 | 23.00 | 16.43 |
| 3. | Y | Y | 3 | 2 | 13.00 | 11.00 | 22.00 | 8.00 | 4.00 | 1.00 | 12.00 | 10.14 |
| 4. | N | Y | 3 | 2 | 25.00 | 26.00 | 6.00 | 22.00 | 18.00 | 14.00 | 25.00 | 19.43 |
| 5. | Y | N | 3 | 1 | 3.00 | 1.00 | 18.00 | 2.00 | 7.00 | 10.00 | 10.00 | **7.29** |
| 6. | N | N | 3 | 1 | 22.00 | 20.00 | 5.00 | 17.00 | 21.00 | 22.00 | 24.00 | 18.71 |
| 7. | Y | Y | 3 | 1 | 11.00 | 9.00 | 24.00 | 7.00 | 3.00 | 7.00 | 11.00 | 10.29 |
| 8. | N | Y | 3 | 1 | 26.00 | 28.00 | 9.00 | 21.00 | 17.00 | 20.00 | 26.00 | 21.00 |
| 9. | Y | N | 2 | 2 | 5.00 | 2.00 | 25.00 | 10.00 | 16.00 | 5.00 | 2.00 | **9.29** |
| 10. | N | N | 2 | 2 | 15.00 | 18.00 | 12.00 | 24.00 | 27.00 | 15.00 | 15.00 | 18.00 |
| 11. | Y | Y | 2 | 2 | 9.00 | 10.00 | 27.00 | 14.00 | 13.00 | 2.00 | 3.00 | 11.14 |
| 12. | N | Y | 2 | 2 | 21.00 | 25.00 | 11.00 | 28.00 | 26.00 | 13.00 | 16.00 | 20.00 |
| 13. | Y | N | 2 | 1 | 6.00 | 4.00 | 26.00 | 9.00 | 15.00 | 11.00 | 1.00 | 10.29 |
| 14. | N | N | 2 | 1 | 17.00 | 16.00 | 13.00 | 23.00 | 28.00 | 21.00 | 17.00 | 19.29 |
| 15. | Y | Y | 2 | 1 | 12.00 | 12.00 | 28.00 | 13.00 | 11.00 | 8.00 | 5.00 | 12.71 |
| 16. | N | Y | 2 | 1 | 20.00 | 22.00 | 14.00 | 27.00 | 25.00 | 19.00 | 18.00 | 20.71 |
| 17. | Y | N | 1 | 2 | 2.00 | 6.00 | 16.00 | 5.00 | 10.00 | 6.00 | 4.00 | **7.00** |
| 18. | N | N | 1 | 2 | 16.00 | 19.00 | 2.00 | 18.00 | 23.00 | 18.00 | 19.00 | 16.43 |
| 19. | Y | Y | 1 | 2 | 8.00 | 8.00 | 21.00 | 12.00 | 6.00 | 3.00 | 7.00 | **9.29** |
| 20. | N | Y | 1 | 2 | 24.00 | 24.00 | 8.00 | 26.00 | 19.00 | 17.00 | 22.00 | 20.00 |
| 21. | Y | N | 1 | 1 | 1.00 | 5.00 | 15.00 | 4.00 | 9.00 | 12.00 | 8.00 | **7.71** |
| 22. | N | N | 1 | 1 | 18.00 | 17.00 | 3.00 | 19.00 | 24.00 | 24.00 | 20.00 | 17.86 |
| 23. | Y | Y | 1 | 1 | 7.00 | 7.00 | 20.00 | 11.00 | 5.00 | 9.00 | 6.00 | **9.29** |
| 24. | N | Y | 1 | 1 | 23.00 | 23.00 | 7.00 | 25.00 | 20.00 | 23.00 | 21.00 | 20.29 |
| 25. | Y | N | 4 | - | 10.00 | 13.00 | 19.00 | 1.00 | 2.00 | 26.00 | 13.00 | 12.00 |
| 26. | N | N | 4 | - | 27.00 | 21.00 | 1.00 | 15.00 | 14.00 | 28.00 | 27.00 | 19.00 |
| 27. | Y | Y | 4 | - | 14.00 | 14.00 | 23.00 | 6.00 | 1.00 | 25.00 | 14.00 | 13.86 |
| 28. | N | Y | 4 | - | 28.00 | 27.00 | 10.00 | 20.00 | 12.00 | 27.00 | 28.00 | 21.71 |

**Notes**: (1) list of tenors for the calculation of statistics [1/52, 1/12, 0.25, 0.5, 1, 2, 3, 4, 5, 7, 10, 12] (2) STD stands for standard deviation, MMR for max min range, IQR for interquartile range, MAE - mean average error, MAD - maximum absolute difference (in basis points), SRF - short rate fit (an absolute difference between fitted $y(1/52)$ and rebased for 365 days NBP reference rate - in basis points) (2) first five columns indicate: identification number of a weight set, decision to exclude eligible for switch bonds, decision to limit the time domain to 12 years to maturity, weight base: 1 - $W^{oa}$, 2- $W^{vol}$, 3 - $W^{oa} + W^{vol}$ and 4 - all equal weights (3) short rate fit is (4) smoothness is calculated as proposed in Equation **??** (5) the bold marks indicate that a particular system is in top 5.

Table 6: Descriptive statistics of zero coupon rates for different tenors and systems

| | 1/52 | 1/12 | 1/4 | 1/2 | 1 | 2 | 3 | 4 | 5 | 7 | 10 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| mean(1) | 0.0321 | 0.0321 | 0.0321 | 0.0323 | 0.0330 | 0.0348 | 0.0368 | 0.0386 | 0.0401 | 0.0425 | 0.0446 | 0.0454 |
| std(1) | 0.0154 | 0.0155 | 0.0157 | 0.0161 | 0.0165 | 0.0167 | 0.0163 | 0.0158 | 0.0154 | 0.0146 | 0.0140 | 0.0139 |
| max(1) | 0.0659 | 0.0657 | 0.0666 | 0.0705 | 0.0742 | 0.0752 | 0.0742 | 0.0731 | 0.0723 | 0.0722 | 0.0721 | 0.0717 |
| min(1) | 0.0010 | 0.0008 | 0.0005 | 0.0001 | -0.0002 | 0.0007 | 0.0026 | 0.0047 | 0.0066 | 0.0098 | 0.0118 | 0.0117 |
| Q1(1) | 0.0152 | 0.0152 | 0.0151 | 0.0151 | 0.0156 | 0.0171 | 0.0198 | 0.0226 | 0.0252 | 0.0292 | 0.0323 | 0.0331 |
| median(1) | 0.0355 | 0.0359 | 0.0369 | 0.0382 | 0.0395 | 0.0399 | 0.0405 | 0.0417 | 0.0430 | 0.0454 | 0.0469 | 0.0480 |
| Q3(1) | 0.0456 | 0.0453 | 0.0447 | 0.0445 | 0.0454 | 0.0478 | 0.0504 | 0.0525 | 0.0542 | 0.0562 | 0.0574 | 0.0578 |
| mean(28) | 0.0318 | 0.0316 | 0.0314 | 0.0316 | 0.0326 | 0.0348 | 0.0368 | 0.0386 | 0.0401 | 0.0425 | 0.0445 | 0.0453 |
| std(28) | 0.0155 | 0.0158 | 0.0163 | 0.0166 | 0.0168 | 0.0167 | 0.0163 | 0.0158 | 0.0154 | 0.0147 | 0.0141 | 0.0140 |
| max(28) | 0.0660 | 0.0686 | 0.0769 | 0.0779 | 0.0752 | 0.0744 | 0.0738 | 0.0731 | 0.0729 | 0.0730 | 0.0731 | 0.0730 |
| min(28) | 0.0008 | 0.0007 | 0.0005 | 0.0002 | -0.0001 | 0.0007 | 0.0026 | 0.0046 | 0.0067 | 0.0098 | 0.0116 | 0.0110 |
| Q1(28) | 0.0152 | 0.0148 | 0.0147 | 0.0148 | 0.0153 | 0.0170 | 0.0198 | 0.0227 | 0.0253 | 0.0292 | 0.0320 | 0.0327 |
| median(28) | 0.0344 | 0.0348 | 0.0356 | 0.0369 | 0.0389 | 0.0400 | 0.0409 | 0.0419 | 0.0430 | 0.0452 | 0.0469 | 0.0480 |
| Q3(28) | 0.0454 | 0.0447 | 0.0439 | 0.0439 | 0.0450 | 0.0478 | 0.0504 | 0.0525 | 0.0542 | 0.0562 | 0.0573 | 0.0577 |

**Notes**: (1) tenors (in years) are in columns (2) statistics with label *1* are calculated for the curves obtained in the highest ranked system and with label *28* - the lowest ranked.

To summarise, we have found heuristically a class of weighting system for Polish government bonds yield curve to be used in NSS estimation which significantly improves the fit and smoothness as compared to the traditional approach of all equal weight. This **class** has three core characteristics:

- at least the same weight for the short end of the curve as a sum for all other tenors of bonds
- exclusion of *eligible-for-switch* bonds from the estimation
- bonds' weights based on at least outstanding amounts (in the best systems we had either $W^{oa}$ or $W^{oa} + W^{vol}$)

## Estimation results for the best weight system

In this section, we present and discuss the results of Nelson-Siegel-Svenssson estimation of Polish government yield curves for the period of 2005:01-2020:06 using the highest ranked weighting system in Table 5, namely the one labelled *1*. Table 6 provides detailed statistics of time series of zero coupon rates for selected 12 tenors in two systems: the highest (label *1*) and the lowest (label *28*) ranked in that table and Figure 30 in Appendix provides time series of MAE errors in these two.

It is worth noting that the mean (and median) curve in system *1* is lightly higher in the tenors up to and including 1Y than in system *28*, which is not surprising in the light of previous discussion on *switch bonds* exclusion due to existence of a non-negative valued *switch option*. On the other hand in the same - short - segments standard deviation is lower in the high ranked system. The highest standard deviation readings in both systems sit in the 1-2Y region, whereas the lowest are for the long end (12Y), which is desirable and was observed in the reality.

Visual inspection of such produced yield curves confirms the advantages of the chosen weighing system. The yield curves are smooth, start at or very close to NBP reference rates, most of them have flat asymptotic long end and cross time single tenor volatility looks tamed. The same qualitative results have been achieved when fitting NSS curve with the assumption of the zero coupon rates being continuously compounded.

Forward and spot rates are not directly observable on the market, hence in order to present *goodness-of-fit* of one particular curve estimated for a single date we have to either (1) calculate implied (predicted) *ytm* of a series of bonds that were subject to fixing on that date using the estimated spot NSS curve or (2) calculate hypothetical curve called *par curve*, by introduction of synthetic securities which are priced at par (100% of nominal value) using the spot NSS by changing the coupon rate. These coupon rates are in fact *par yields*. When we combine this two approaches by plotting a par curve and colouring the dots that represent single bonds in line with the difference between the predicted and observed *ytm* it would be clearly visible that the curve fits inside bid

Table 7: NSS parameters yearly averages

| | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 | 2011 | 2012 |
|---|---|---|---|---|---|---|---|---|
| $\beta_0$ | 0.0833 | 0.0772 | 0.0673 | 0.0600 | 0.0544 | 0.0650 | 0.0788 | 0.0756 |
| $\beta_1$ | -0.0291 | -0.0360 | -0.0228 | -0.0022 | -0.0163 | -0.0297 | -0.0361 | -0.0287 |
| $\beta_2$ | -0.0364 | 0.0086 | 0.0347 | 0.0158 | 0.0280 | 0.0312 | -0.0193 | -0.0428 |
| $\beta_3$ | -0.1016 | -0.0836 | -0.0411 | 0.0016 | 0.0157 | -0.0003 | -0.0978 | -0.1367 |
| $\tau_1$ | 1.37 | 2.95 | 3.94 | 1.17 | 2.56 | 3.72 | 2.74 | 2.48 |
| $\tau_2$ | 24.40 | 34.30 | 24.89 | 35.33 | 28.67 | 37.67 | 50.40 | 93.40 |

| | 2013 | 2014 | 2015 | 2016 | 2017 | 2018 | 2019 | 2020 H1 |
|---|---|---|---|---|---|---|---|---|
| $\beta_0$ | 0.0671 | 0.0719 | 0.0707 | 0.0771 | 0.0751 | 0.0833 | 0.0767 | 0.0798 |
| $\beta_1$ | -0.0371 | -0.0476 | -0.0546 | -0.0618 | -0.0599 | -0.0680 | -0.0614 | -0.0703 |
| $\beta_2$ | -0.0344 | -0.0437 | -0.0422 | -0.0537 | -0.0559 | -0.0735 | -0.0523 | -0.0629 |
| $\beta_3$ | -0.0986 | -0.0734 | -0.1010 | -0.0916 | -0.1723 | -0.3900 | -0.1599 | -0.3687 |
| $\tau_1$ | 2.46 | 2.00 | 2.29 | 2.10 | 1.70 | 1.83 | 2.24 | 2.76 |
| $\tau_2$ | 63.42 | 17.76 | 18.51 | 12.97 | 35.01 | 77.97 | 23.72 | 72.50 |

ask spread, except for short term bonds (*switch bonds*), which are excluded from estimation in this system of weights (grey coloured). The designed features of the weight system and filtering rules allow for very realistic construction of the curve with benchmark *on-the-run* issues (i.e. DS0725) being priced almost exactly on the curve and small, illiquid bonds are trading dearer to the curve, as they sit in long horizon portfolios already and there is no active ongoing price discovery mechanism (i.e. WS0429).

Table 7 and Figure 31 in the Appendix inspect the NSS parameters estimated for Polish curve. The parameters $\beta_0$ and $\beta_1$ are volatile in separation but as a sum behave almost in line with NBP reference rates, because the system of weights here put double stress on the short term rate fit. The parameter $\beta_2$ responsible for the convexity of the first hump seems to have experienced two periods in the 15-year history we deal with. First one ends approximately in 2011 during which we observe higher readings and higher volatility of $\beta_2$. In the second part of the time series $\beta_2$ is less volatile and the readings are on average lower. The parameter $\tau_1$ which determines the position of this first hump hovers between 0.3 and 10, but yearly average - between 1.37 and 3.94. The parameters which govern the second hump: $\beta_3$ and $\tau_2$ are very often taking extreme values, especially in the periods of rapid descent of long interest rates. Very high readings of $\tau_2$ in some dates indicate that the second hump is not crucial in estimating the curve with parsimonious form.

## Expectations Hypothesis domain in Poland

The carefully estimated time series of the Polish government yield curves will now be used to *extract* market expectations of the future interest rates via calculation of implied forward rate structure which, in turn, is instrumental in pure expectations hypothesis testing. This section provides the results of extensive tests of pure expectations hypothesis and will help to provide evidence for our claim that *Pure Expectations Hypothesis does not hold universally.* In general, we would like find sufficient evidence to reject the null hypothesis that forward rates are unbiased predictors of the future spot rates in Poland, which in turn will give rise to hypothesising on risk premia existence and their structure estimation in the future research on that matter.

Our staring point is the search for non-zero term premia structure's existence is a short recall of our yield curves' static properties. Table 8 shows selected descriptive statistics of the zero coupon rates in Poland in the whole dataset calculated for 3 and 6 months and from 1 to 10 years for each year. Additionally we report these statistics for level, slope and curvature of the yield curve, the potential candidates for the state variables in the forecasting models. Couple of patterns transpire there: (1) the most volatile interest rates are those in segment of 2-3Y, (2) volatility decreases in the sector 3-10Y to the lowest levels on the curve, (3) persistence (of different lags) follow the pattern observed for the volatility. But what is striking is that the mean yields are strictly increasing, with $2Y - 0.25Y$ spread of 26 basis points, $5Y - 0.25Y$ - 89 basis points and the long term bonds spread $10Y - 0.25Y$ - 123 basis points. It is hard to explain, in absence of any hypothetical term or risk premia, because during the considered period monetary policy was in easing or natural bias and the interest rates were, generally, in a strong downward trend. Our intuition calls for two possible strands of justification. First, the markets are very poor in forming expectations of future paths of interest rates. Second, the market expectations are formed *correctly* with a consideration of monotonically increasing risk premia. In this chapter we will deal with the first intuition, whereas in the next we will check if hypothesising on risk term premia is plausible for Polish government bonds.

Table 8: Descriptive statistics of NSS fitted yields in Poland, 2005:01-2020:06

| | mean | std | max | min | $\rho(1)$ | $\rho(12)$ | $\rho(24)$ | $\rho(36)$ |
|---|---|---|---|---|---|---|---|---|
| 3 months | 0.0322 | 0.0158 | 0.0651 | 0.0007 | 0.9732 | 0.7016 | 0.5291 | 0.4631 |
| 6 months | 0.0324 | 0.0161 | 0.0652 | 0.0005 | 0.9743 | 0.7297 | 0.5617 | 0.4754 |
| 1 year | 0.0330 | 0.0166 | 0.0674 | 0.0005 | 0.9752 | 0.7644 | 0.6026 | 0.4860 |
| 2 years | 0.0348 | 0.0167 | 0.0683 | 0.0017 | 0.9752 | 0.7917 | 0.6367 | 0.4902 |
| 3 years | 0.0368 | 0.0164 | 0.0675 | 0.0037 | 0.9748 | 0.7954 | 0.6423 | 0.4855 |
| 4 years | 0.0386 | 0.0159 | 0.0667 | 0.0059 | 0.9744 | 0.7888 | 0.6356 | 0.4750 |
| 5 years | 0.0401 | 0.0154 | 0.0659 | 0.0081 | 0.9740 | 0.7775 | 0.6239 | 0.4613 |
| 6 years | 0.0414 | 0.0150 | 0.0654 | 0.0100 | 0.9738 | 0.7645 | 0.6113 | 0.4464 |
| 7 years | 0.0425 | 0.0147 | 0.0649 | 0.0116 | 0.9737 | 0.7518 | 0.6002 | 0.4320 |
| 8 years | 0.0433 | 0.0144 | 0.0646 | 0.0128 | 0.9738 | 0.7405 | 0.5914 | 0.4192 |
| 9 years | 0.0440 | 0.0142 | 0.0644 | 0.0137 | 0.9739 | 0.7310 | 0.5852 | 0.4084 |
| 10 years | 0.0446 | 0.0141 | 0.0644 | 0.0143 | 0.9741 | 0.7235 | 0.5813 | 0.3998 |
| Level | 0.0454 | 0.0139 | 0.0654 | 0.0148 | 0.9745 | 0.7137 | 0.5788 | 0.3885 |
| Slope | 0.0122 | 0.0073 | 0.0244 | -0.0052 | 0.9507 | 0.0473 | -0.1275 | -0.0181 |
| Curvature | 0.0034 | 0.0049 | 0.0125 | -0.0089 | 0.9270 | 0.5950 | 0.4466 | 0.2665 |

Notes (1) NSS fit using weight system labelled *1*.

Prior to proceeding directly into testing, two remarks are technically important. First, we have shown evidence that Polish interest rates are unit root processes (at least in the period under consideration) hence usually the regressions used to test expectation hypotheses would incorporate spreads to some contemporary observed rate, i.e. 1Y rate, instead of nominal levels. This is in line with majority of research conducted for liquid markets, despite a few authors positing that the US rates are stationary (cf. Sarno et al. (2007)). Second, we will use beginning of month yield curves instead of daily data set, which again is in line with the approach in vast majority of studies. Inference in daily data of the processes that are highly persistent may prove to be arduous and weary, without any improvement on the statistical significance of the results.

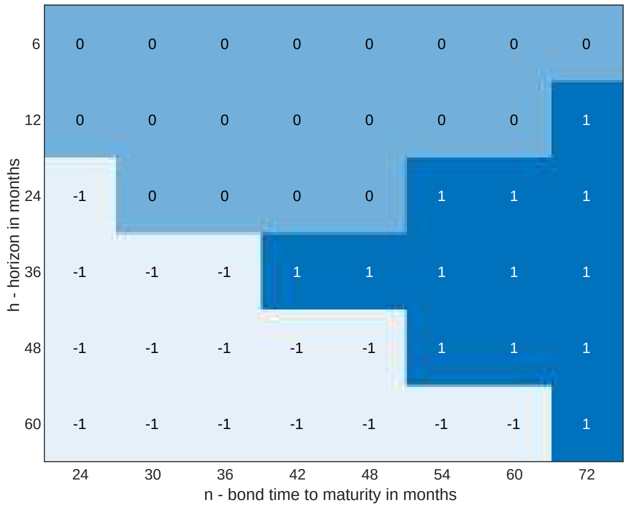
## Excess return on term premium regressions

The so called in the literature *term premium regressions* as performed by Fama & Bliss (1987) are the first of the two classic models to consider when testing of pure expectations hypothesis (PEH). Recall from the literature review that the regressions are of a form:

$$rx_{t+h}^{(n)} = \alpha + \beta \left( f_t^{t+h,t+n} - y_t^{(h)} \right) + \epsilon_{t+h}^{(n)} \tag{9}$$

where $rx_{t+h}^{(n)}$ is an excess return (over $h$-year yield observed at time $t$) of the investment in $n$-year bond at time $t$ with disinvestment after $h$-years (investment horizon), $f_t^{t+h,t+n}$ is a forward rate observed (implied) at $t$ and good for discounting to time $t+h$ a cash-flow occurring at $t+n$. Note that the choice of the maturities of bonds and the length of investment horizons is somehow limited by the length of our data series. Having 15.5 year history (186 months) we could have tried to test PEH in combinations of horizon and length of a bond like $h = 6$ months and $n = 180$ months but then we would be left with just six observations to infer from, which - obviously - is not plausible. We have chosen vectors of tested horizons and bond maturities so that to have the analysed time series length of variables least of approximately 2/3-rds of the maximum data scope. Hence the maximum bond we consider is of 6-year maturity. Selected pairs $(h, n)$ should satisfy $\forall_{(h,n)} h < n$ as explained in Chapter 1, translating here to the maximum horizon in a vector $\mathbf{h}$ being less then 6-years, as we are interested in excess returns in investments in bonds for shorter periods than their maturity. In consequence of these constraints we have chosen the following vectors: of horizons - $\mathbf{h} = [6, 12, 24, 36, 48, 60]$ and of bond maturities $\mathbf{n} = [24, 30, 36, 42, 48, 54, 60, 72]$ in months.

Figure 11: Pure expectations hypothesis: regression of type 1 and 2 (Fama & Bliss) for Poland




Notes: (1) *0* indicates that there is no sufficient evidence to reject null hypothesis of PEH, (2) *1* - sufficient evidence (with at 0.05 significance) to reject PEH, (3) *-1* - not applicable (i.e. a horizon is longer or equal bond's maturity) (4) NSS fit using weight system labelled *1*.

Table 12 reports all of the details of all the regressions (32) performed for different selected pairs $(h, n)$ such that $\forall_{(h,n)} h < n$. Our null hypothesis is that the coefficient $\hat{\beta} = 0$, because PEH claims that nothing should forecasts returns including observed at $t$ spread of forward rates over some shorter bond. As in all our hypothesis testing in this paper we assume significance level of 0.05. Due to major overlapping of periods in our time series, before inference, we correct standard errors for heteroscedasticity (heterogeneity of variances) and autocorrelation (henceforth: HAC) proposed by Newey & West (1987) using Bartlett kernels, in which we assumed 3-year lag period[16]. Coefficients $\beta$ are increasing within the same horizon regression along the tenors of bonds, so in general are also the standard errors of these estimators. Bold estimates of $\beta$ with their corresponding p-values implied by these HAC corrected standard errors call for rejection of null hypothesis in the following cases:

1. 6-year bond for all horizons starting and including 12 months ($[12, 24, 36, 48, 60]$)
2. 5-year bond for all horizons starting and including 24 months ($[24, 36, 48, 60]$)
3. 5.5-year bond for all horizons starting and including 24 months ($[24, 36, 48, 60]$)
4. 4-year and 3.5-year bonds for 36-month horizon

Obviously, F-statistic test calls for rejection of null hypothesis that all the coefficients except intercept in a given regression are zero for the same pairs $(h, n)$, since we have only one regressor and the same null hypothesis when testing for significance of $\beta$ estimator. The Figure 11 shows these on a *heatmap*, from which we originate an assumption that the longer - not tested here - maturities

[16]as during 36 months autocorrelations of all tenors of fitted Polish yield curve fall below 0.5

with even short investment horizons would also call for the rejection of null hypothesis. Generally, we posit here that PEH holds for bonds up to 4 years and investment horizons up to 24 months, in any other combinations evidence for rejecting PEH is stronger the longer the bond and the longer the horizon are.

For the pairs of $(h, n)$ where we have collected sufficient evidence to reject the PEH we are not sure what is the root cause of this rejection, as $R^2$ values are in the range of $4.7 - 45.5\%$ and this share of explained variance increases with longer tenors of bonds within the same considered horizon. It means that on average only a third of the variance of excess return is explained by the volatility of the term premium defined as in this particular regression as a spread between corresponding forward rate and a short rate in line in maturity with the investment horizon assumed. Hence it leaves us with two thirds of variance unexplained and hints that potentially variable or fixed term premia are one of the variables to consider in increasing the $R^2$ readings.

We have also performed robustness checks of the above-mentioned results using previously prepared 28 systems of weights (the details are reported in Figure 32 in the Appendix). Coefficients, HAC standard errors and, in consequence, p-values differ slightly between the systems, but the final decisions regarding null hypothesis (at 0.05 significance level) are almost the same or all 27 as for the first system labelled *1* with one exception of for pair (24, 54), which coefficient $\beta$ has in some systems p-value greater than 0.05.

In summary, having analysed the results of regressions of excess returns over corresponding forward-to-spot term premia we found first argument that the PEH indeed does not hold universally across maturities and investment horizons. PEH holds with 0.95 confidence level only for short term bonds and short horizons, and this result is robust for changes of fitted yields from 28 different weight systems.

## Realized change in the spot rate on term premium regressions

We consider here yet another classic regression from Fama & Bliss (1987) sometimes referred to as *forecasts of the change in the spot rate.*

$$y_{t+h}^{(n)} - y_t^{(h)} = \alpha + \beta \left( f_t^{t+h,t+n} - y_t^{(h)} \right) + \epsilon_{t+h}^{(n)} \tag{10}$$

where all the symbols have the same meaning as in the previous section and $y_{t+h}^{(n)}$ is a zero coupon rate of $n$-year bond (tenor is calculated at time $t$) observed at time $t+h$, hence in the future from point $t$ in time. We have the same *regressor* as before but the *regressand* this time is a realised spread of spot rates. For the PEH to hold we obviously would like that this spread is explained fully by term premium, hence the null hypothesis is: $H_0 : \beta = 1$.

The results of these regressions run for the same pairs of horizon and bond maturity $(h, n)$ as for the term premium regressions are shown in Table 13. Yet again the standard errors are of HAC type, but we have a fundamentally different null hypothesis which is reflected in p-value calculations. The domain where null hypothesis meets strong evidence against it is of the same exact form as for the term premium regressions. What is more, the results are robust for changes in system of weights, again with one exception of a pair (24, 54), for which in some systems we read p-value of $\beta$ higher than 0.05 (see Figure 33 in Appendix). Share of explained variance in total variance for the regressions with p-value of $\beta$ estimates lower than 0.05 is very weak - in the range of $0.1 - 19.7\%$[17],

[17] if we exclude the regressions where a simple model with intercept only was equally good - judging by p-value of F-statistics

Figure 12: Type 1 regression: a modo Fama & Bliss (1987) -Term premium regressions, Poland 2005:01-2020:06

| | h | n | $\alpha$ | SE($\alpha$) | p-val | $\beta$ | SE($\beta$) | p-val | $R^2$ | $R^2$ adj. | obs | RMSE | F-stat | F p-val | $\rho(6)$ | $\rho(12)$ | $\rho(24)$ | $\rho(36)$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\mathcal{R}_1$ | 6.0000 | 24.0000 | 0.0098 | 0.0029 | 0.0004 | 0.1443 | 0.5059 | 0.3878 | 0.0020 | -0.0050 | 162.0000 | 0.0159 | 0.2700 | 0.6030 | 0.2200 | -0.1300 | -0.1300 | 0.0200 |
| $\mathcal{R}_2$ | 6.0000 | 30.0000 | 0.0131 | 0.0044 | 0.0013 | 0.1785 | 0.5579 | 0.3745 | 0.0020 | -0.0050 | 156.0000 | 0.0218 | 0.2600 | 0.6096 | 0.1800 | -0.1400 | -0.0900 | 0.0700 |
| $\mathcal{R}_3$ | 6.0000 | 36.0000 | 0.0154 | 0.0061 | 0.0055 | 0.2408 | 0.6225 | 0.3494 | 0.0020 | -0.0050 | 150.0000 | 0.0280 | 0.3300 | 0.5688 | 0.1500 | -0.1600 | -0.0500 | 0.1100 |
| $\mathcal{R}_4$ | 6.0000 | 42.0000 | 0.0173 | 0.0078 | 0.0130 | 0.2296 | 0.7101 | 0.3732 | 0.0010 | -0.0060 | 144.0000 | 0.0344 | 0.2100 | 0.6479 | 0.1500 | -0.1600 | -0.0100 | 0.1000 |
| $\mathcal{R}_5$ | 6.0000 | 48.0000 | 0.0203 | 0.0093 | 0.0145 | 0.3175 | 0.7506 | 0.3361 | 0.0020 | -0.0050 | 138.0000 | 0.0404 | 0.3100 | 0.5777 | 0.1300 | -0.1800 | 0.0300 | 0.1000 |
| $\mathcal{R}_6$ | 6.0000 | 54.0000 | 0.0225 | 0.0112 | 0.0223 | 0.4821 | 0.8143 | 0.2769 | 0.0040 | -0.0030 | 132.0000 | 0.0470 | 0.5700 | 0.4529 | 0.1300 | -0.2000 | 0.1000 | 0.0700 |
| $\mathcal{R}_7$ | 6.0000 | 60.0000 | 0.0230 | 0.0135 | 0.0436 | 0.6604 | 0.8998 | 0.2315 | 0.0070 | -0.0010 | 126.0000 | 0.0545 | 0.8400 | 0.3615 | 0.1400 | -0.2200 | 0.1300 | 0.0600 |
| $\mathcal{R}_8$ | 6.0000 | 72.0000 | 0.0261 | 0.0192 | 0.0878 | 0.8458 | 1.0426 | 0.2086 | 0.0080 | 0.0000 | 114.0000 | 0.0680 | 0.9400 | 0.3334 | 0.1400 | -0.1700 | 0.0900 | 0.0900 |
| $\mathcal{R}_9$ | 12.0000 | 24.0000 | 0.0069 | 0.0017 | 0.0000 | -0.0895 | 0.2845 | 0.6234 | 0.0020 | -0.0040 | 162.0000 | 0.0078 | 0.2900 | 0.5900 | 0.4900 | -0.0600 | -0.2300 | 0.0300 |
| $\mathcal{R}_{10}$ | 12.0000 | 30.0000 | 0.0102 | 0.0026 | 0.0000 | -0.0295 | 0.3538 | 0.5333 | 0.0000 | -0.0060 | 156.0000 | 0.0118 | 0.0200 | 0.8956 | 0.4800 | -0.0400 | -0.1700 | 0.0600 |
| $\mathcal{R}_{11}$ | 12.0000 | 36.0000 | 0.0124 | 0.0037 | 0.0004 | 0.1056 | 0.4138 | 0.3993 | 0.0010 | -0.0060 | 150.0000 | 0.0159 | 0.1400 | 0.7088 | 0.4800 | -0.0200 | -0.1000 | 0.1000 |
| $\mathcal{R}_{12}$ | 12.0000 | 42.0000 | 0.0140 | 0.0046 | 0.0011 | 0.1941 | 0.4797 | 0.3428 | 0.0020 | -0.0050 | 144.0000 | 0.0200 | 0.3200 | 0.5721 | 0.4800 | 0.0100 | -0.0400 | 0.0900 |
| $\mathcal{R}_{13}$ | 12.0000 | 48.0000 | 0.0149 | 0.0056 | 0.0041 | 0.4316 | 0.5529 | 0.2175 | 0.0090 | 0.0010 | 138.0000 | 0.0243 | 1.1700 | 0.2817 | 0.4800 | 0.0100 | -0.0100 | 0.1100 |
| $\mathcal{R}_{14}$ | 12.0000 | 54.0000 | 0.0151 | 0.0069 | 0.0139 | 0.8270 | 0.6751 | 0.1103 | 0.0260 | 0.0180 | 132.0000 | 0.0283 | 3.4500 | 0.0654 | 0.4800 | 0.0000 | 0.0500 | 0.0900 |
| $\mathcal{R}_{15}$ | 12.0000 | 60.0000 | 0.0143 | 0.0082 | 0.0403 | 1.2199 | 0.8233 | 0.0692 | 0.0460 | 0.0390 | 126.0000 | 0.0328 | 6.0400 | 0.0154 | 0.4800 | -0.0200 | 0.0900 | 0.0800 |
| $\mathcal{R}_{16}$ | 12.0000 | 72.0000 | 0.0119 | 0.0111 | 0.1403 | **1.9337** | 1.0601 | 0.0341 | 0.0820 | 0.0740 | 114.0000 | 0.0427 | 10.0100 | 0.0020 | 0.4800 | 0.0000 | 0.0900 | 0.1300 |
| $\mathcal{R}_{17}$ | 24.0000 | 30.0000 | 0.0036 | 0.0007 | 0.0000 | -0.0891 | 0.1020 | 0.8086 | 0.0150 | 0.0090 | 156.0000 | 0.0025 | 2.3500 | 0.1270 | 0.7200 | 0.2900 | -0.1400 | -0.0100 |
| $\mathcal{R}_{18}$ | 24.0000 | 36.0000 | 0.0065 | 0.0014 | 0.0000 | -0.0196 | 0.1792 | 0.5435 | 0.0000 | -0.0070 | 150.0000 | 0.0052 | 0.0300 | 0.8622 | 0.7600 | 0.3600 | -0.0900 | 0.0400 |
| $\mathcal{R}_{19}$ | 24.0000 | 42.0000 | 0.0086 | 0.0020 | 0.0000 | 0.1135 | 0.2492 | 0.3244 | 0.0030 | -0.0040 | 144.0000 | 0.0081 | 0.4600 | 0.4978 | 0.7700 | 0.4100 | -0.0200 | 0.0500 |
| $\mathcal{R}_{20}$ | 24.0000 | 48.0000 | 0.0098 | 0.0028 | 0.0002 | 0.3390 | 0.3089 | 0.1362 | 0.0180 | 0.0110 | 138.0000 | 0.0110 | 2.4900 | 0.1169 | 0.7800 | 0.4300 | -0.0100 | 0.0500 |
| $\mathcal{R}_{21}$ | 24.0000 | 54.0000 | 0.0101 | 0.0036 | 0.0029 | **0.6585** | 0.3954 | 0.0479 | 0.0470 | 0.0400 | 132.0000 | 0.0140 | 6.4300 | 0.0124 | 0.7700 | 0.4200 | -0.0100 | 0.0500 |
| $\mathcal{R}_{22}$ | 24.0000 | 60.0000 | 0.0098 | 0.0045 | 0.0151 | **1.0033** | 0.4706 | 0.0165 | 0.0830 | 0.0760 | 126.0000 | 0.0169 | 11.2900 | 0.0010 | 0.7500 | 0.3900 | -0.0100 | 0.0700 |
| $\mathcal{R}_{23}$ | 24.0000 | 72.0000 | 0.0099 | 0.0071 | 0.0831 | **1.6001** | 0.5068 | 0.0008 | 0.1510 | 0.1430 | 114.0000 | 0.0220 | 19.8500 | 0.0000 | 0.7500 | 0.4300 | 0.0200 | 0.1300 |
| $\mathcal{R}_{24}$ | 36.0000 | 42.0000 | 0.0020 | 0.0007 | 0.0029 | **0.1809** | 0.0646 | 0.0026 | 0.1090 | 0.1030 | 144.0000 | 0.0019 | 17.3500 | 0.0001 | 0.7800 | 0.4000 | 0.0500 | -0.1500 |
| $\mathcal{R}_{25}$ | 36.0000 | 48.0000 | 0.0034 | 0.0016 | 0.0167 | **0.4418** | 0.1319 | 0.0004 | 0.1690 | 0.1630 | 138.0000 | 0.0039 | 27.6200 | 0.0000 | 0.7800 | 0.4500 | 0.0700 | -0.1600 |
| $\mathcal{R}_{26}$ | 36.0000 | 54.0000 | 0.0040 | 0.0026 | 0.0585 | **0.7852** | 0.2216 | 0.0002 | 0.2430 | 0.2370 | 132.0000 | 0.0059 | 41.6900 | 0.0000 | 0.7800 | 0.4500 | 0.0400 | -0.1900 |
| $\mathcal{R}_{27}$ | 36.0000 | 60.0000 | 0.0043 | 0.0035 | 0.1120 | **1.1260** | 0.2979 | 0.0001 | 0.2980 | 0.2920 | 126.0000 | 0.0080 | 52.6200 | 0.0000 | 0.7800 | 0.4500 | 0.0200 | -0.1900 |
| $\mathcal{R}_{28}$ | 36.0000 | 72.0000 | 0.0042 | 0.0058 | 0.2321 | **1.7795** | 0.4698 | 0.0001 | 0.3860 | 0.3800 | 114.0000 | 0.0119 | 70.4000 | 0.0000 | 0.7600 | 0.4300 | -0.0100 | -0.1100 |
| $\mathcal{R}_{29}$ | 48.0000 | 54.0000 | 0.0016 | 0.0002 | 0.0000 | **0.2553** | 0.0307 | 0.0000 | 0.4000 | 0.3950 | 132.0000 | 0.0012 | 86.6000 | 0.0000 | 0.7800 | 0.5200 | 0.1100 | -0.1800 |
| $\mathcal{R}_{30}$ | 48.0000 | 60.0000 | 0.0030 | 0.0006 | 0.0000 | **0.5323** | 0.0592 | 0.0000 | 0.4320 | 0.4280 | 126.0000 | 0.0026 | 94.3500 | 0.0000 | 0.7500 | 0.4500 | 0.0500 | -0.1800 |
| $\mathcal{R}_{31}$ | 48.0000 | 72.0000 | 0.0054 | 0.0025 | 0.0149 | **1.0644** | 0.2007 | 0.0000 | 0.4550 | 0.4500 | 114.0000 | 0.0057 | 93.5800 | 0.0000 | 0.7400 | 0.4000 | -0.0500 | -0.0900 |
| $\mathcal{R}_{32}$ | 60.0000 | 72.0000 | 0.0039 | 0.0009 | 0.0000 | **0.3871** | 0.0658 | 0.0000 | 0.3310 | 0.3250 | 114.0000 | 0.0025 | 55.4400 | 0.0000 | 0.8000 | 0.4800 | -0.0300 | -0.2300 |

Regression: $rx_{t+h}^{(n)} = \alpha + \beta\left(f_t^{t+h,t+n} - y_t^{(h)}\right) + \epsilon_{t+h}^{(n)}$

Null Hypothesis: $\hat{\beta} = 0$ (PEH claims that nothing should forecasts returns)

Standard errors are calculated with heteroscedasticity and aurocorrelation correction a la Newley-West (Bartlett kernel)

Bold estimates of $\beta$ have lower p-value than 0.05 (our assumed significance level) and call for rejection of null hypothesis

$\mathcal{R}$. indicates that F-test found evidence to reject null hypothesis that the model with no independent variables fits the data as well as your model.

Figure 13: Type 2 regression: a modo Fama & Bliss (1987) - Forecasts of the change in the spot rate, Poland 2005:01-2020:06)

| | h | n | $\alpha$ | SE($\alpha$) | p-val | $\beta$ | SE($\beta$) | p-val | $R^2$ | $R^2$ adj. | obs | RMSE | F-stat | F p-val | $\rho(6)$ | $\rho(12)$ | $\rho(24)$ | $\rho(36)$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\mathcal{R}_1$ | 6.0000 | 24.0000 | -0.0032 | 0.0010 | 0.0000 | 0.9466 | 0.1666 | 0.3742 | 0.4020 | 0.3990 | 162.0000 | 0.0052 | 107.6900 | 0.0000 | 0.2200 | -0.1300 | -0.1300 | 0.0200 |
| $\mathcal{R}_2$ | 6.0000 | 30.0000 | -0.0032 | 0.0011 | 0.0000 | 0.9487 | 0.1372 | 0.3544 | 0.4420 | 0.4380 | 156.0000 | 0.0054 | 121.7600 | 0.0000 | 0.1900 | -0.1400 | -0.1000 | 0.0700 |
| $\mathcal{R}_3$ | 6.0000 | 36.0000 | -0.0029 | 0.0012 | 0.0000 | 0.9443 | 0.1221 | 0.3242 | 0.4670 | 0.4640 | 150.0000 | 0.0055 | 129.7800 | 0.0000 | 0.1600 | -0.1500 | -0.0600 | 0.1100 |
| $\mathcal{R}_4$ | 6.0000 | 42.0000 | -0.0027 | 0.0013 | 0.0000 | 0.9534 | 0.1160 | 0.3440 | 0.4880 | 0.4850 | 144.0000 | 0.0056 | 135.4500 | 0.0000 | 0.1500 | -0.1500 | -0.0100 | 0.1000 |
| $\mathcal{R}_5$ | 6.0000 | 48.0000 | -0.0027 | 0.0013 | 0.0000 | 0.9459 | 0.1044 | 0.3022 | 0.5110 | 0.5070 | 138.0000 | 0.0056 | 142.1700 | 0.0000 | 0.1300 | -0.1700 | 0.0300 | 0.1100 |
| $\mathcal{R}_6$ | 6.0000 | 54.0000 | -0.0026 | 0.0014 | 0.0000 | 0.9306 | 0.0986 | 0.2409 | 0.5240 | 0.5200 | 132.0000 | 0.0057 | 142.9300 | 0.0000 | 0.1300 | -0.1900 | 0.1000 | 0.0800 |
| $\mathcal{R}_7$ | 6.0000 | 60.0000 | -0.0023 | 0.0015 | 0.0000 | 0.9174 | 0.0965 | 0.1960 | 0.5300 | 0.5260 | 126.0000 | 0.0059 | 139.6000 | 0.0000 | 0.1400 | -0.2100 | 0.1200 | 0.0600 |
| $\mathcal{R}_8$ | 6.0000 | 72.0000 | -0.0021 | 0.0017 | 0.0000 | 0.9134 | 0.0906 | 0.1697 | 0.5620 | 0.5580 | 114.0000 | 0.0060 | 143.7600 | 0.0000 | 0.1400 | -0.1700 | 0.0800 | 0.1000 |
| $\mathcal{R}_9$ | 12.0000 | 24.0000 | -0.0068 | 0.0016 | 0.0000 | 1.0784 | 0.2818 | 0.6096 | 0.2120 | 0.2070 | 162.0000 | 0.0078 | 42.9200 | 0.0000 | 0.4900 | -0.0500 | -0.2300 | 0.0300 |
| $\mathcal{R}_{10}$ | 12.0000 | 30.0000 | -0.0066 | 0.0017 | 0.0000 | 1.0092 | 0.2328 | 0.5158 | 0.2300 | 0.2250 | 156.0000 | 0.0078 | 46.0800 | 0.0000 | 0.4900 | -0.0400 | -0.1700 | 0.0700 |
| $\mathcal{R}_{11}$ | 12.0000 | 36.0000 | -0.0061 | 0.0018 | 0.0000 | 0.9385 | 0.2035 | 0.3812 | 0.2330 | 0.2280 | 150.0000 | 0.0079 | 45.0300 | 0.0000 | 0.4800 | -0.0200 | -0.1100 | 0.1000 |
| $\mathcal{R}_{12}$ | 12.0000 | 42.0000 | -0.0054 | 0.0018 | 0.0000 | 0.9155 | 0.1878 | 0.3264 | 0.2430 | 0.2380 | 144.0000 | 0.0079 | 45.6500 | 0.0000 | 0.4900 | 0.0100 | -0.0400 | 0.0900 |
| $\mathcal{R}_{13}$ | 12.0000 | 48.0000 | -0.0048 | 0.0018 | 0.0000 | 0.8520 | 0.1791 | 0.2043 | 0.2370 | 0.2310 | 138.0000 | 0.0080 | 42.2400 | 0.0000 | 0.4800 | 0.0200 | -0.0100 | 0.1100 |
| $\mathcal{R}_{14}$ | 12.0000 | 54.0000 | -0.0041 | 0.0019 | 0.0000 | 0.7626 | 0.1860 | 0.1009 | 0.2230 | 0.2170 | 132.0000 | 0.0079 | 37.4000 | 0.0000 | 0.4800 | 0.0100 | 0.0500 | 0.1000 |
| $\mathcal{R}_{15}$ | 12.0000 | 60.0000 | -0.0034 | 0.0020 | 0.0000 | 0.6970 | 0.1972 | 0.0622 | 0.2110 | 0.2040 | 126.0000 | 0.0080 | 33.0800 | 0.0000 | 0.4900 | -0.0100 | 0.0900 | 0.0900 |
| $\mathcal{R}_{16}$ | 12.0000 | 72.0000 | -0.0022 | 0.0021 | 0.0000 | 0.6201 | 0.2008 | **0.0292** | 0.1970 | 0.1900 | 114.0000 | 0.0083 | 27.4500 | 0.0000 | 0.4800 | 0.0000 | 0.0800 | 0.1400 |
| $\mathcal{R}_{17}$ | 24.0000 | 30.0000 | -0.0142 | 0.0029 | 0.0000 | 1.3358 | 0.4031 | 0.7976 | 0.1790 | 0.1740 | 156.0000 | 0.0098 | 33.5500 | 0.0000 | 0.7300 | 0.2900 | -0.1400 | -0.0100 |
| $\mathcal{R}_{18}$ | 24.0000 | 36.0000 | -0.0127 | 0.0028 | 0.0000 | 1.0243 | 0.3550 | 0.5273 | 0.1240 | 0.1180 | 150.0000 | 0.0103 | 20.9800 | 0.0000 | 0.7600 | 0.3600 | -0.0900 | 0.0400 |
| $\mathcal{R}_{19}$ | 24.0000 | 42.0000 | -0.0113 | 0.0027 | 0.0000 | 0.8394 | 0.3294 | 0.3129 | 0.0930 | 0.0860 | 144.0000 | 0.0107 | 14.4900 | 0.0002 | 0.7700 | 0.4100 | -0.0300 | 0.0500 |
| $\mathcal{R}_{20}$ | 24.0000 | 48.0000 | -0.0096 | 0.0028 | 0.0000 | 0.6573 | 0.3059 | 0.1312 | 0.0660 | 0.0590 | 138.0000 | 0.0109 | 9.5700 | 0.0024 | 0.7800 | 0.4300 | -0.0100 | 0.0500 |
| $\mathcal{R}_{21}$ | 24.0000 | 54.0000 | -0.0079 | 0.0029 | 0.0000 | 0.4756 | 0.3125 | **0.0467** | 0.0400 | 0.0320 | 132.0000 | 0.0111 | 5.3900 | 0.0219 | 0.7700 | 0.4200 | -0.0100 | 0.0500 |
| $\mathcal{R}_{22}$ | 24.0000 | 60.0000 | -0.0064 | 0.0030 | 0.0000 | 0.3385 | 0.3093 | **0.0162** | 0.0230 | 0.0160 | 126.0000 | 0.0111 | 2.9800 | 0.0868 | 0.7500 | 0.3900 | -0.0100 | 0.0700 |
| $\mathcal{R}_{23}$ | 24.0000 | 72.0000 | -0.0048 | 0.0035 | 0.0000 | 0.2143 | 0.2474 | **0.0007** | 0.0130 | 0.0040 | 114.0000 | 0.0108 | 1.4800 | 0.2266 | 0.7600 | 0.4400 | 0.0200 | 0.1300 |
| $\mathcal{R}_{24}$ | 36.0000 | 42.0000 | -0.0120 | 0.0044 | 0.0000 | -0.0890 | 0.3862 | **0.0024** | 0.0010 | -0.0060 | 144.0000 | 0.0112 | 0.1200 | 0.7302 | 0.7800 | 0.4000 | 0.0500 | -0.1500 |
| $\mathcal{R}_{25}$ | 36.0000 | 48.0000 | -0.0100 | 0.0048 | 0.0000 | -0.3217 | 0.3944 | **0.0004** | 0.0120 | 0.0050 | 138.0000 | 0.0116 | 1.6700 | 0.1985 | 0.7800 | 0.4400 | 0.0700 | -0.1600 |
| $\mathcal{R}_{26}$ | 36.0000 | 54.0000 | -0.0079 | 0.0051 | 0.0000 | -0.5579 | 0.4424 | **0.0002** | 0.0400 | 0.0330 | 132.0000 | 0.0117 | 5.4200 | 0.0215 | 0.7800 | 0.4500 | 0.0400 | -0.1900 |
| $\mathcal{R}_{27}$ | 36.0000 | 60.0000 | -0.0063 | 0.0052 | 0.0000 | -0.6697 | 0.4459 | **0.0001** | 0.0640 | 0.0570 | 126.0000 | 0.0118 | 8.5500 | 0.0041 | 0.7700 | 0.4500 | 0.0200 | -0.1900 |
| $\mathcal{R}_{28}$ | 36.0000 | 72.0000 | -0.0041 | 0.0058 | 0.0000 | -0.7503 | 0.4655 | **0.0001** | 0.1040 | 0.0960 | 114.0000 | 0.0116 | 13.0100 | 0.0005 | 0.7600 | 0.4300 | 0.0000 | -0.1100 |
| $\mathcal{R}_{29}$ | 48.0000 | 54.0000 | -0.0129 | 0.0017 | 0.0000 | -1.0174 | 0.2389 | **0.0000** | 0.1480 | 0.1410 | 132.0000 | 0.0096 | 22.5100 | 0.0000 | 0.7800 | 0.5200 | 0.1100 | -0.1800 |
| $\mathcal{R}_{30}$ | 48.0000 | 60.0000 | -0.0121 | 0.0025 | 0.0000 | -1.0980 | 0.2304 | **0.0000** | 0.1750 | 0.1690 | 126.0000 | 0.0101 | 26.3900 | 0.0000 | 0.7500 | 0.4500 | 0.0500 | -0.1800 |
| $\mathcal{R}_{31}$ | 48.0000 | 72.0000 | -0.0107 | 0.0049 | 0.0000 | -1.0895 | 0.3929 | **0.0000** | 0.1880 | 0.1810 | 114.0000 | 0.0110 | 25.9300 | 0.0000 | 0.7400 | 0.4000 | -0.0400 | -0.0900 |
| $\mathcal{R}_{32}$ | 60.0000 | 72.0000 | -0.0192 | 0.0046 | 0.0000 | -0.9071 | 0.3212 | **0.0000** | 0.1030 | 0.0950 | 114.0000 | 0.0121 | 12.8400 | 0.0005 | 0.8000 | 0.4800 | -0.0300 | -0.2300 |

Regression: $y_{t+h}^{(n)} - y_t^{(h)} = \alpha + \beta\left(f_t^{t+h,t+n} - y_t^{(h)}\right) + \epsilon_{t+h}^{(n)}$,

Null Hypothesis: $\hat{\beta} = 1$ (PEH claims that nothing should forecasts returns)

Standard errors are calculated with heteroscedasticity and aurocorrelation correction a la Newley-West (Bartlett kernel)

Bold estimates of $\beta$ have lower p-value than 0.05 (our assumed significance level) and call for rejection of null hypothesis

$\mathcal{R}$. indicates that F-test found evidence to reject null hypothesis that the model with no independent variables fits the data as well as your model.

suggesting other than term premia variables influencing our regressand.

In summary, the realised change in the spot rate on term premium regressions gave qualitatively similar results though the $R^2$ values are approximately half the ones estimated for the first type of Fama and Bliss regressions. The domain where PEH holds (or at least there is no sufficient evidence that it does not) is broader than in Fama & Bliss (1987) or Campbell & Shiller (1991), but still for medium and long term bonds PEH is rejected.

## One year excess return on average one year forward rates regressions

In this subsection we present the results for Poland of almost classic *ten shape* regressions proposed by Cochrane & Piazzesi (2005) to strengthen evidence against expectations hypothesis, by finding a single *return forecasting factor* - a combination of a series of one year forward rates and one year spot rate that explains the expected return of all bonds. In this pursue we follow the authors' proposal closely.

The key model here is a series of regressions on one year excess return of $2, 3, 4$ and $5$ year bonds (separately), where regressors are four consequitive one year forward rates:

$$rx_{t+1}^{(n)} = \beta_0^{(n)} + \beta_1^{(n)} y_t^1 + \beta_2^{(n)} f_t^{t+1,t+2} + \beta_3^{(n)} f_t^{t+2,t+3} + \beta_4^{(n)} f_t^{t+3,t+4} + \beta_5^{(n)} f_t^{t+4,t+5} + \epsilon_{t+1}^{(n)} \tag{11}$$

These models are not estimated directly but in restricted form in two-stepped regression. First, we run a regression of the average (across maturities) excess return on all forward rates and the one year spot:

$$\frac{\sum_{n=2}^{5} rx_{t+1}^{(n)}}{4} = \gamma_0 + \gamma_1 y_t^1 + \gamma_2 f_t^{t+1,t+2} + \gamma_3 f_t^{t+2,t+3} + \gamma_4 f_t^{t+3,t+4} + \gamma_5 f_t^{t+4,t+5} + \bar{\epsilon}_{t+1} \tag{12}$$

to extract vector of coefficients $\gamma$. The regressand there is an arithmetic average of four excess returns of investments into $2, 3, 4, 5$-year bonds over one year spot rate observed at the beginning of a hypothetical investments. The results of the first step are summarized in the upper panel of Table 9. $R^2$ is 0.35, which is precisely what the authors received for US data in their original article.
In the second step we run 4 regressions for $n = 2, 3, 4, 5$, to find coefficients: $b_n$

$$rx_{t+1}^{(n)} = b_n \left( \gamma_0 + \gamma_1 y_t^1 + \gamma_2 f_t^{t+1,t+2} + \gamma_3 f_t^{t+2,t+3} + \gamma_4 f_t^{t+3,t+4} + \gamma_5 f_t^{t+4,t+5} \right) + \epsilon_{t+1}^{(n)} \tag{13}$$

$$\text{or} \quad rx_{t+1}^{(n)} = b_n \left( \gamma^T \mathbf{f}_t \right) + \epsilon_{t+1}^{(n)} \tag{14}$$

where $\mathbf{f}_t = [1, f_t^{t+1,t+2}, f_t^{t+2,t+3}, f_t^{t+3,t+4}, f_t^{t+4,t+5}]$. Obviously the coefficients $\gamma_n$ and $b_n$ are not separately identified, hence Cochrane & Piazzesi (2005) propose to normalise $b_n$ to have $\sum_{n=2}^{5} b_n = 1$. This step is not necessary since at the end we are interested in the pattern (supposedly a *tent*) that emerges from the cross product of both vectors of parameters, and we skip it for our inference from Polish data.

The vector $\gamma$ consists of coefficients with greater absolute value than the corresponding values found by Cochrane & Piazzesi (2005) for $n = \{4, 5\}$. The shape that slowly transpires for Polish government bonds is not resembling a *tent*, rather a *seagull*. The shape may be characteristic to the country data and, in particular, the trends and cycles of monetary policy the timespan of data covers. Cochrane & Piazzesi (2005) covered 40 years of data from 1963 to 2003, during which period rates hit double digit percentage points in the seventies and into eighties, whereas our data set is placed in absolute terms in much benign interest rates environment and has no impact of a serious tightening of monetary policy. Moreover not the shape as such is important, but the fact that these linear combinations exist with statistical significance and the $R^2$ in the range of $33.1 - 46.6\%$, which is roughly double the reads we have reported for the Fama and Bliss regressions type 2 (Table 13).

Table 9: One year excess return on average one year forward rates regressions, a modo Cochrane (2005) , Poland 2005:01-2020:06

First step regression

| | $\gamma_0$ | $\gamma_1$ | $\gamma_2$ | $\gamma_3$ | $\gamma_4$ | $\gamma_5$ |
|---|---|---|---|---|---|---|
| coeff | -0.0460 | 0.4069 | 1.3191 | 4.4494 | -22.5294 | 17.3856 |
| SE | 0.0180 | 1.8330 | 11.9305 | 31.0120 | 34.8548 | 14.1698 |
| p-val | 0.4816 | 0.6580 | 0.9064 | 1.0000 | 0.0000 | 1.0000 |

Second step regression

| n | $b_n$ | SE(hac) | p-val | $R^2$ |
|---|---|---|---|---|
| 2 | 0.8144 | 0.1088 | 0.0000 | 0.3310 |
| 3 | 1.2088 | 0.1563 | 0.0000 | 0.3490 |
| 4 | 1.5780 | 0.2281 | 0.0000 | 0.3460 |
| 5 | 1.8485 | 0.2737 | 0.0000 | 0.4660 |

Note: Standard errors (of the second step regression) are calculated with heteroscedasticity and aurocorrelation correction a la Newley-West

Table 10: Multiregressions of excess one year return on *return-forecasting error* and *Fama & Bliss term premium*, Poland 2005:01-2020:06

| n | $b_n$ | SE(hac) | p-val | $c_n$ | SE(hac) | p-val | $R^2$ |
|---|---|---|---|---|---|---|---|
| $n=2$ | 0.8211 | 0.1058 | 0.0000 | -0.0433 | 0.4745 | 0.5363 | 0.3350 |
| $n=3$ | 1.1991 | 0.1305 | 0.0000 | 0.0333 | 0.4795 | 0.4723 | 0.3460 |
| $n=4$ | 1.5064 | 0.1948 | 0.0000 | 0.1752 | 0.5990 | 0.3849 | 0.3340 |
| $n=5$ | 1.0961 | 0.5984 | 0.0335 | 1.5006 | 0.8250 | 0.0345 | 0.4350 |

Note: Standard errors are calculated with heteroscedasticity and aurocorrelation correction a la Newley-West

The robustness check of regression coefficients and the shapes they form are documented in Figure 34 in the Appendix, where we clearly see that even with relatively small diversification in yield nominal values in particular 28 systems the shape of regressions coefficients is still the same for all bond maturities under consideration but the shapes have sometimes more pronounced *left wing of the seagull* at 3-year maturity.

In order to document even stronger arguments against PEH we run also a multiregression on both the so called *return-forecasting error* $b_n\left(\gamma^T \mathbf{f}_t\right)$ and the Fama Bliss term premium $\left(f_t^{t+1,t+n} - y_t^1\right)$ for $h = 12$ months as regressors:

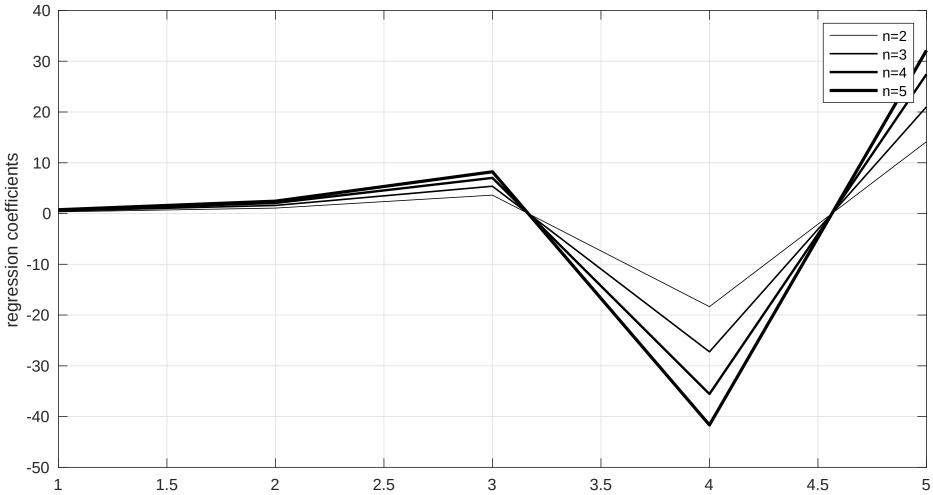
$$rx_{t+1}^{(n)} = b_n\left(\gamma^T \mathbf{f}_t\right) + c_n\left(f_t^{t+1,t+n} - y_t^1\right) + \epsilon_{t+1}^{(n)} \tag{15}$$

Table 10 reports these regressions' results for each bond maturity $n = \{2, 3, 4, 5\}$. Strikingly, coefficients $c_n$ in front of the *Fama & Bliss term premium* are not statistically significant except for the case of 5-year bond. Reported $R^2$ are generally sightly lower than in single factor (restricted) model, with minor improvement only occurring for 2-year bond (from 33.1% to 33.5%). In Fama Bliss term premium regression the coefficients where in absolute terms slightly higher except again for $n = 5$. Having this picture in mind, we may conclude that the tests proposed by Cochrane & Piazzesi (2005) are extending the area of PEH rejection further in to shorter horizons and bond maturities.

## Rolling realised returns on term premia regressions

In what follows, we present the outcomes of regression proposed by Thornton (2006) and dubbed by the author as: *conventional*. The idea is very simple, and test the wide spread assumption of effective bond markets, that there should not exist a quantitatively significant difference between the results of a strategy of rolling short term bonds and another one of investing in a long term

Table 11: Regression coefficients of one-year excess returns on forward rates, Poland 2005:01-2020:06




| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $n = 2$ | 0.3314 | 1.0744 | 3.6238 | -18.3490 | 14.1596 |
| $n = 3$ | 0.4919 | 1.5946 | 5.3786 | -27.2341 | 21.0161 |
| $n = 4$ | 0.6421 | 2.0816 | 7.0212 | -35.5511 | 27.4342 |
| $n = 5$ | 0.7522 | 2.4384 | 8.2246 | -41.6449 | 32.1366 |

Notes: (1) Standard errors are calculated with heteroscedasticity and autocorrelation correction a la Newley-West (2) Restricted (2 stepped) model

bond for the same period as the cumulated length of the first strategy.

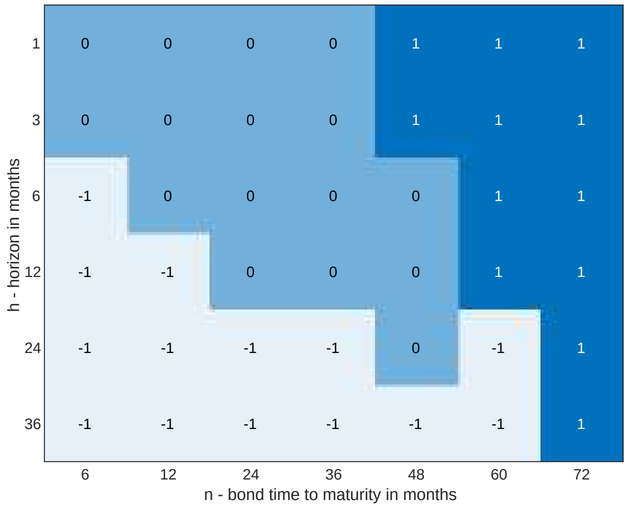
Regressions are performed for different combinations of $n$ and $h$, where we try to explain variability of an average *realised* $h$ period rates in $n$ tenor with the *observed and implied by the market* spread between spot rates feasible for discounting in a long $n$ and short $h$ tenors. We obviously need that $k \equiv \frac{n}{h} \in \mathbb{Z}$ because the two strategies have to have the same horizons, without any gaps or lags. Since the interest rates in Poland are *near-root* processes we also incorporate spreads to short term yields $y_t^{(h)}$ in both: regressand and regressor. The model has the following form:

$$\frac{1}{k}\sum_{i=0}^{k-1} y_{t+i\times h}^{(h)} - y_t^{(h)} = \alpha + \beta\left(y_t^{(n)} - y_t^{(h)}\right) + \epsilon_t \tag{16}$$

Notice that the rolling return is averaged in $h$-frequencies. For the null hypothesis that PEH holds we require: $H_0 : \beta = 1$. We have decided to choose short term investment length vector to be $\mathbf{h} = \{1, 3, 6, 12, 24, 36\}$ and bonds' maturities vector: $\mathbf{n} = \{6, 12, 24, 36, 48, 60, 72\}$. Obviously not all of the pairs (h,n) are feasible, because we have to comply with two rules: (1) $h < n$ and (2) above-mentioned: $\frac{n}{h} \in \mathbb{Z}$.

The results of these regressions for various pairs of $(h, n)$ and the decisions on PEH hypothesis with this regard are revealed in Table 16 and in Figure 14. We have found enough evidence to reject PEH for medium to longer bonds (4-6 years or more) across all potential horizons we considered. In these cases the $R^2$ are significantly different from zero which suggest that the slope of the yield curve has predictive power for the short short term rates. This predictive power, however diminishes as horizon is increased of the bond has longer maturity. The shorter the bond the shorter horizon staring from which PEH is rejected with 0.95 confidence level. $\beta$ coefficients within the same short term horizon $h$ decrease in value along with bond maturities, except for the first three entries for $h = \{1, 3, 6\}$ months. The $H_0$ decisions were checked for robustness to weight system changes and the results are revealed in Figure 35 in the Appendix. The regressions of strategies of rolling 3, 6 or 12 months investments for 4 years in many weighting systems would call for rejection of PEH in approximately half of the systems and in the system labelled *1* rolling investments of 6 and 12 months for 4 years are on the verge of rejection of null hypothesis (0.0552 and 0.0713 p-values).

Figure 14: Pure expectations hypothesis testing: conventional regression a modo Thornton (2006) for Poland



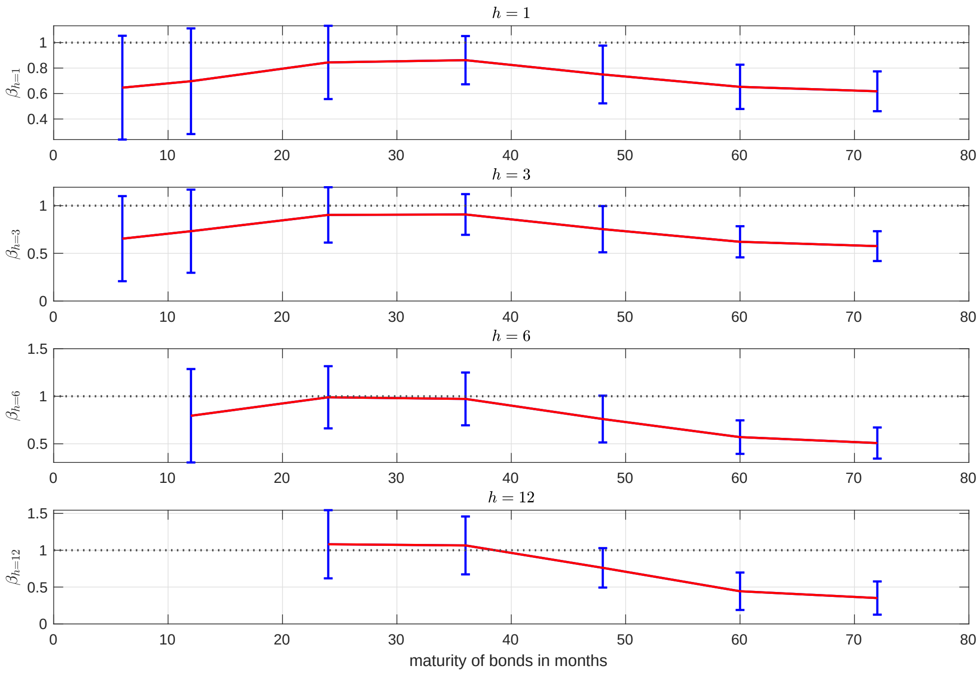

Notes: (1) *0* indicates that there is no sufficient evidence to reject null hypothesis of PEH, (2) *1* - sufficient evidence (with at 0.05 significance) to reject PEH, (3) *-1* - not applicable (i.e. a horizon is longer or equal bond's maturity) (4) NSS fit using weight system labelled *1*.

Figure 15 shows $\beta$ coefficients for various horizons ($1, 3, 6$ and 12 months) with their confidence bands at 0.95. The shapes observed for US data (Thornton (2006), Campbell & Shiller (1991)) are similar but the ones for Poland seem to be horizontal reflections of their American counterparts and the level is generally much closer to 1. Sufficient evidence to reject PEH ($\beta = 1$) at $0.05 - 0.06$ significance starts with 4-year bonds for all terms of short term rolled investments. In consequence, our results qualitatively differ from the original paper in that, EH in Poland works relatively good in the short-to-middle segments but not for longer bonds.[18]

Overall, the regressions presented in this subsection further confirm the inference from previous four, that PEH works fine in Poland for horizons up to 12 months *and* for bonds with maturities up to 36 months (included).

[18]whereas Thornton (2006) posit that *EH works better at the short and long ends of the maturity spectrum and less in the middle*

Figure 15: Smile and smirk in coefficients of *conventional* regressions a modo Thornton (2006) for Poland




Notes: (1) NSS fit using weight system labelled *1*.

## Realised spread on pro rata temporis current spread regressions

The last regression we will run is the one proposed again by Thornton (2006) and called: *contrarian*. Here we explain the realised spread on holding $n$ long bond for $h$ period over the original rate $y_t^{(n)}$ by the observed spread between the two spot rates scaled pro rata temporis by $h/(n-h)$:

$$y_{t+h}^{(n-h)} - y_t^{(n)} = \alpha + \beta\left(\frac{h}{n-h}\right)\left(y_t^{(n)} - y_t^{(h)}\right) + \epsilon_t \tag{17}$$

The null hypothesis is again $H_0 : \beta = 1$. As the name suggests these regressions are instrumental in showing and documenting the so-called *Campbell-Schiller paradox* (cf. Campbell & Shiller (1991)). The paradox in its original form says that the slope of the term structure (our regressor in the model) almost always: (1) gives a forecasts in the **wrong** direction for the **short-term** changes in the **long-term** bond yields and (2) gives a forecast in the **right** direction for the **long-term** changes in the **short-term** rates.

Figure 17 reports $\beta$ coefficients for various horizons $(3, 6, 12, 24$ and $36$ months) with their confidence bands at 0.95. Again the results for Poland differ from the ones obtained for American bonds by Thornton (2006) or Campbell & Shiller (1991). The *wrong signs* (as the authors called them) of coefficients are only visible and statistically different from 1 for horizons starting with $h = 36$ months and they indeed are decreasing with maturity of analysed bond, not like in Thornton (2006) for almost all combinations of $(h, n)$. For the *Campbell-Schiller paradox* to be present in Polish data we would have needed negative $\beta$ coefficients for the pairs of $(h, n)$ where EH does not hold and this is certainly not the case. Though, we have to admit that our samples are relatively small to the ones available for US (approximately 1/3rd to 1/4th in length), and the future assessment of *Campbell-Schiller paradox* existance in Polish bond data may change as time series grow.

Figure 16: Rolling realised returns on term premia regressions, Poland 2005:01-2020:06

| | h | n | $\alpha$ | SE($\alpha$) | p-val | $\beta$ | SE($\beta$) | p-val | $R^2$ | $R^2$ adj. | obs | RMSE | F-stat | F p-val | $\rho(6)$ | $\rho(12)$ | $\rho(24)$ | $\rho(36)$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\mathcal{R}_1$ | 1.0000 | 6.0000 | -0.0009 | 0.0005 | 0.0000 | 0.6461 | 0.2486 | 0.0773 | 0.2340 | 0.2300 | 179.0000 | 0.0025 | 54.1900 | 0.0000 | 0.4400 | -0.0100 | -0.2100 | -0.1200 |
| $\mathcal{R}_2$ | 1.0000 | 12.0000 | -0.0021 | 0.0009 | 0.0000 | 0.6967 | 0.2532 | 0.1155 | 0.2550 | 0.2510 | 173.0000 | 0.0044 | 58.5700 | 0.0000 | 0.5800 | 0.0000 | -0.3200 | -0.1100 |
| $\mathcal{R}_3$ | 1.0000 | 24.0000 | -0.0053 | 0.0014 | 0.0000 | 0.8444 | 0.1756 | 0.1877 | 0.3580 | 0.3540 | 161.0000 | 0.0061 | 88.6800 | 0.0000 | 0.6800 | 0.1500 | -0.4500 | -0.0800 |
| $\mathcal{R}_4$ | 1.0000 | 36.0000 | -0.0083 | 0.0021 | 0.0000 | 0.8621 | 0.1156 | 0.1165 | 0.4300 | 0.4270 | 149.0000 | 0.0064 | 111.1200 | 0.0000 | 0.7000 | 0.2700 | -0.3100 | -0.1300 |
| $\mathcal{R}_5$ | 1.0000 | 48.0000 | -0.0106 | 0.0028 | 0.0000 | **0.7497** | 0.1383 | 0.0352 | 0.4150 | 0.4110 | 137.0000 | 0.0064 | 95.9200 | 0.0000 | 0.7400 | 0.3900 | -0.1000 | -0.1400 |
| $\mathcal{R}_6$ | 1.0000 | 60.0000 | -0.0130 | 0.0029 | 0.0000 | **0.6525** | 0.1061 | 0.0005 | 0.3650 | 0.3600 | 125.0000 | 0.0068 | 70.6600 | 0.0000 | 0.7500 | 0.4100 | 0.0200 | -0.1100 |
| $\mathcal{R}_7$ | 1.0000 | 72.0000 | -0.0155 | 0.0037 | 0.0000 | **0.6175** | 0.0954 | 0.0000 | 0.3230 | 0.3170 | 113.0000 | 0.0076 | 52.9600 | 0.0000 | 0.8200 | 0.5300 | 0.0600 | -0.0200 |
| $\mathcal{R}_8$ | 3.0000 | 6.0000 | -0.0006 | 0.0003 | 0.0000 | 0.6539 | 0.2721 | 0.1017 | 0.1870 | 0.1820 | 179.0000 | 0.0016 | 40.7300 | 0.0000 | 0.3900 | 0.0000 | -0.1700 | -0.1000 |
| $\mathcal{R}_9$ | 3.0000 | 12.0000 | -0.0018 | 0.0007 | 0.0000 | 0.7323 | 0.2659 | 0.1571 | 0.2290 | 0.2250 | 173.0000 | 0.0037 | 50.8400 | 0.0000 | 0.5300 | -0.0100 | -0.2900 | -0.1100 |
| $\mathcal{R}_{10}$ | 3.0000 | 24.0000 | -0.0051 | 0.0013 | 0.0000 | 0.9030 | 0.1769 | 0.2918 | 0.3360 | 0.3320 | 161.0000 | 0.0057 | 80.5400 | 0.0000 | 0.6500 | 0.1300 | -0.4100 | -0.0600 |
| $\mathcal{R}_{11}$ | 3.0000 | 36.0000 | -0.0082 | 0.0021 | 0.0000 | 0.9076 | 0.1302 | 0.2389 | 0.3940 | 0.3900 | 149.0000 | 0.0063 | 95.7400 | 0.0000 | 0.6900 | 0.2800 | -0.2600 | -0.1000 |
| $\mathcal{R}_{12}$ | 3.0000 | 48.0000 | -0.0102 | 0.0028 | 0.0000 | **0.7535** | 0.1480 | 0.0479 | 0.3480 | 0.3430 | 137.0000 | 0.0065 | 72.0500 | 0.0000 | 0.7500 | 0.4200 | -0.0400 | -0.1100 |
| $\mathcal{R}_{13}$ | 3.0000 | 60.0000 | -0.0123 | 0.0029 | 0.0000 | **0.6210** | 0.0995 | 0.0001 | 0.2800 | 0.2740 | 125.0000 | 0.0071 | 47.9100 | 0.0000 | 0.7700 | 0.4400 | 0.0500 | -0.1000 |
| $\mathcal{R}_{14}$ | 3.0000 | 72.0000 | -0.0148 | 0.0038 | 0.0000 | **0.5756** | 0.0953 | 0.0000 | 0.2420 | 0.2350 | 113.0000 | 0.0078 | 35.3600 | 0.0000 | 0.8300 | 0.5400 | 0.0700 | -0.0200 |
| $\mathcal{R}_{15}$ | 6.0000 | 12.0000 | -0.0013 | 0.0005 | 0.0000 | 0.7951 | 0.2995 | 0.2470 | 0.1920 | 0.1870 | 173.0000 | 0.0026 | 40.6700 | 0.0000 | 0.4000 | -0.0600 | -0.2200 | -0.1000 |
| $\mathcal{R}_{16}$ | 6.0000 | 24.0000 | -0.0047 | 0.0011 | 0.0000 | 0.9892 | 0.1993 | 0.4783 | 0.3050 | 0.3000 | 161.0000 | 0.0051 | 69.7200 | 0.0000 | 0.5900 | 0.1000 | -0.3500 | -0.0400 |
| $\mathcal{R}_{17}$ | 6.0000 | 36.0000 | -0.0078 | 0.0020 | 0.0000 | 0.9721 | 0.1692 | 0.4345 | 0.3470 | 0.3430 | 149.0000 | 0.0061 | 78.2200 | 0.0000 | 0.6800 | 0.2900 | -0.1900 | -0.0700 |
| $\mathcal{R}_{18}$ | 6.0000 | 48.0000 | -0.0096 | 0.0027 | 0.0000 | 0.7605 | 0.1500 | 0.0552 | 0.2710 | 0.2660 | 137.0000 | 0.0067 | 50.1900 | 0.0000 | 0.7500 | 0.4400 | 0.0200 | -0.0900 |
| $\mathcal{R}_{19}$ | 6.0000 | 60.0000 | -0.0114 | 0.0030 | 0.0000 | **0.5705** | 0.1074 | 0.0000 | 0.1860 | 0.1800 | 125.0000 | 0.0073 | 28.1900 | 0.0000 | 0.7800 | 0.4700 | 0.0800 | -0.0800 |
| $\mathcal{R}_{20}$ | 6.0000 | 72.0000 | -0.0137 | 0.0039 | 0.0000 | **0.5082** | 0.0997 | 0.0000 | 0.1530 | 0.1450 | 113.0000 | 0.0081 | 20.0500 | 0.0000 | 0.8300 | 0.5400 | 0.0900 | -0.0200 |
| $\mathcal{R}_{21}$ | 12.0000 | 24.0000 | -0.0034 | 0.0008 | 0.0000 | 1.0810 | 0.2820 | 0.6130 | 0.2120 | 0.2070 | 161.0000 | 0.0039 | 42.7700 | 0.0000 | 0.4900 | -0.0600 | -0.2300 | 0.0300 |
| $\mathcal{R}_{22}$ | 12.0000 | 36.0000 | -0.0066 | 0.0015 | 0.0000 | 1.0651 | 0.2395 | 0.6072 | 0.2650 | 0.2600 | 149.0000 | 0.0055 | 53.1100 | 0.0000 | 0.6700 | 0.2700 | -0.1300 | 0.0100 |
| $\mathcal{R}_{23}$ | 12.0000 | 48.0000 | -0.0081 | 0.0023 | 0.0000 | 0.7610 | 0.1630 | 0.0713 | 0.1720 | 0.1660 | 137.0000 | 0.0066 | 28.0200 | 0.0000 | 0.7700 | 0.4700 | 0.0900 | -0.0600 |
| $\mathcal{R}_{24}$ | 12.0000 | 60.0000 | -0.0093 | 0.0028 | 0.0000 | **0.4434** | 0.1548 | 0.0002 | 0.0750 | 0.0680 | 125.0000 | 0.0074 | 10.0100 | 0.0020 | 0.7900 | 0.4800 | 0.1000 | -0.0700 |
| $\mathcal{R}_{25}$ | 12.0000 | 72.0000 | -0.0114 | 0.0036 | 0.0000 | **0.3511** | 0.1374 | 0.0000 | 0.0520 | 0.0430 | 113.0000 | 0.0083 | 6.0900 | 0.0151 | 0.8300 | 0.5300 | 0.0900 | -0.0200 |
| $\mathcal{R}_{26}$ | 24.0000 | 48.0000 | -0.0048 | 0.0014 | 0.0000 | 0.6578 | 0.3105 | 0.1353 | 0.0650 | 0.0590 | 137.0000 | 0.0055 | 9.4500 | 0.0026 | 0.7800 | 0.4300 | -0.0100 | 0.0500 |
| $\mathcal{R}_{27}$ | 24.0000 | 72.0000 | -0.0069 | 0.0028 | 0.0000 | **-0.0357** | 0.2376 | 0.0000 | 0.0000 | -0.0090 | 113.0000 | 0.0074 | 0.0400 | 0.8445 | 0.8200 | 0.5000 | -0.0200 | 0.0300 |
| $\mathcal{R}_{28}$ | 36.0000 | 72.0000 | -0.0021 | 0.0029 | 0.0000 | **-0.7587** | 0.4756 | 0.0001 | 0.1060 | 0.0970 | 113.0000 | 0.0058 | 13.1000 | 0.0004 | 0.7600 | 0.4300 | 0.0000 | -0.1100 |

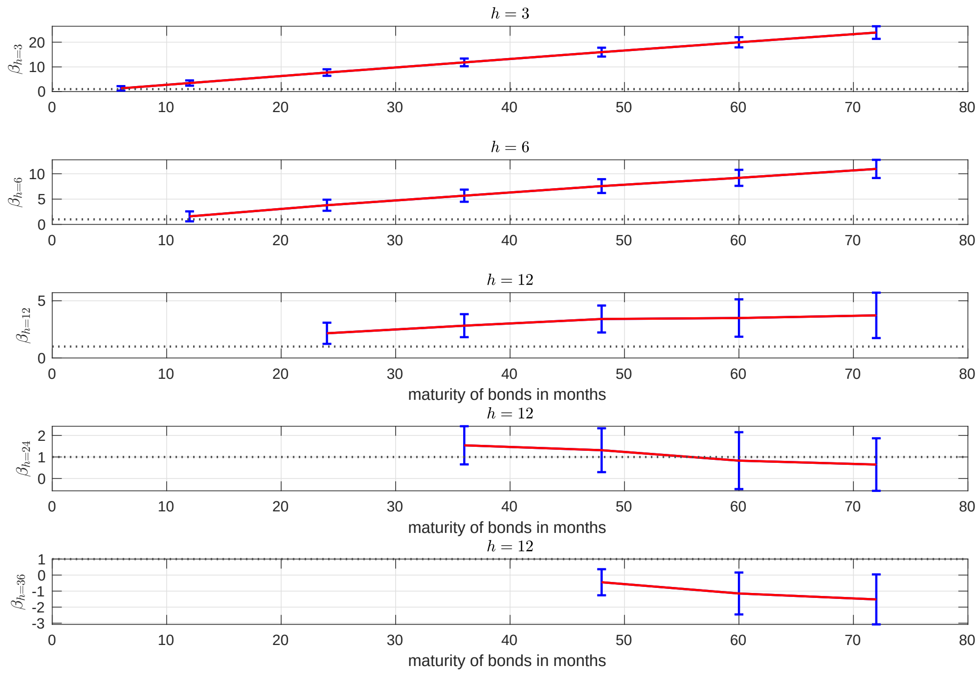
Regression: $\frac{1}{k}\sum_{i=0}^{k-1} y_{t+i\times h}^{(h)} - y_t^{(h)} = \alpha + \beta\left(y_t^{(n)} - y_t^{(h)}\right) + \epsilon_t$

Null hypothesis (PEH holds): $H_0 : \beta = 1$.

Standard errors are calculated with heteroscedasticity and aurocorrelation correction a la Newley-West

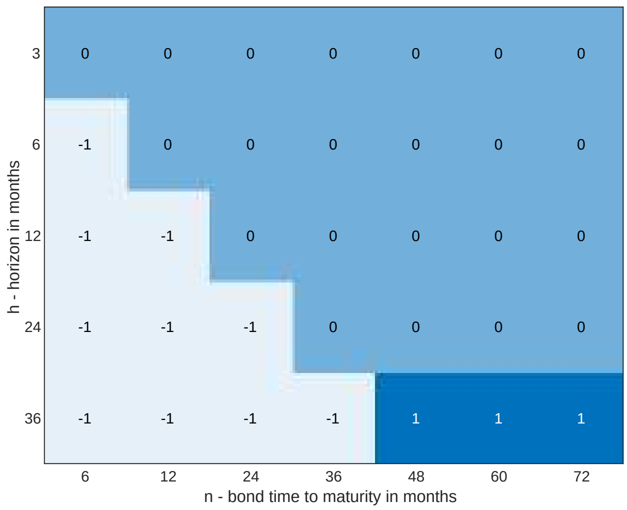
Bold estimates of $\beta$ have lower p-value than 0.05 (our assumed significance level) and call for rejection of null hypothesis

Figure 17: Smirks in coefficients of *contrarian* regressions a modo Thornton (2006) for Poland




Notes: (1) NSS fit using weight system labelled *1*.

Figure 18: Pure expectations hypothesis testing: contrarian regression a modo Thornton (2006) for Poland




Notes: (1) *0* indicates that there is no sufficient evidence to reject null hypothesis of PEH, (2) *1* - sufficient evidence (with at 0.05 significance) to reject PEH, (3) *-1* - not applicable (i.e. a horizon is longer or equal bond's maturity) (4) NSS fit using weight system labelled *1*.

## Summary

We have collected more than ten stylised facts on Polish government bonds market, which were consequently used in filtering and weight system design. These facts revealed importance of liquidity in bonds lifespan, heterogeneity of liquidity in different yield curve segments, bid-ask spreads behaviour as well as price distortion in the very short end of the curve due to *switch auctions*. We found that in our data maximum timespan all segment-wise average yield time series are *trend stationary* when corrected for long term variance, which will have important implications in the next chapter on

Figure 19: Realised spread on pro rata temporis current spread regressions, Poland 2005:01-2020:06

| | h | n | $\alpha$ | SE($\alpha$) | p-val | $\beta$ | SE($\beta$) | p-val | $R^2$ | $R^2$ adj. | obs | RMSE | F-stat | F p-val | $\rho(6)$ | $\rho(12)$ | $\rho(24)$ | $\rho(36)$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\mathcal{R}_1$ | 3.0000 | 6.0000 | -0.0011 | 0.0006 | 0.0000 | 1.3078 | 0.5443 | 0.7141 | 0.1870 | 0.1820 | 179.0000 | 0.0031 | 40.7300 | 0.0000 | 0.3900 | 0.0000 | -0.1700 | -0.1000 |
| $\mathcal{R}_2$ | 3.0000 | 12.0000 | -0.0013 | 0.0005 | 0.0000 | 3.4576 | 0.6499 | 0.9999 | 0.4860 | 0.4830 | 173.0000 | 0.0033 | 161.7800 | 0.0000 | 0.2800 | -0.0400 | -0.1200 | -0.0700 |
| $\mathcal{R}_3$ | 3.0000 | 24.0000 | -0.0016 | 0.0006 | 0.0000 | 7.7072 | 0.8059 | 1.0000 | 0.6500 | 0.6480 | 161.0000 | 0.0036 | 295.3900 | 0.0000 | 0.1300 | -0.0800 | -0.0300 | 0.0000 |
| $\mathcal{R}_4$ | 3.0000 | 36.0000 | -0.0016 | 0.0007 | 0.0000 | 11.8230 | 0.9522 | 1.0000 | 0.7040 | 0.7020 | 149.0000 | 0.0039 | 349.4500 | 0.0000 | 0.0800 | -0.1000 | 0.0100 | 0.0500 |
| $\mathcal{R}_5$ | 3.0000 | 48.0000 | -0.0016 | 0.0008 | 0.0000 | 15.9722 | 1.0931 | 1.0000 | 0.7410 | 0.7390 | 137.0000 | 0.0040 | 385.8200 | 0.0000 | 0.0600 | -0.1200 | 0.0600 | 0.0400 |
| $\mathcal{R}_6$ | 3.0000 | 60.0000 | -0.0015 | 0.0009 | 0.0000 | 19.9699 | 1.2577 | 1.0000 | 0.7600 | 0.7580 | 125.0000 | 0.0042 | 390.2700 | 0.0000 | 0.0700 | -0.1700 | 0.1000 | 0.0000 |
| $\mathcal{R}_7$ | 3.0000 | 72.0000 | -0.0013 | 0.0010 | 0.0000 | 23.9073 | 1.5550 | 1.0000 | 0.7880 | 0.7860 | 113.0000 | 0.0041 | 412.9000 | 0.0000 | 0.0300 | -0.1300 | 0.0000 | 0.0300 |
| $\mathcal{R}_8$ | 6.0000 | 12.0000 | -0.0026 | 0.0009 | 0.0000 | 1.5902 | 0.5990 | 0.8378 | 0.1920 | 0.1870 | 173.0000 | 0.0052 | 40.6700 | 0.0000 | 0.4000 | -0.0600 | -0.2200 | -0.1000 |
| $\mathcal{R}_9$ | 6.0000 | 24.0000 | -0.0032 | 0.0010 | 0.0000 | 3.7914 | 0.6661 | 1.0000 | 0.4030 | 0.3990 | 161.0000 | 0.0053 | 107.1400 | 0.0000 | 0.2200 | -0.1300 | -0.1300 | 0.0200 |
| $\mathcal{R}_{10}$ | 6.0000 | 36.0000 | -0.0029 | 0.0012 | 0.0000 | 5.6738 | 0.7323 | 1.0000 | 0.4670 | 0.4640 | 149.0000 | 0.0055 | 128.9900 | 0.0000 | 0.1600 | -0.1500 | -0.0600 | 0.1100 |
| $\mathcal{R}_{11}$ | 6.0000 | 48.0000 | -0.0027 | 0.0013 | 0.0000 | 7.5696 | 0.8309 | 1.0000 | 0.5130 | 0.5090 | 137.0000 | 0.0056 | 142.0400 | 0.0000 | 0.1300 | -0.1700 | 0.0400 | 0.1000 |
| $\mathcal{R}_{12}$ | 6.0000 | 60.0000 | -0.0023 | 0.0015 | 0.0000 | 9.1994 | 0.9651 | 1.0000 | 0.5300 | 0.5260 | 125.0000 | 0.0059 | 138.6800 | 0.0000 | 0.1400 | -0.2200 | 0.1200 | 0.0600 |
| $\mathcal{R}_{13}$ | 6.0000 | 72.0000 | -0.0020 | 0.0017 | 0.0000 | 10.9614 | 1.0944 | 1.0000 | 0.5660 | 0.5620 | 113.0000 | 0.0059 | 144.6500 | 0.0000 | 0.1200 | -0.1600 | 0.0600 | 0.1000 |
| $\mathcal{R}_{14}$ | 12.0000 | 24.0000 | -0.0068 | 0.0016 | 0.0000 | 2.1619 | 0.5641 | 0.9803 | 0.2120 | 0.2070 | 161.0000 | 0.0078 | 42.7700 | 0.0000 | 0.4900 | -0.0600 | -0.2300 | 0.0300 |
| $\mathcal{R}_{15}$ | 12.0000 | 36.0000 | -0.0061 | 0.0018 | 0.0000 | 2.8259 | 0.6131 | 0.9986 | 0.2340 | 0.2290 | 149.0000 | 0.0079 | 44.8600 | 0.0000 | 0.4800 | -0.0200 | -0.1100 | 0.1000 |
| $\mathcal{R}_{16}$ | 12.0000 | 48.0000 | -0.0048 | 0.0018 | 0.0000 | 3.4089 | 0.7187 | 0.9996 | 0.2370 | 0.2310 | 137.0000 | 0.0080 | 41.8900 | 0.0000 | 0.4800 | 0.0200 | -0.0100 | 0.1100 |
| $\mathcal{R}_{17}$ | 12.0000 | 60.0000 | -0.0034 | 0.0020 | 0.0000 | 3.4952 | 0.9971 | 0.9938 | 0.2100 | 0.2040 | 125.0000 | 0.0080 | 32.7100 | 0.0000 | 0.4900 | -0.0100 | 0.0900 | 0.0900 |
| $\mathcal{R}_{18}$ | 12.0000 | 72.0000 | -0.0022 | 0.0021 | 0.0000 | 3.7274 | 1.2100 | 0.9879 | 0.1970 | 0.1900 | 113.0000 | 0.0083 | 27.2400 | 0.0000 | 0.4800 | 0.0000 | 0.0800 | 0.1400 |
| $\mathcal{R}_{19}$ | 24.0000 | 36.0000 | -0.0128 | 0.0028 | 0.0000 | 1.5429 | 0.5404 | 0.8424 | 0.1240 | 0.1180 | 149.0000 | 0.0104 | 20.8100 | 0.0000 | 0.7600 | 0.3600 | -0.0900 | 0.0400 |
| $\mathcal{R}_{20}$ | 24.0000 | 48.0000 | -0.0096 | 0.0028 | 0.0000 | 1.3157 | 0.6211 | 0.6944 | 0.0650 | 0.0590 | 137.0000 | 0.0109 | 9.4500 | 0.0026 | 0.7800 | 0.4300 | -0.0100 | 0.0500 |
| $\mathcal{R}_{21}$ | 24.0000 | 60.0000 | -0.0064 | 0.0030 | 0.0000 | 0.8309 | 0.8058 | 0.4169 | 0.0220 | 0.0140 | 125.0000 | 0.0111 | 2.8100 | 0.0962 | 0.7400 | 0.3900 | -0.0100 | 0.0700 |
| $\mathcal{R}_{22}$ | 24.0000 | 72.0000 | -0.0048 | 0.0035 | 0.0000 | 0.6490 | 0.7442 | 0.3186 | 0.0130 | 0.0040 | 113.0000 | 0.0109 | 1.4800 | 0.2256 | 0.7500 | 0.4400 | 0.0200 | 0.1300 |
| $\mathcal{R}_{23}$ | 36.0000 | 48.0000 | -0.0100 | 0.0047 | 0.0000 | **-0.4458** | 0.4962 | 0.0018 | 0.0130 | 0.0060 | 137.0000 | 0.0116 | 1.7800 | 0.1847 | 0.7800 | 0.4400 | 0.0600 | -0.1700 |
| $\mathcal{R}_{24}$ | 36.0000 | 60.0000 | -0.0062 | 0.0054 | 0.0000 | **-1.1470** | 0.7974 | 0.0035 | 0.0670 | 0.0590 | 125.0000 | 0.0118 | 8.8300 | 0.0036 | 0.7700 | 0.4400 | 0.0100 | -0.1900 |
| $\mathcal{R}_{25}$ | 36.0000 | 72.0000 | -0.0041 | 0.0058 | 0.0000 | **-1.5174** | 0.9512 | 0.0041 | 0.1060 | 0.0970 | 113.0000 | 0.0117 | 13.1000 | 0.0004 | 0.7600 | 0.4300 | 0.0000 | -0.1100 |

Regression: $y_{t+h}^{(n-h)} - y_t^{(n)} = \alpha + \beta \left(\frac{h}{n-h}\right) \left(y_t^{(n)} - y_t^{(h)}\right) + \epsilon_t$

Null hypothesis (PEH holds): $H_0 : \beta = 1$.

Standard errors are calculated with heteroscedasticity and aurocorrelation correction a la Newley-West

Bold estimates of $\beta$ have lower p-value than 0.05 (our assumed significance level) and call for rejection of null hypothesis expectations hypothesis testing.

Moreover, we argued that as for the ultra short end (1/52 years) rate we should use NBP bill rebased rate (which importance in the default free instruments market in PLN cannot be overlooked). We developed here very efficient algorithm which allowed for relatively quick estimations of 28 systems with approximately 4 thousand days each. We have tested 28 different weight systems and rank them in the space of *goodness-of-fit* and *smoothness* and confirmed that there is a class of weights that systematically gives better results than the classic approach of all equal weights. The highest ranked systems have at least the same weight for the short end of the curve as a sum for all other tenors of bonds, *eligible-for-switch* bonds were excluded from the estimation and weights were be based on at least outstanding amounts.

The evidence presented in the last section suggests that indeed PEH does not hold universally for Polish government bonds yield curve. Contrary to the US data and research where PEH is almost always rejected, we have found that for Poland there is a limited domain where PEH cannot be ruled out. The scope where *pure expectations hypothesis* probably holds in Poland is bounded by (1) the investment horizon of approximately 12 months and (joined condition) and (2) by maturity of the bond of circa 36 months. It is still unclear (as in the bewildering variety of research) what causes the rejection of PEH for all other combinations of horizon length and maturity: existence of some kind of risk premia[19] or unexpected excess yield (we have only a mixture of these two contained in $\beta$ coefficient estimators). This leads us to further yield structure decomposition and possible hypothesis that the yield curve spans all information relevant for forecasting future yields and returns and no variables other than the current curve are needed, which we analyse in the future, interlinked research.

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[19] i.e. EH with constant risk premium, EH with time-varying risk premium

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# APPENDIX. AUXILIARY FIGURES. DETAILED TABLES

Figure 20: Selected characteristics of Polish fixed coupon government bonds traded on BondSpot in 2005:01-2020:06 - short term bonds < 1.5 Y

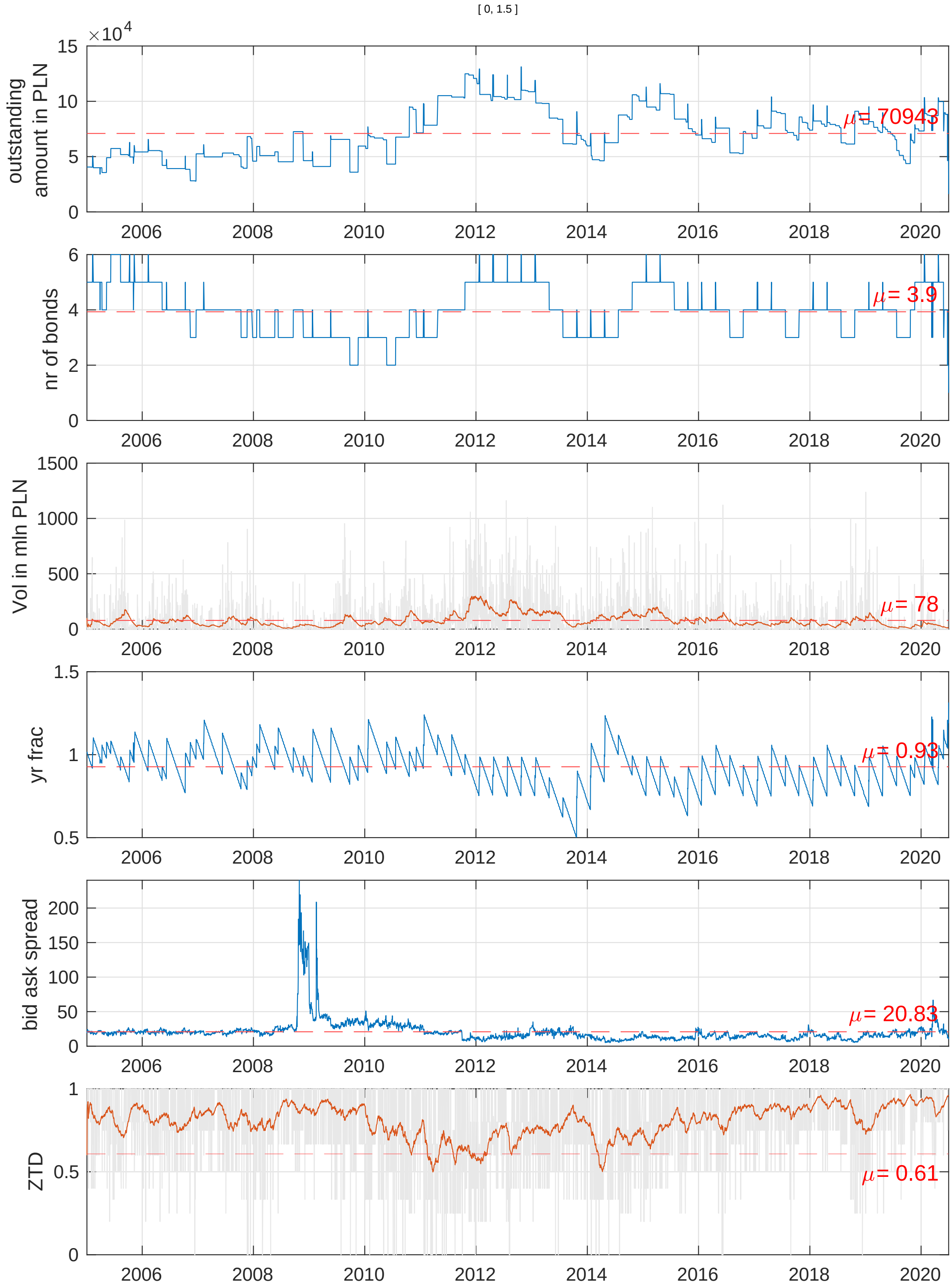


Figure 21: Selected characteristics of Polish fixed coupon government bonds traded on BondSpot in 2005:01-2020:06 - bonds with time to maturity between 1.5 and 3.5

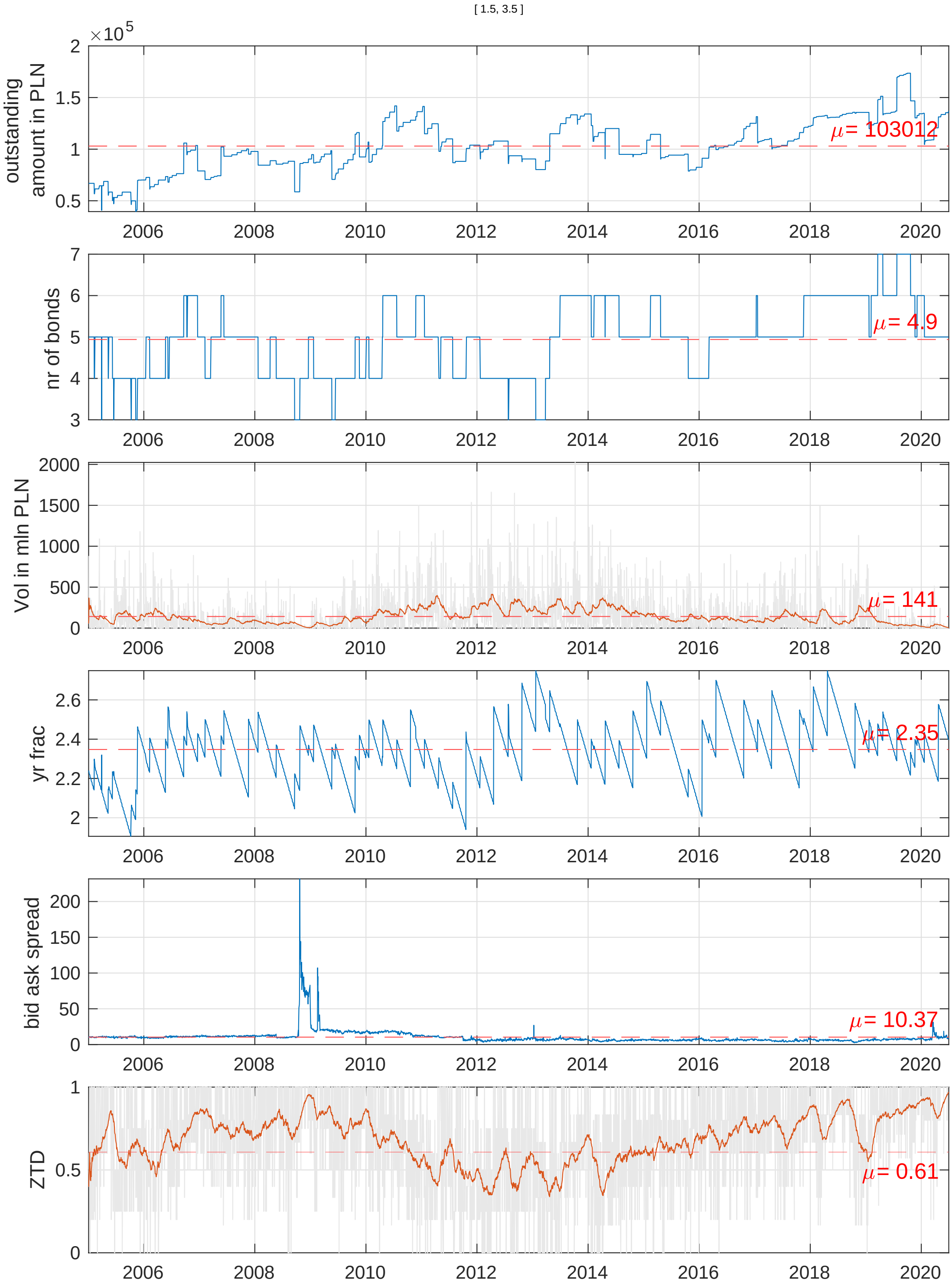


Figure 22: Selected characteristics of Polish fixed coupon government bonds traded on BondSpot in 2005:01-2020:06 - bonds with time to maturity between 3.5 and 6.0

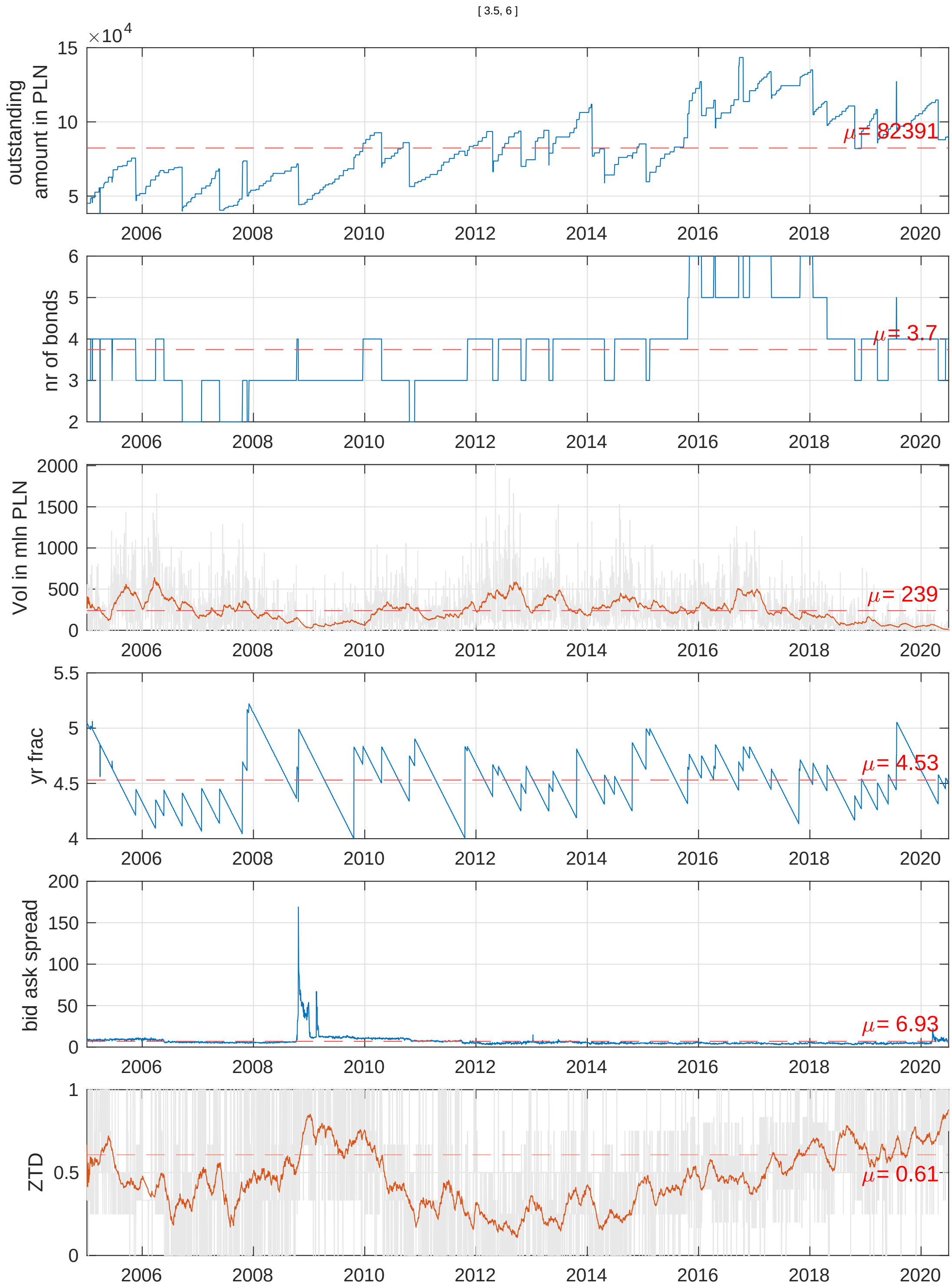


Figure 23: Selected characteristics of Polish fixed coupon government bonds traded on BondSpot in 2005:01-2020:06- bonds with time to maturity between 6.0 and 12.0

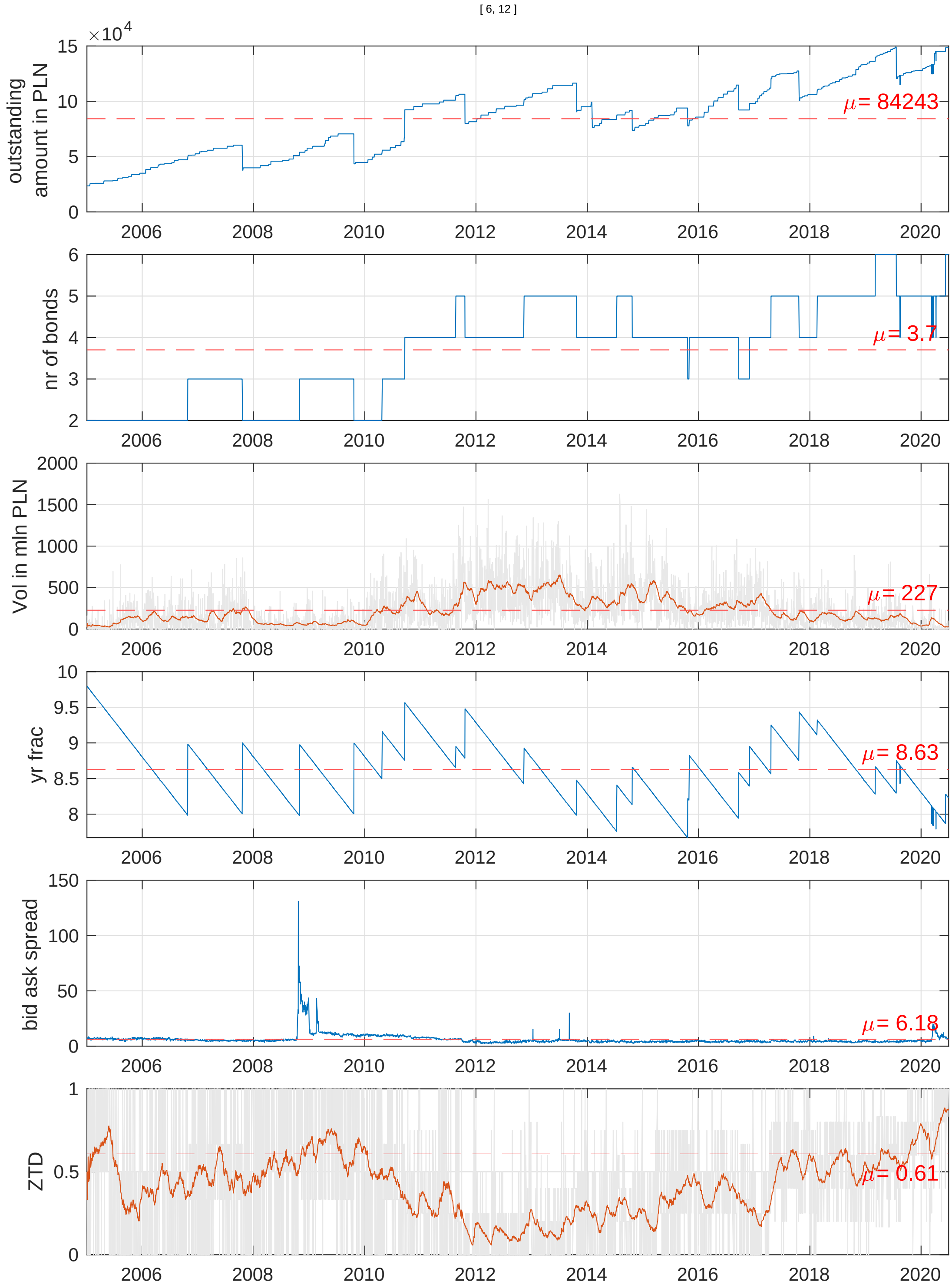


Figure 24: Selected characteristics of Polish fixed coupon government bonds traded on BondSpot in 2005:01-2020:06 - ultra long bonds > 12 Y

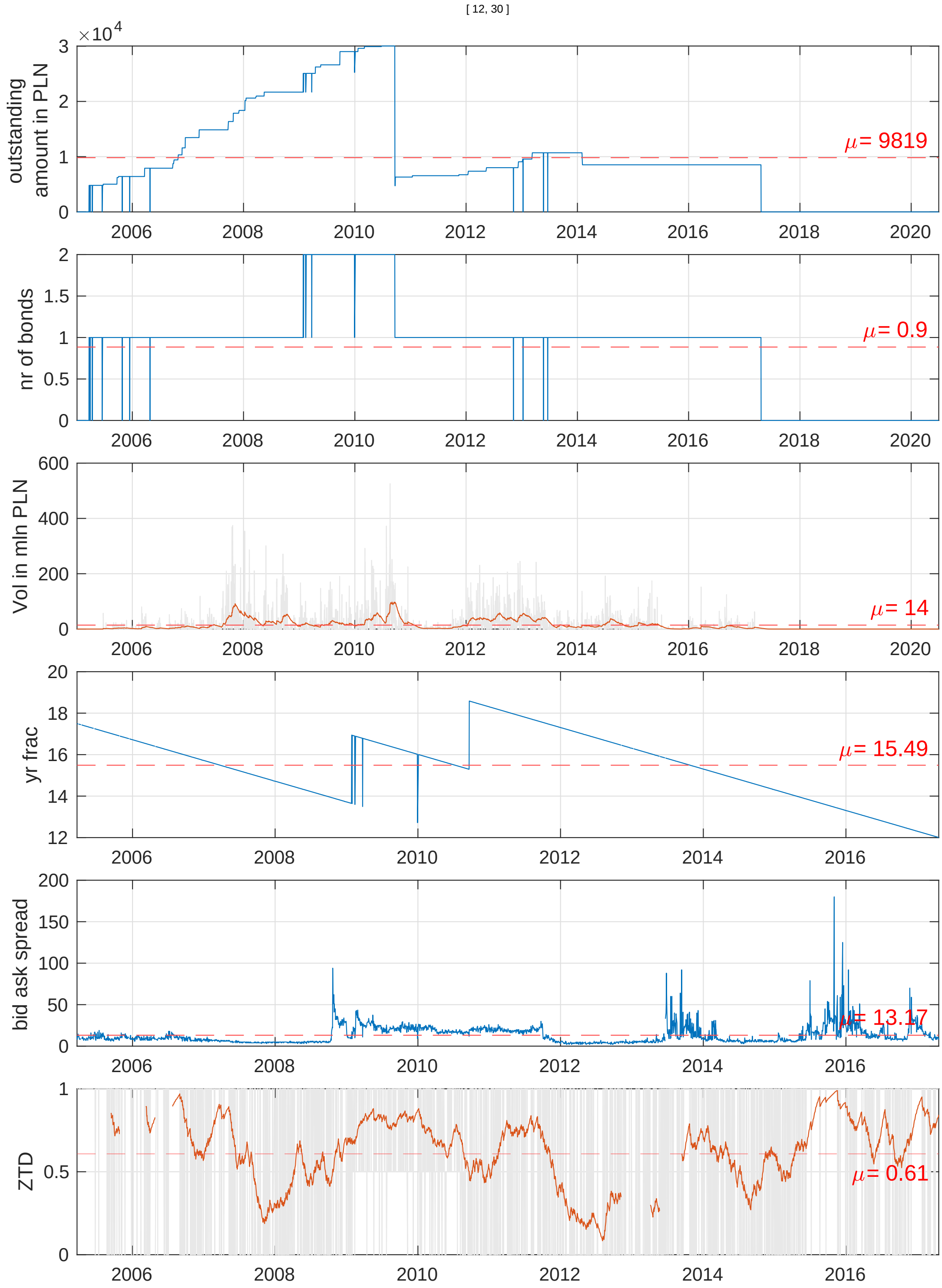
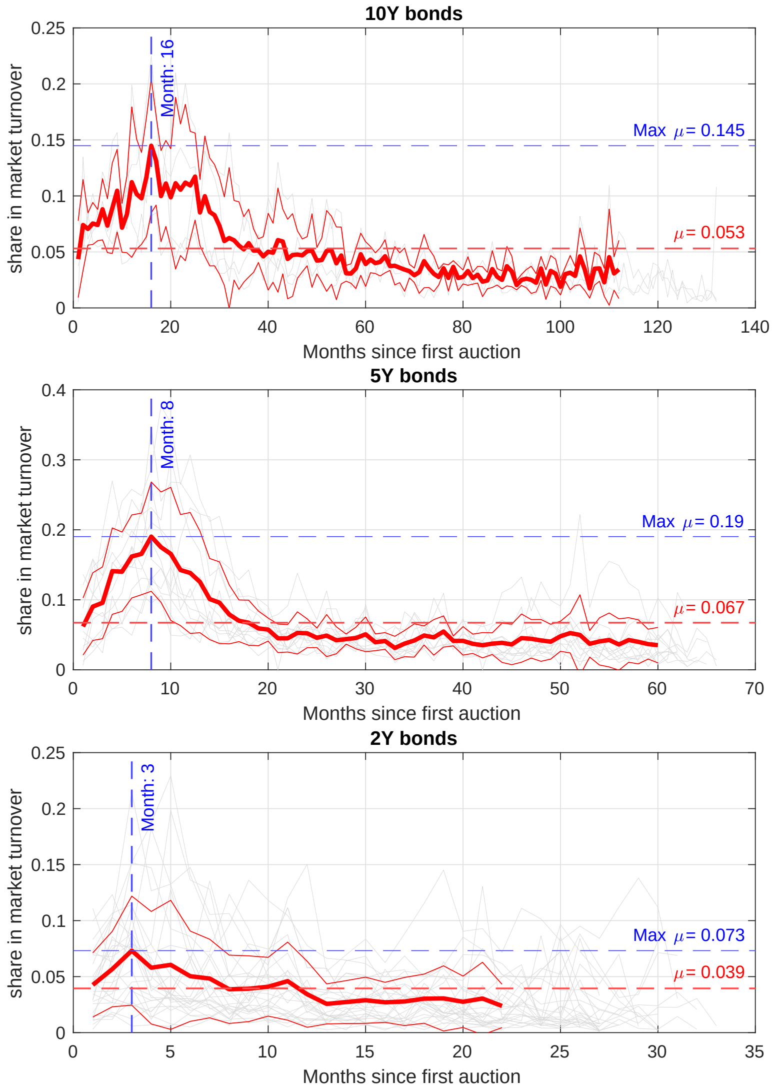
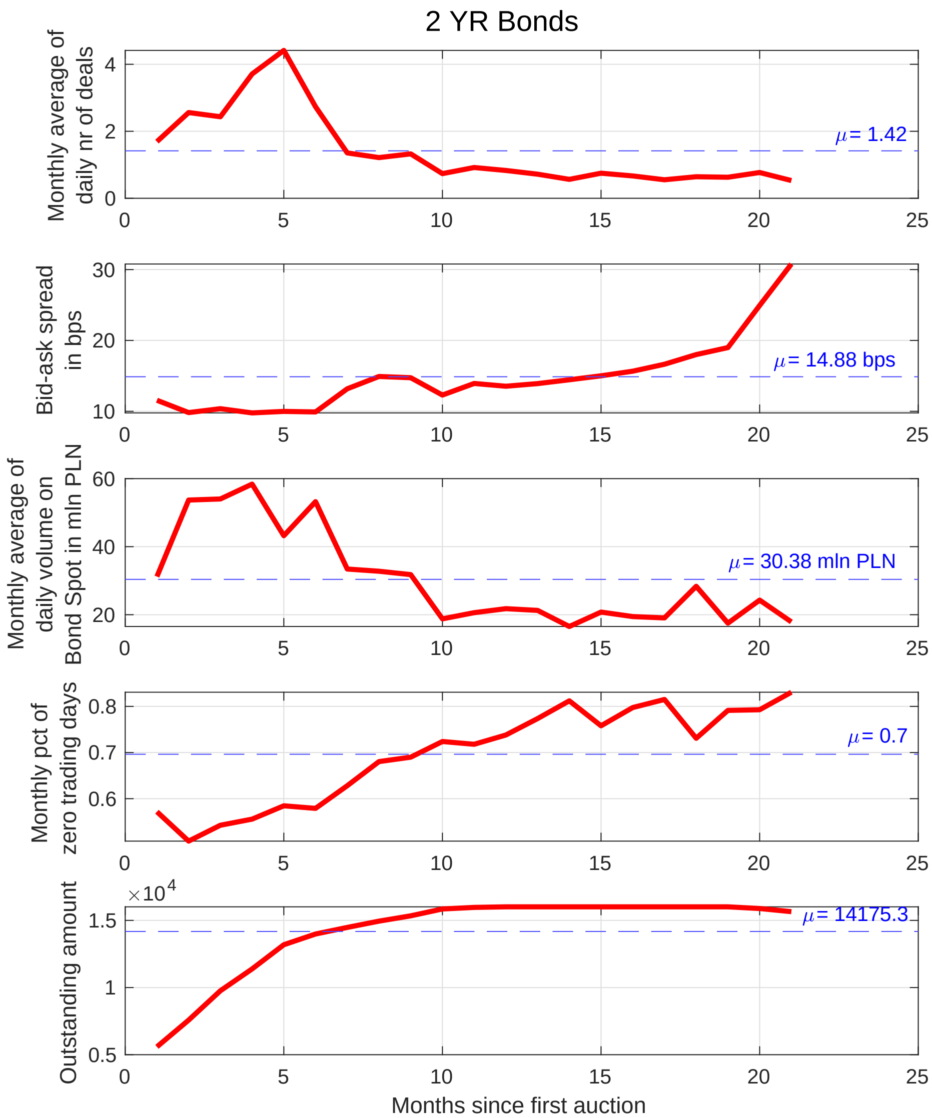


Figure 25: Averaged turnover share of fixed coupon government bonds of 2, 5, 10Y types during their lifespan



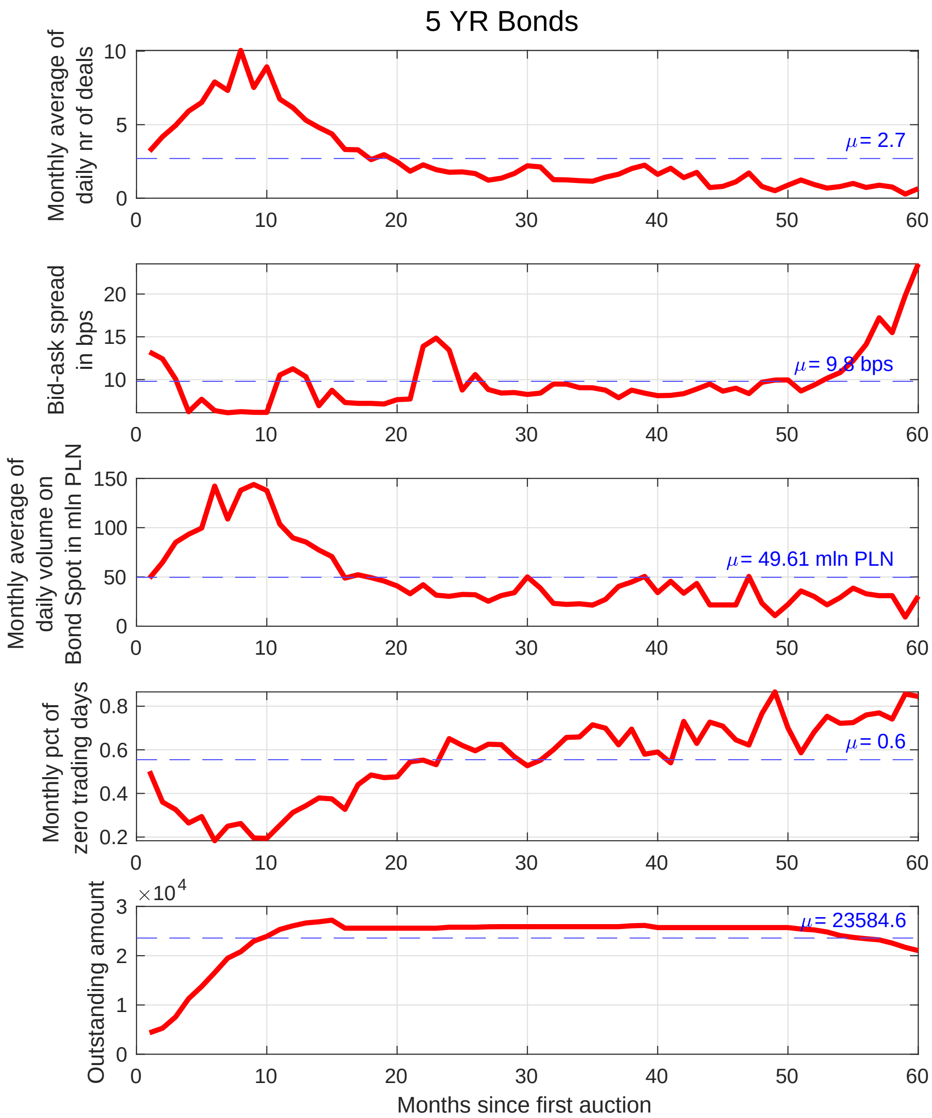

Notes: (1) thinner red lines indicate one standard deviation up and down from the mean on a certain month, (2) (2) Only bonds with full life history contained in the period 2005.Q1: 2020.Q3 were taken into consideration.

Figure 26: Selected averaged liquidity measures of 2Y fixed coupon government bonds



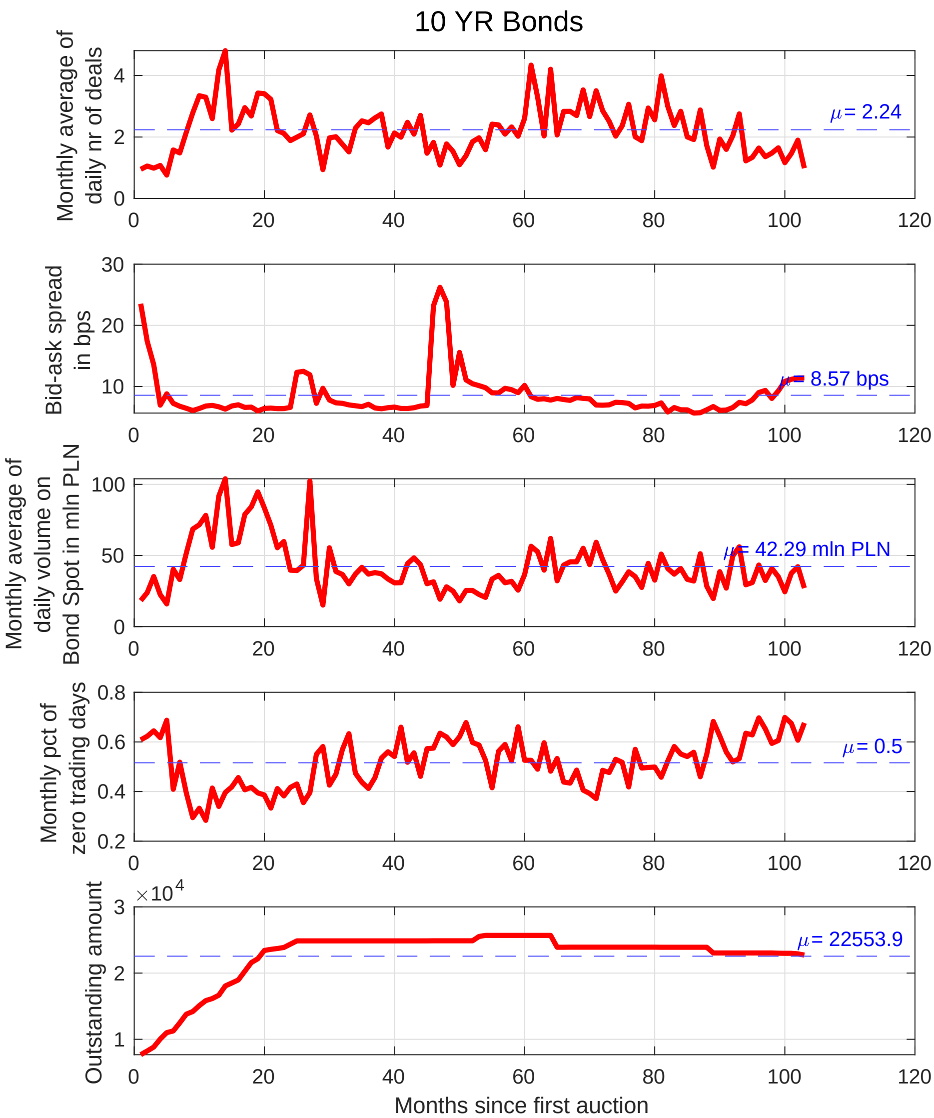

Notes: (1) Only bonds with full life history contained in the period 2005.Q1: 2020.Q3 were taken into consideration.

Figure 27: Selected averaged liquidity measures of 5Y fixed coupon government bonds



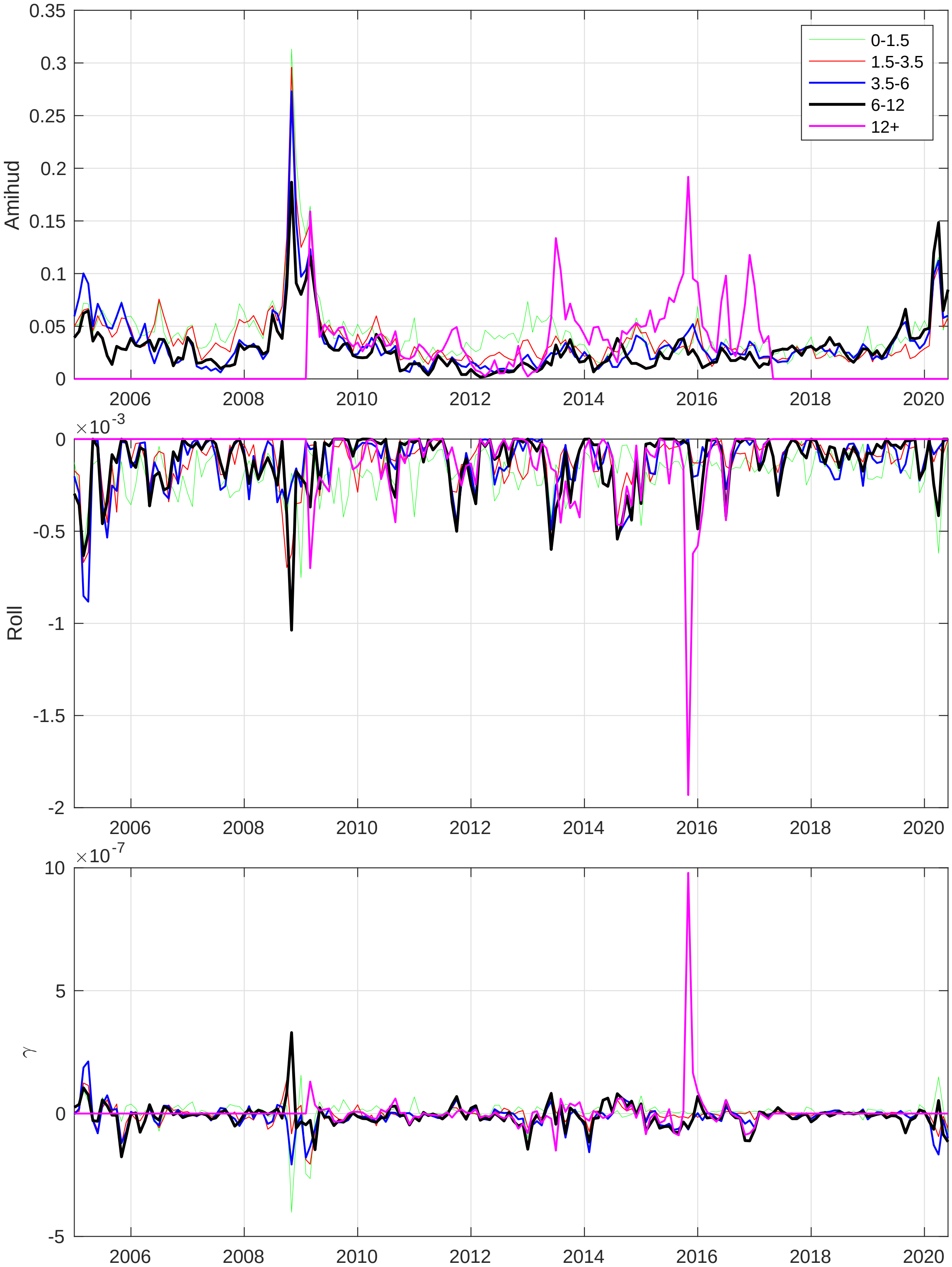

Notes: (1) Only bonds with full life history contained in the period 2005.Q1: 2020.Q3 were taken into consideration.

Figure 28: Selected averaged liquidity measures of 10Y fixed coupon government bonds




Notes: (1) Only bonds with full life history contained in the period 2005.Q1: 2020.Q3 were taken into consideration.

Figure 29: Selected liquidity measures of Polish fixed coupon government bonds traded on BondSpot in 2005:01-2020:06 - by segments (in years)



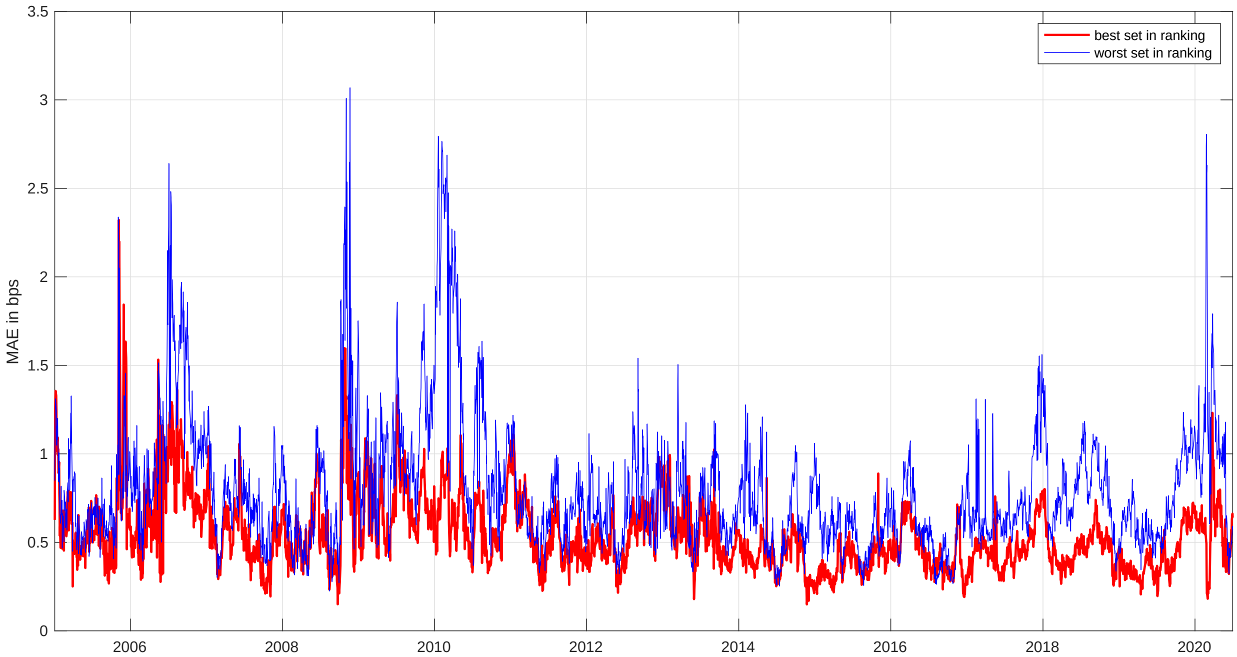

Notes: Amihud's illiquidity measure (yield change as a proxy of return, volume taken from BS, when ZTD: volume of 0.1 was imputed to avoid division by zero) , Roll's effective spread measure, and $\gamma$ a measure proposed by Bao et al. (2011)

Table 12: Descriptive statistics of *switch spread* of Polish fixed coupon government bonds in 2005:01-2020:06

| Segment | mean | std |
|---|---|---|
| $[0.2, 0.3]$ | 22.35 | 17.15 |
| $(0.3, 0.4]$ | 23.95 | 20.37 |
| $(0.4, 0.5]$ | 21.49 | 19.03 |
| $(0.5, 0.6]$ | 10.06 | 20.50 |
| $(0.6, 0.7]$ | 10.35 | 18.83 |
| $(0.7, 0.8]$ | 6.41 | 16.75 |
| $(0.8, 0.9]$ | 4.74 | 14.72 |
| $(0.9, 1.0]$ | 4.15 | 12.73 |

**Notes**: (1) Switch spread is a difference between linearly interpolated rate between NBP rate and the average *ytm* in the segment $[1.0, 1.5]$ and the *ytm* of a particular shorter than 1.25 years bonds

Figure 30: Timeseries of the best and worst performer in the ranking

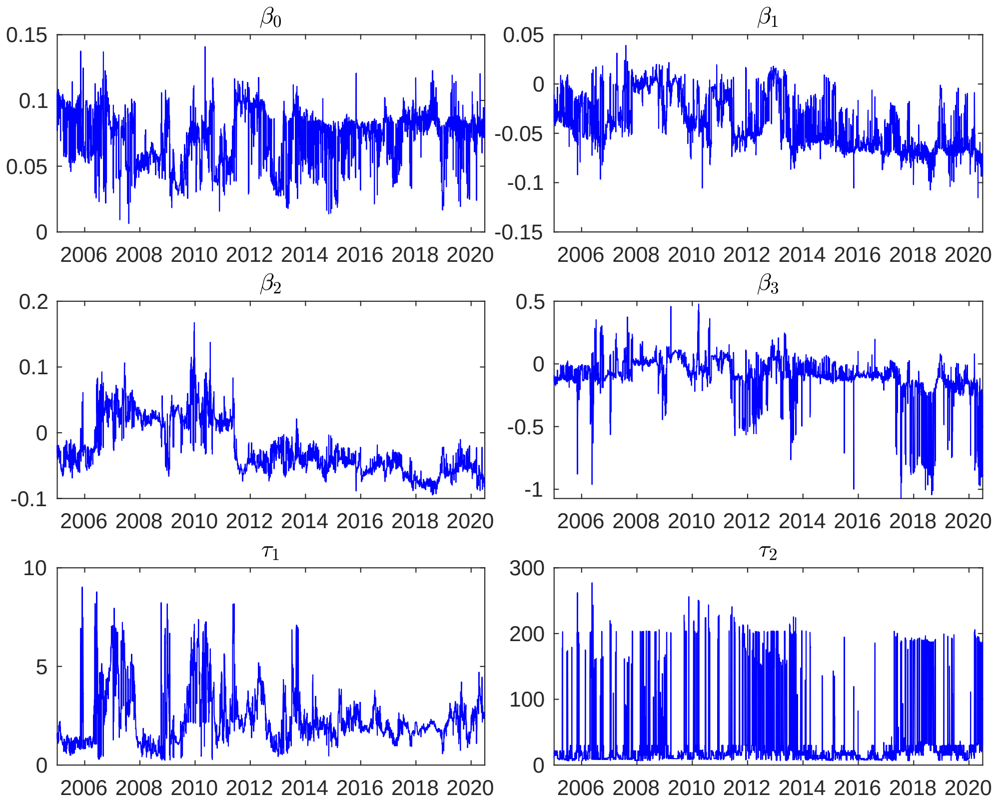



Notes: (1) the highest and the lowest ranked weighting system in Table **??** were (2) data timespan: 2005:01-2020:06

**Input:** $\forall t$ prices $p_{i,t}$, weights $W_{i,t}$, rates $r_t^{nbp}$ and a list starting values $\hat{\Theta}$
**Output:** time series of optimal $\Theta^*$ for each $t$
**for** $t \in DS$ **do**
  **retrieve** from database for date $t$: $p_i$ and chosen set of weights $W_i$ for every $i$-bond and $W^{nbp}$, $r^{nbp}$
  **for** $s \in \hat{\Theta}$ **do**
    **while** *tolerance conditions not met* **do**
      **retrieve** cash-flow schedule $\boxplus_i$ for each bond $i$
      **calculate** $P_i(\Theta_k)$ for each bond using $\boxplus_i$ and current iteration spot rates $y(x;\Theta_k)$ , $R(\Theta_k)$
      **calculate** objective function value in $k$-th iteration
    **end**
    **return** $\Theta_s^*$
  **end**
  **choose** $\Theta_s^*$ with the lowest objective function's value ($\mathcal{O}$)
  **store** $\Theta^* \equiv \min_s \mathcal{O}(\Theta_s^*)$ for the date $t$
**end**

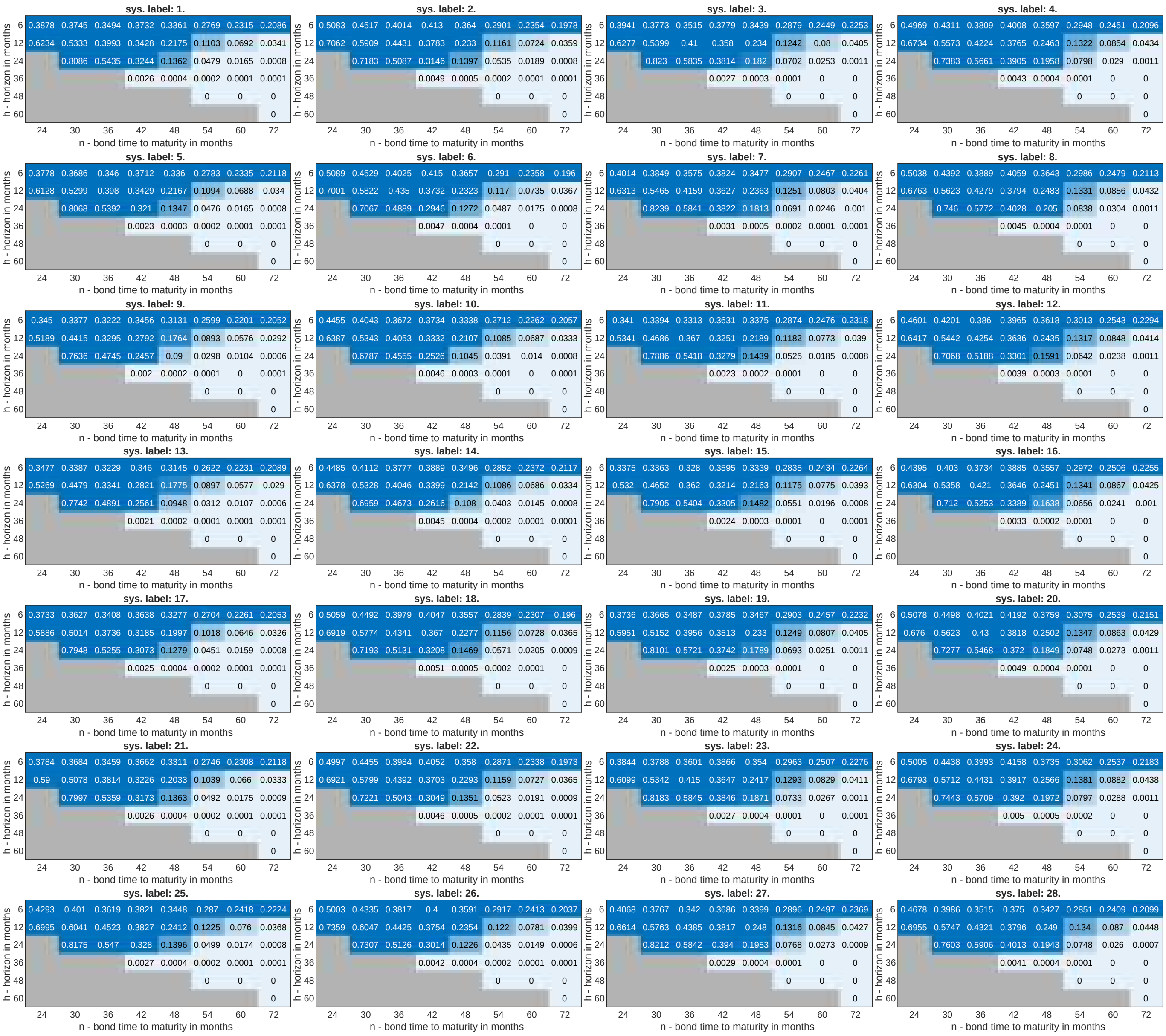
**Algorithm 2:** Calculating time series of the optimal $\Theta^*$ (**slower** version)

Figure 31: NSS parameters in estimated Polish zero coupon yield curve



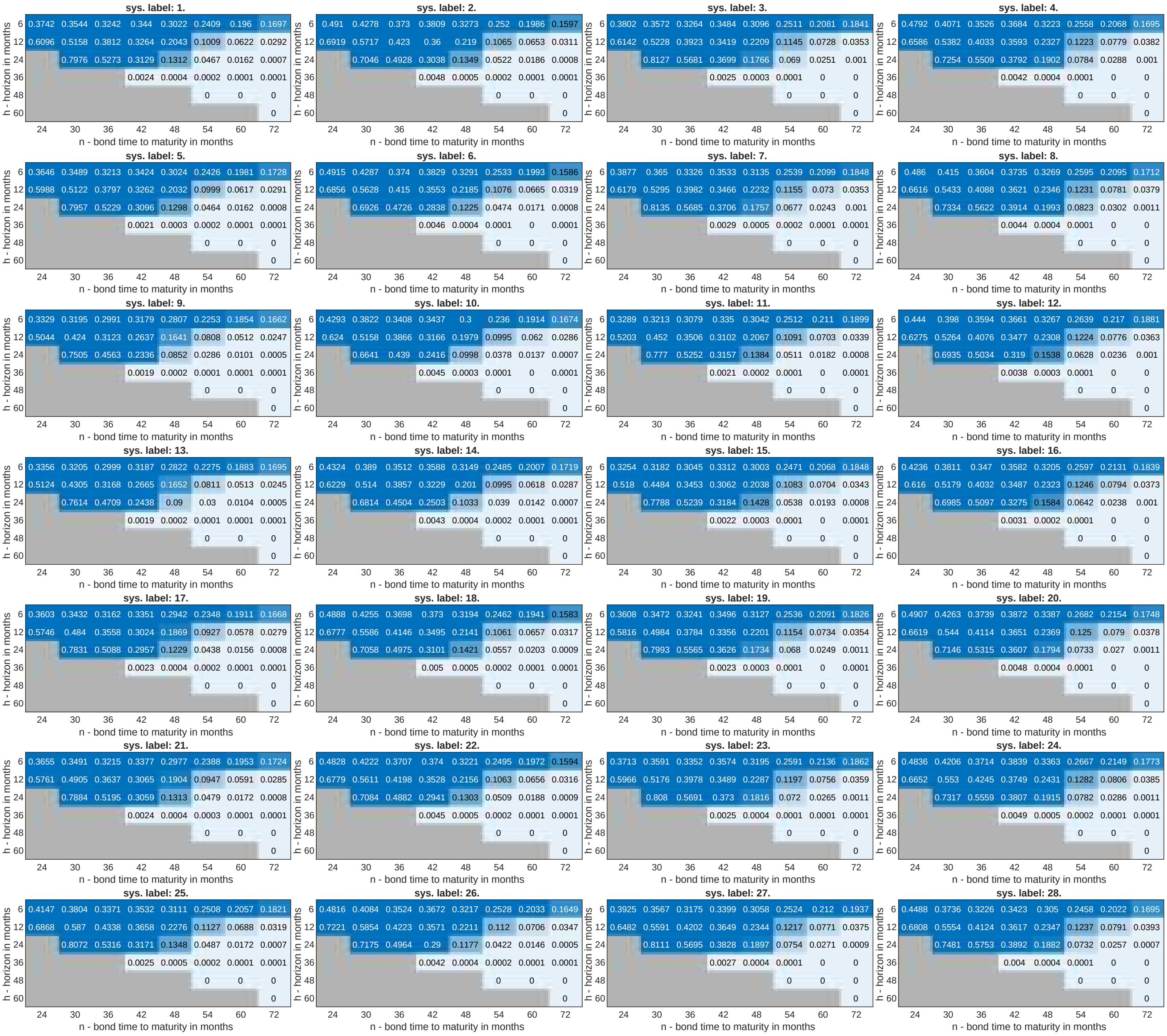

Notes: the highest ranked weight system labelled *1* was used

Figure 32: Robustness checks on null hypothesis decisions by system of weights - regression type 1 *Fama & Bliss*



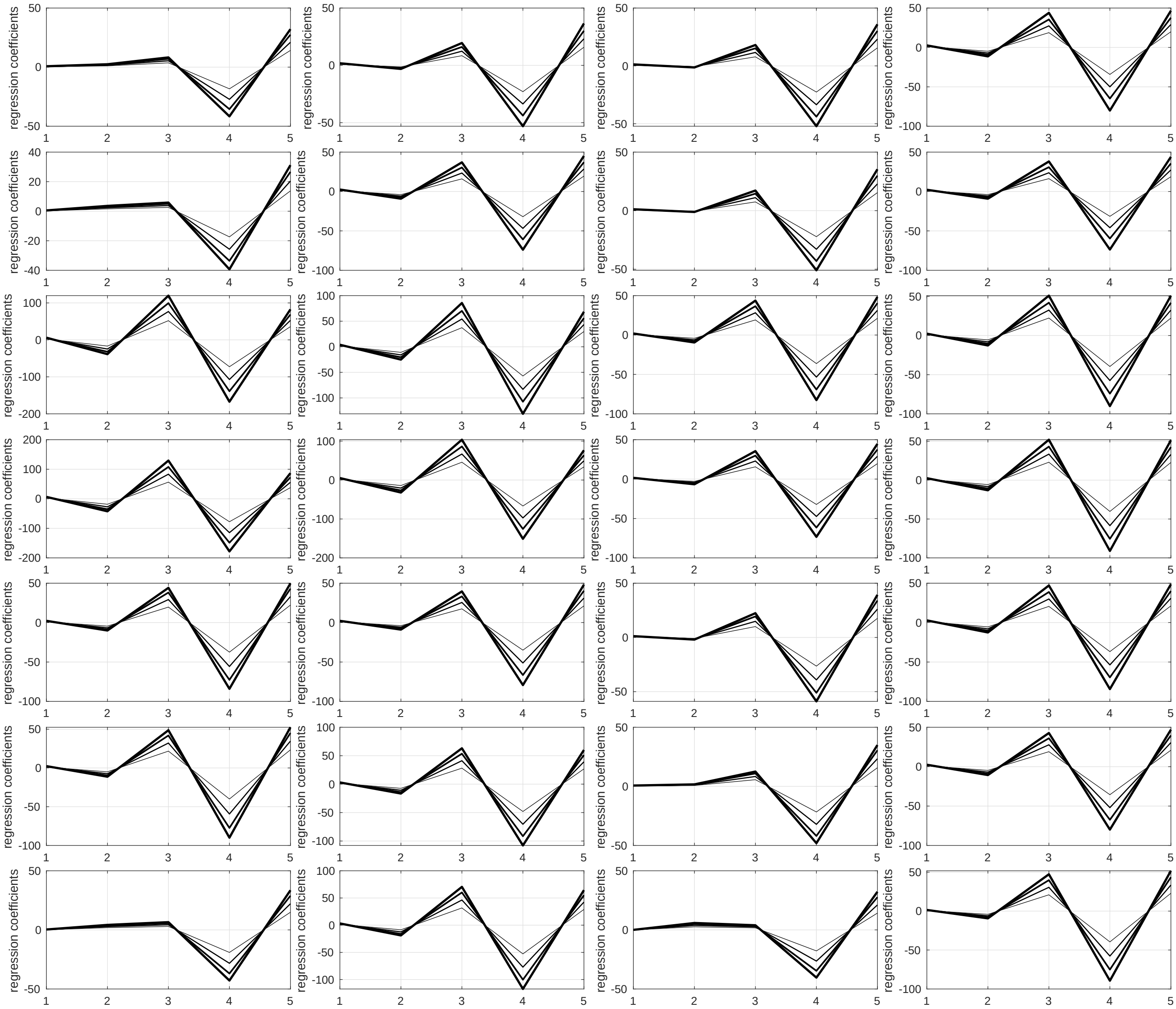

Notes: (1) p-values are reported in the boxes (2) grey areas indicate non-feasible choices of pairs (h,n) (3) system labels are as in the ranking.

Figure 33: Robustness checks on null hypothesis decisions by system of weights - regression type 2 *Fama & Bliss*




Notes: (1) p-values are reported in the boxes (2) grey areas indicate non-feasible choices of pairs (h,n) (3) system labels are as in the ranking.

Figure 34: Regression coefficients of one year excess returns on forward rates - *Cochrane & Piazzesi*

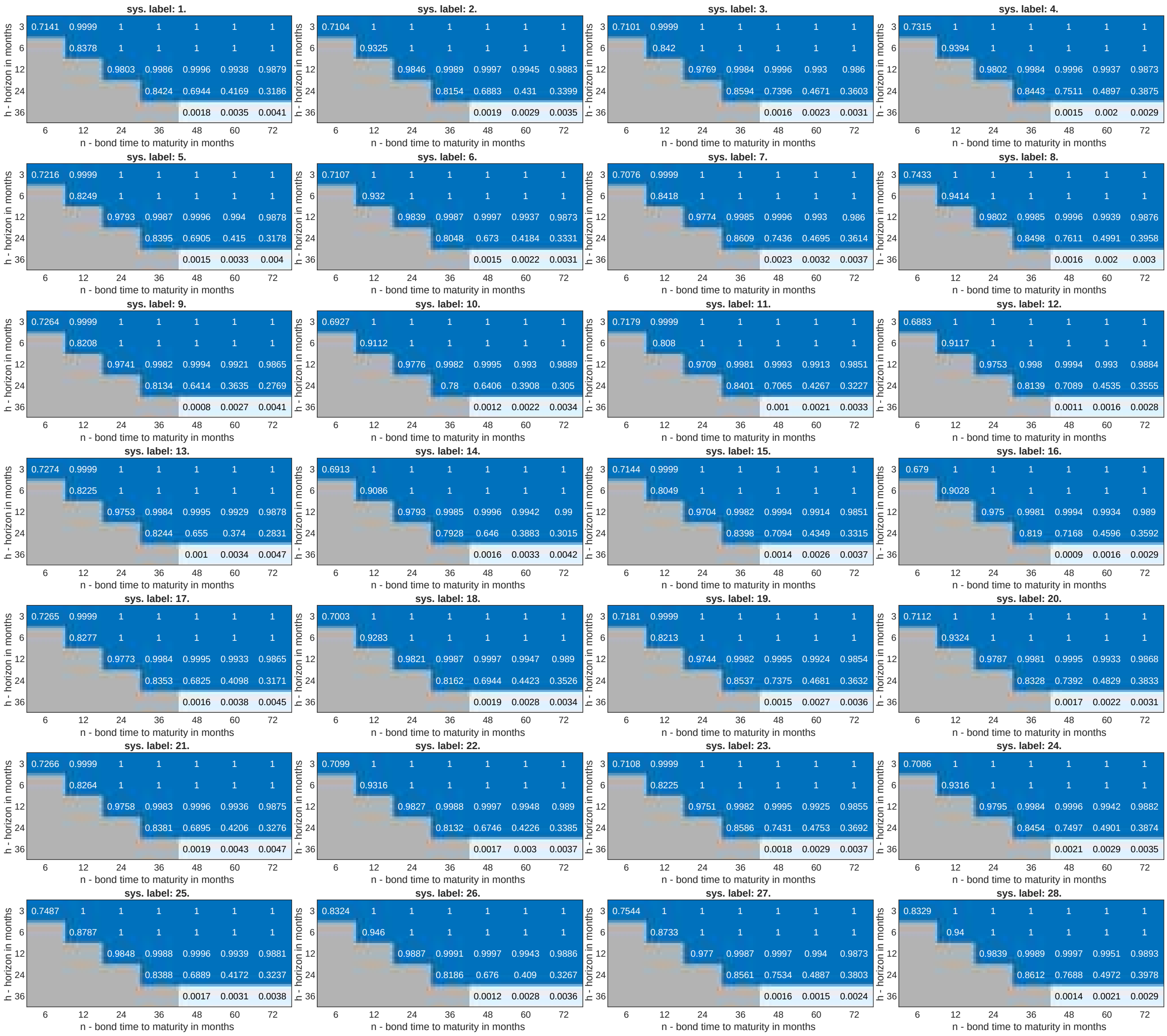


Notes: (1) p-values are reported in the boxes (2) grey areas indicate non-feasible choices of pairs (h,n) (3) system labels are as in the ranking.

Figure 35: Robustness checks on null hypothesis decisions by system of weights in rolling realised returns on term premia regressions - *Thornton* - conventional


Notes: (1) p-values are reported in the boxes (2) grey areas indicate non-feasible choices of pairs (h,n) (3) system labels are as in the ranking.

Figure 36: Robustness checks on null hypothesis decisions by system of weights in rolling realised returns on term premia regressions - *Thornton* - contrarian




Notes: (1) p-values are reported in the boxes (2) grey areas indicate non-feasible choices of pairs (h,n) (3) system labels are as in the ranking.