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Optimal federal transfers during uncoordinated response to a pandemic

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Abstract
An outbreak of a deadly disease pushes policymakers to depress economic activity due to externalities associated with individual behavior. Sometimes, these decisions are left to local authorities (e.g., states). This creates another externality, as the outbreak doesn't respect states' boundaries. A strategic Pigouvian subsidy that rewards states which depress their economies more than the average corrects that externality by creating a race-to-the-bottom type of response. In a symmetric equilibrium nobody receives a subsidy, but the allocation is efficient. If states are concerned about unequal burden of the lockdown costs, but cannot easily issue new debt to finance transfer payments, then lock-downs will be insufficient in some areas and excessive in others. When that's the case, federal stimulus checks can limit the extent of local outbreaks.

Keywords:
Covid-19; strategic Pigouvian taxation; fiscal federalism; free-riding; race-to-the-bottom

JEL Classification
H77, H21, H23, I19

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1 Introduction

In response to the COVID-19 pandemic, countries imposed various types of restrictions on economic activity to slow down the outbreak. In the United States those restrictions were largely left out to individual states. Since the virus does not recognize state and country borders, and complete border closure between states is practically impossible, this created a clear externality: one state could implement a very restrictive policy that limited contacts between people and created a recession in that state, only to have the virus spread because neighboring states would not do the same. While the federal government does not have the authority to close businesses and impose shelter-in-place at the state or county level, its power of the purse is substantial. This paper investigates, theoretically, how that power can be used to incentivize local authorities to implement stricter lock-downs.

I start with a model of endowment economy where policymakers in individual states face a trade-off between fighting the pandemic and maintaining sufficient level of economic activity.[1] Not surprisingly, in the presence of inter-state externality, the efficient allocation features deeper recession and fewer infections than the outcome of the uncoordinated response of individual state governors. The efficient allocation can be implemented with a federal tax τ on the difference between a state's output and the country's average, i.e. not necessarily on the economic activity itself. In a symmetric Nash equilibrium with identical states, all states' outputs are identical, so *in equilibrium no state receives a transfer*. The very presence of the policy, however, gives each state governor an incentive to depress the economy more then they otherwise would, by generating a race-to-the-bottom type of response which implements the optimal allocation. The logic is very similar to the one that we know from the tax competition literature (Wilson, 1986): states compete for transfers from the federal government by depressing their economies.

Next, I consider a production economy with heterogenous households, similar to the economy in Michaud and Rothert (2018). The key element of the heterogeneity is that some households are employed in jobs that contribute more to the spread of the disease. By the same logic as in the endowment economy, without coordination the local authorities do not sufficiently reduce the hours worked in sectors that contribute the most to the spread of the disease.

Additionally, the framework with heterogenous households offers new insights related to income and consumption inequality. The incentives of local authorities to implement lockdowns that are sufficiently

[1] Eichenbaum et al. (2020) and numerous other studies show that a recession is an optimal policy in the presence of a potentially deadly pandemic.

strict from the epidemiological perspective depend on the degree to which those authorities are able to redistribute resources within their constituencies. If state governors are not able to redistribute income within their states (due e.g. to state's fiscal situation), then they won't sufficiently limit hours worked in sectors that contribute the most to the spread of the disease, even if there are no inter-state externalities in the spread of the virus. In that situation, a federal lump-sum transfer to the poorest households can reduce the spread of the disease within each state and in the whole country, because it will incentivize local governments to implement stricter lockdown policies. The effectiveness of such transfer in reducing the overall health costs of the pandemic depends on the difference in the degree to which the poorest households' and richest households' employment activities contribute to the spread of the disease. The federal transfers play a bigger role if the sectors that contribute the most to the disease spread employ mostly the lower income households.

Both frameworks analyze the trade-offs between *cumulative* economic and health outcomes over the course of a pandemic, rather than the day-to-day management of the outbreak. A separate contribution of the paper is to derive the conditions under which the compartmentalized epidemiological SIR model (Kermack and McKendrick, 1927) implies a convex relationship between those two, thereby allowing the use of standard tools from the optimization theory.

The paper is motivated by three empirical facts. Two of them have to do with policy coordination (or, rather, lack thereof) and epidemiological spillovers. First, during the first wave of the pandemic, we have observed a substantial heterogeneity in policy response across the U.S. states. Rothert et al. (2020) report that by end of April and during the whole month of May and June about 40% of U.S. population lived in areas that were not subject to stay-at-home orders and about 15% of the population lived in counties that bordered states with different containment measures. The justification to not impose harsher restrictions mostly rested on protecting the individual freedoms and relied on individuals' civic responsibility. In a few cases, however, they explicitly invoked the trade-off between limiting the spread of the disease, and the economic costs to the local businesses, as indicated by the following quote from the governor of North Dakota: "we get to listen to people who think we've locked down too much and people who think we need to lock down more".[2] The clear lack of coordination in the response to the outbreak has been a problem in a situation, when the country (and the whole world) has been trying to control and mitigate the pandemic, and when actions taken in one region affect other regions.

[2] https://www.kvrr.com/2020/04/06/gov-doug-burgum-says-math-is-on-his-side-in-not-ordering-stay-at-home-order/

Second, the evidence that the actions taken in one region indeed affected other regions, is overwhelming. Within the United States, empirical studies suggest that in the early months of the Covid-19 pandemic as much as 10-20% new cases arose from interactions between people from different states (Rothert et al., 2020; Brinkman and Mangum, 2020).[3] Model-based simulations indicate that lax policies in the most lenient states could translate into millions of additional infections in the long-run (Rothert et al., 2020) and that thousands of lives could have been saved if the states had coordinated their lock-down policies or had a greater ability to limit inter-state travel (Renne et al., 2020; Giannone et al., 2020).

The last stylized fact has to do with the role of income redistribution between households within regions, and the individual regions' abilities (or willingness) to do so. We know that lockdown policies do not impact all households equally (Galasso, 2020; Basu et al., 2020), and that people in higher-income brackets and those with college degrees find it easier to work from home (Adams-Prassl et al., 2020). Additionally, Lawrence and Rothert (2021) provide evidence that countries with lower pre-pandemic degree of income redistribution, for a given severity of the current and projected outbreak, imposed less stringent lockdown policies. The empirical evidence therefore indicates that income redistribution and the composition of workforce might play an important role in shaping the lives vs. livelihoods trade-off faced by the policymakers.

1.1 Literature Review

This paper is related to the ongoing research on the economic aspects of the current COVID-19 pandemic: the analysis of the lives vs. livelihoods trade-off and understanding the response of the policymakers. Eichenbaum et al. (2020) and Kaplan et al. (2020) offer earls analyses of the optimal containment in macro frameworks at business-cycle frequency. Gori et al. (2021) analyze the impact of a potentially endemic, serious infectious disease on the capital accumulation in a version of the Solow growth model merged with the classic SIR framework. Finally, Glover et al. (2020) focus on the distributional consequences of lockdowns. Additionally, several studies attempted to understand differences in policy responses across different localities. For example, Allcott et al. (2020) argued that political preferences of a region mattered for that region's response to the pandemic. Painter and Qiu (2020) showed that compliance with lockdown policies is affected by peoples' political beliefs. The main contribution of this paper is the analysis of uncoordinated lock-down policies through the lenses of fiscal federalism.

[3] Other papers documenting the existence of substantial inter-state and inter-regional spillovers include, e.g. Dave et al. (2020) or Eckardt et al. (2020).

In order to justify the modeling framework, I characterize certain properties of the cumulative outcomes in the standard epidemiological SIR model. The problem of non-convexities present in that class of models is frequently pointed out in the literature, e.g. in Boucekkine et al. (2021), Bosi et al. (2021), Federico and Ferrari (2021), or Goenka et al. (2021). This poses a clear challenge for any papers interested in analytical characterization of optimal policies. The focus of this paper is on cumulative outcomes rather than on the day-to-day management of the pandemic. The paper therefore describes the conditions under which those cumulative outcomes are a convex function of the economic activity.

The inter-regional coordination of containment measures adopted to battle Covid-19 has only recently started to gain traction in both theoretical and empirical literature. Early work by Beck and Wagner (2020) focuses on the timing of optimal coordination, rather than on fiscal redistributive policies. Rothert (2021) focused on the strategic interaction between states and the free-riding problem, but abstracted from issues related to within state income redistribution, which is an important part of this paper. Policy coordination, an important motivation for this paper, is also a central topic in papers by Renne et al. (2020), Crucini and O'Flaherty (2020), or Acharya et al. (2020).

The model presented in this paper offers some insights into the effectiveness of federal transfers payments to the poor as a way to induce state governors to implement stricter lockdown policies. This is particularly interesting in the context of the studies that analyzed the unequal consequences of Covid-19. Alon et al. (2020) looked at the impact on gender equality, Barrot et al. (2020) at the uneven impact on different sectors, Glover et al. (2020) at the uneven impact on different age and income groups, and Atolia et al. (2021) at long-run distributional consequences of pandemics in general.

Finally, the paper is closely related to a rich literature on the coordination and competition between regions. That problem has been extensively studied in the context of the tax competition between states and countries in a financially integrated area (Wilson, 1986; Janeba and Wilson, 2011; Chirinko and Wilson, 2017). The general idea is that states undercut each other by lowering capital income tax, in order to attract foreign companies and collect the capital income tax revenues. The game between the states takes the form of a Prisoner's Dilemma, so in the uncooperative equilibrium all states have lower tax rates and lower tax revenues then they would have if they could coordinate. The logic in this paper is very similar: states are willing to depress their economies more if that implies a higher federal transfer.

2 Preliminaries: cumulative health costs in the SIR model

The basic epidemiological model divides the population into three distinct groups: Susceptible (S), Infected (I), and Recovered (R), and is typically referred to as the SIR model.[4] The model was developed and is best suited for analyzing the daily dynamics of an outbreak. The focus of this paper, however, is on cumulative outcomes. I will therefore start by describing how the cumulative outcomes in the SIR model depend on model parameters, and how those parameters typically depend on economic variables and government policies. The key result of this section characterizes the parameter space in which the cumulative health outcomes of the outbreak in the SIR model is an increasing, convex function of the economic activity.

2.1 Dynamics

This section follows Chapter 10 in Murray (2001). For ease of exposition I will focus on the most basic version of the SIR model, with one more group - Deceased (D). Initial population is N. At each time t, an individual can be Susceptible (S), Infected (I), Recovered (R), or Dead (D): $N = S(t) + I(t) + R(t) + D(t)$. Initially, we have $S(0) = S_0 > 0$, $I(0) = I_0 > 0$, $R(0) = 0$, and $D(0) = 0$. The flow of individuals between the four groups is described by the following differential equations:

$$\frac{dS}{dt} = -\beta SI, \tag{2.1}$$

$$\frac{dI}{dt} = \beta SI - \alpha I, \tag{2.2}$$

$$\frac{dR}{dt} = \alpha(1-\delta)I, \tag{2.3}$$

$$\frac{dD}{dt} = \alpha\delta I \tag{2.4}$$

New infections happen when a susceptible person comes in contact with an infected person, which is captured by the product $S \cdot I$ in the first equation above. The rate at which infected individuals either recover or die is α, with fraction δ dying and fraction $1-\delta$ recovering. It is immediate from (2.2) that an epidemic will arise (i.e. $\frac{dI}{dt}\big|_{t=0} > 0$) if and only if $\beta S(0) > \alpha$, which I will assume from now on.

The key parameter in the model, and the main focus of this section is β. It is the rate at which susceptible individuals become infected. There are at least three factors that affect the value of β. The first is the infectiousness of the virus which affects the probability that a close contact[5] will result in a

[4] Various extensions include additional groups, e.g. deceased or exposed.

[5] A "close contact" refers to a situation that creates an epidemiological risk, e.g. being in a distance smaller than 2 meters, or being in the same room for an extended period of time.

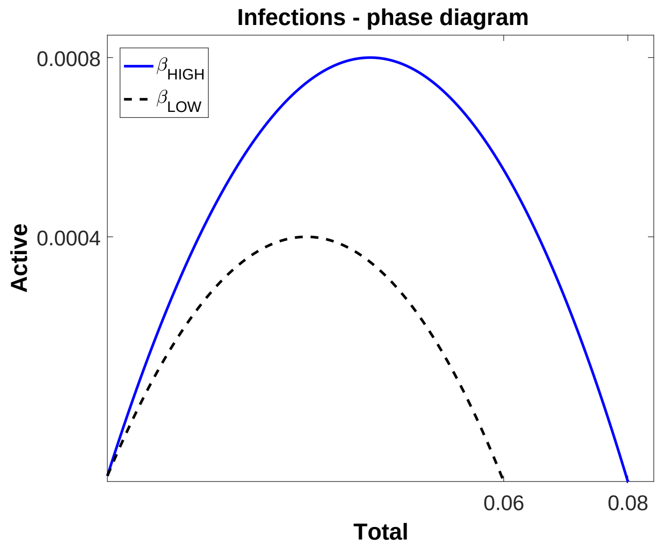


Figure 1: SIR dynamics - phase diagram

NOTES: $N = 1$, $S(0) = 0.99999$, $\alpha = 0.05$, $\beta_{LOW} = 0.0515$, $\beta_{HIGH} = 0.052$.

transmission from one person to another, given the individuals' behavior during such close contacts. The infectiousness is completely beyond individuals' and policymakers' control. The second factor is our behavior in situations involving close contacts. Those include mask-wearing, keeping windows open, avoiding loud talking, etc. The behavior is to a large extent beyond policymakers' control, although certain measures may be mandated by policymakers and enforced by businesses (mask-wearing being a primary example).

The third factor is the probability that a close contact between an infected person and a susceptible person will take place at all. That is the main part of β that the policymakers and individuals can affect. Individuals can choose to have a home-cooked meal instead of going to a restaurant, or watch a movie on Netflix instead of going to a movie theater. In general, they can choose the extent of social interactions they engage in. Policymakers can impose capacity restrictions on restaurants or theaters, order closures of non-essential businesses, ban mass events, etc. The policymakers' restrictions is the main focus of this paper. Those restrictions will, in general, impose a limit on the number of hours that individuals can work and lower the level of economic activity. We will therefore expect that $\beta = g(y)$, where y is the average level of economic activity and $g' > 0$.

Figure 1 shows the phase diagram of the model for two different levels of β — high and low. The horizontal axis plots the cumulative infections, which are defined as $CI \equiv N - S$, while the vertical axis plots the currently infected individuals, I. The key message from the figure is that both cumulative infections and the infection peak are increasing in β, and therefore in the level of economic activity.

2.2 Cumulative outcomes

The optimal control in the SIR model is challenging due to non-convexities present in equation (2.1) that drives the dynamics of new infections. This makes the model analytically intractacable and most studies that focus on optimal policies are quantitative and rely on computational methods. We can, however, sacrifice some of the insights from the dynamics of the model, and focus on the cumulative outcomes instead. Those are much more tractable, while still containing important information about the health costs of the whole pandemic. *Ceteris paribus*, the cumulative health cost of the pandemic will be proportional to the total number of people infected and to the total number of people that died. This section will show that, under certain assumptions, those cumulative health outcomes are a convex (and, of course, increasing) function of the economic activity.

Cumulative infections and infection peak Let $S_\infty := \lim_{t\to\infty} S(t)$, be the number of susceptible individuals when the pandemic is over. Then, the cumulative infections at $t = \infty$ are given by $CI = N - S_\infty$, and total deaths become $D_\infty = \delta\,(N - S_\infty)$. One can show (Murray, 2001) that S_∞ is defined implicitly as the root of the following equation:

$$S_\infty = S_0 \cdot e^{\frac{\beta(S_\infty - N)}{\alpha}}. \tag{2.5}$$

If the probability of dying depends on the capacity of the healthcare system, then the cumulative health costs may also depend on the pace at which new infections are increasing. One statistic that helps capture the burden that the pandemic puts on the healthcare system is the peak of the infection curve, i.e. the maximum level reached by active infections (Bonneuil, 2021; Loertscher and Muir, 2021). The number of infections, $I(t)$, reaches its maximum when $\frac{dI}{dS} = 0$. Murray (2001) shows that this happens when $S(t) = \frac{\alpha}{\beta}$ and that:

$$I_{max} = \frac{\alpha}{\beta}\log\frac{\alpha}{\beta S_0} - \frac{\alpha}{\beta} + N \tag{2.6}$$

Figure 2 presents the cumulative outcomes in the SIR model, as a function of β. The top left panel shows the cumulative infections, the top right panel shows the peak of the infection curve, the bottom left panel shows the cumulative deaths, while the bottom right panel plots the second derivative of the cumulative infections w.r.t. β. The figure illustrates the main result formalized in Proposition 2.1 below: when β is

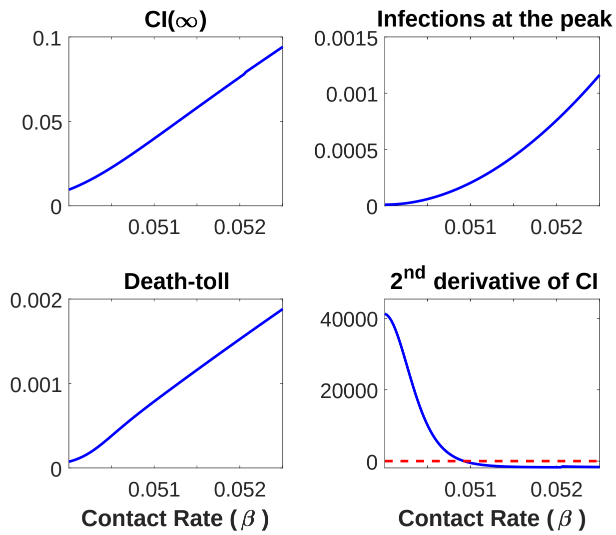


Figure 2: Cumulative outcomes in the SIR model

NOTES: $N = 1$, $S(0) = 0.99999$, $\alpha = 0.05$, $\delta = 0.02$.

small and initially almost everyone is susceptible ($S(0) \approx N$), the cumulative health costs are an increasing and convex function of β.

Proposition 2.1. *Fix α and let β and S_0 be such that $S_0 \cdot \beta > \alpha$. Let $CI(\beta) := N - S_\infty(\beta)$ where $S_\infty(\beta)$ solves (2.5), and let $I_{max}(\beta)$ be given by (2.6). Then:*

1. *$CI'(\beta) > 0$ and $I'_{max}(\beta) > 0$ for all $\beta > \frac{\alpha}{S_0}$;*

2. *$I''_{max}(\beta) \geq 0 \iff \beta \leq \alpha\frac{\sqrt{e}}{S_0}$;*

3. *$\lim_{S_0 \to N} \lim_{\beta \searrow \frac{\alpha}{S_0}} CI''(\beta) > 0$.*

Proof. 1. First, note that $CI'(\beta) = -S'_\infty(\beta)$. Next, taking the log of the equation above, and multiplying both sides by α, we will get that $0 = \alpha \log S_0 - \alpha \log S_\infty + \beta (S_\infty - N)$, which then implies that $0 = -\frac{\alpha}{S_\infty} dS_\infty + (S_\infty - N)\, d\beta + \beta dS_\infty$, and hence:

$$\frac{dS_\infty}{d\beta} = \frac{S_\infty - N}{\frac{\alpha}{S_\infty} - \beta} < 0 \qquad \Rightarrow \qquad \frac{dCI}{d\beta} = \frac{N - S_\infty}{\frac{\alpha}{S_\infty} - \beta} > 0$$

where the sign of each expression follows from the fact that $S_\infty < N$ and I reaches maximum when $S = \frac{\alpha}{\beta}$, so $\beta < \frac{\alpha}{S_\infty}$. We also have: $I'_{max}(\beta) = \frac{\alpha}{\beta^2} \log \frac{\beta S_0}{\alpha} + \frac{\alpha}{\beta^2} - \frac{\alpha}{\beta^2 S_0} \frac{\beta S_0}{\alpha} \frac{\alpha}{\beta} = \frac{\alpha}{\beta^2} \log \frac{\beta S_0}{\alpha} > 0$, where the last inequality follows from the fact that $\beta > \frac{\alpha}{S_0}$.

2. Taking 2nd derivative of I_{max} w.r.t. β we get:

$$\frac{d\frac{dI_{max}}{d\beta}}{d\beta} = -2\frac{\alpha}{\beta^3}\cdot\log\frac{S_0}{\alpha/\beta} + \frac{\alpha}{\beta^2}\frac{S_0}{\alpha}\cdot\frac{\alpha}{\beta S_0} = -2\frac{\alpha}{\beta^3}\cdot\log\frac{S_0}{\alpha/\beta} + \frac{\alpha}{\beta^3} = \frac{\alpha}{\beta^3}\cdot\left[1-2\log\frac{S_0}{\alpha/\beta}\right]$$

Hence, $I''_{max}(\beta) \geq 0 \iff \frac{1}{2} \geq \log\frac{S_0}{\alpha/\beta}$, which is equivalent to $\beta \leq \alpha\frac{\sqrt{e}}{S_0}$

3. See Appendix for the algebra.

□

2.2.1 Cumulative outcomes and economic restrictions

Restrictions on economic activity can, to some extent, reduce β. Suppose then, that β is a function of economic activity:

$$\beta = g(y)$$

where y is the overall level of economic activity. Different studies consider different forms of the function $g(\cdot)$. For example, in Eichenbaum et al. (2020) we have $\beta = \beta_0 + \beta_1\ell^S\ell^I + \beta_2 c^S c^I$, where ℓ denotes hours worked, c denotes consumption, and S and I denote susceptible and infected individuals. In Gollier (2020), who allows for a more nuanced interaction involving confinements, we have $\beta_{ij} = \alpha(\beta_c\cdot b_i + \beta_f(1-b_i))(1-b_j)$, where j is a susceptible person, i is an infected person, subscripts c and f stand for confined and free, respectively, and b is a fraction of people confined, and $0 < \beta_c < \beta_f$.[6] In a multi-group SIR model by Acemoglu et al. (2020) the frequency with which an infected person from group j sheds the virus onto a susceptible person in group k is modeled as $\beta_{j,k} = \beta\cdot(1-L_j)(1-L_k)$, where L_j denotes the extent to which lockdown policy affects group j. Overall, in most specifications, the parameter β takes a form of either a linear or a strictly convex (quadratic) function of the economic activity, because it affects the frequency of closed contacts from both sides - susceptible and infected, which enter (2.1) in a multiplicative way. In the remainder of the paper I would therefore assume that $g' > 0$ and $g'' \geq 0$, which would imply that cumulative health costs of the pandemic are an increasing and convex function of the average level of economic activity.

[6] Gollier (2020) also considers another group that is quarantined and allows for imperfect effectiveness of confinement measures, which I omit here to simplify the description.

3 Uncoordinated response and inter-state redistribution

Equipped with the results from previous section, I will now turn to the analysis of the trade-off between lives and livelihoods in a fiscal union, when each region imposes its own lockdowns, but when the outbreak in one region affects the evolution and the cumulative health costs in other regions.[7]

I begin the analysis with a simple example that illustrates how a strategic system of inter-state taxes and transfers can incentivize the state governments to implement stricter lockdows, bringing the resulting allocation closer to the optimum. In order to keep the focus of the analysis on the interaction between the individual states and the federal government, I will consider a static environment, where both the economic and the health outcomes should be interpreted as cumulative outcomes occurring over the whole duration of the pandemic.

Aside from transparency of the analysis, there is additional reason behind this choice of modeling. Any special fiscal transfers, like the Covid-19 stimulus checks, arrive at a rather low frequency (once every few months, at best). Epidemiological models, on the other hand, study the dynamics of the new infections at much higher, usually daily, frequency. The real-life changes in restrictions take place at frequency somewhat lower than changes in infections (Aspri et al., 2021), but probably higher than fiscal decisions. Hence, there is an inherent frequency mismatch between the epidemiological models, and the macroeconomic models studying the effects of fiscal policies. The workaround that I chose in this paper is to shift the focus on the cumulative health outcomes, rather than on the day-to-day management of the pandemic.[8]

3.1 Regional response to the epidemic

The utility of a stand-in household in state s is given by

$$u(c_s) - h(p_s)$$

where c_s is the household's level of consumption, while p_s denotes the overall, cumulative health cost of the outbreak, as described in the previous section. It is a summary statistic that captures the total number of

[7] Rothert et al. (2020) develop a spatial version of an SIR model and show that such spillover effects are quantitatively important: lax policies in most lenient states translate into millions of additional infections in other parts of the U.S. in the long-run.

[8] Similar choices can be found in the literature on pollution externalities, e.g. in Hutchinson and Kennedy (2008), Silva and Caplan (1997), or in Neilson and Kim (2001). Boucekkine et al. (2020) is a notable recent exception that introduces dynamics into a model of spatial diffusion.

infections, hospitalizations, and deaths. As shown in Section 2, those cumulative health costs depend on the frequency with which, over the course of the pandemic, susceptible individuals come in close contact with infected individuals and become themselves infected. That frequency itself is assumed to be an increasing, weakly convex function of the economic activity. This leads to the following, epidemiological constraint, that describes how the cumulative health outcomes depend on the average level of economic activity over the course of the outbreak:

$$p_s = \bar{p} + g\left(y_s\right) + \kappa \frac{1}{S} \sum_{s'} p_{s'} \tag{3.1}$$

where y_s is the economic activity in state s, and κ measures the degree to which the pandemic health costs spill over between states. Those spillovers capture at least two factors. The first one is the epidemiological spillover — a susceptible person in one state can catch the virus by coming into a close contact with an infected person from a different state. Numerous empirical studies documented that those spillovers can be substantial (Rothert et al., 2020; Eckardt et al., 2020; Renne et al., 2020). The second factor has to do with the capacity of the healthcare system and the availability of medical equipment. A severe outbreak in one state increases the demand for medical equipment (e.g. ventilators or personal protective equipment), making it more scarce in other states thereby increasing the overall number of infections and death toll from the pandemic in the rest of the country. Since the model is supposed to capture health outcomes and average trade-offs over the course of the whole pandemic, the term on the right-hand side includes also the state's own health costs. The justification is that the more severe outbreak in a state that spills over to the neighboring states can then accelerate the state's own infections a few weeks later.

All states produce a homogeneous good, so without federal redistribution of income, the resource constraint for each state s is simply given by:

$$c_s = y_s.$$

I assume that the three functions u, h, and g are continuous, twice continuously differentiable, and strictly increasing, u is strictly concave, h is strictly convex, and g is convex. The convexity of g follows from the analysis in Section 2.

3.2 Optimal allocation

The federal social planner solves the following problem:

$$\max_{(c_s, y_s, p_s)_{s=1}^{S}} \sum_{s=1}^{S} \left[u(c_s) - h(p_s)\right]$$

subject to:

$$p_s \geq \bar{p} + g(y_s) + \kappa \frac{1}{S} \sum_{s'} p_{s'} \tag{3.2}$$

$$\sum_s c_s \leq \sum_s y_s \tag{3.3}$$

Since the interdependence of the cumulative health outcomes between states is already captured by (3.2), the disutility term $h(\cdot)$ is assumed separable between states. Let λ_s denote the Lagrange multiplier on (3.2). Standard algebra yields that the optimal allocation will have to satisfy the following necessary first order condition:

$$u'(c_s) = \lambda_s g'(y_s), \qquad i = 1, ..., S, \tag{3.4}$$

where

$$\lambda_s = h'(p_s) + \frac{\kappa}{S} \sum_{s'} \lambda_{s'} = 0, \qquad i = 1, ..., S \tag{3.5}$$

Since all states are identical, and the objective function is strictly concave, the optimal allocation will be symmetric with $c_s = c_{s'}$, $y_s = y_{s'} = y^*$, and $p_s = p_{s'} = p^*$, for all s, s'. It then follows that $\lambda_s = \lambda_{s'}$ for all s, s' and that $c_s = y_s = y^*$ for all s. Equations (3.4)-(3.5) then imply that:

$$h'(p^*) = (1 - \kappa) \frac{u'(y^*)}{g'(y^*)} \tag{3.6}$$

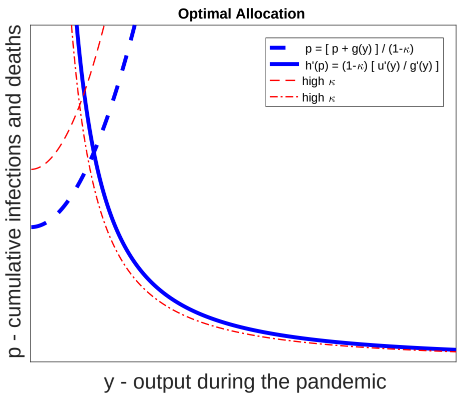
In addition to (3.6), the characterization of the optimal allocation is completed by equation (3.2) with the imposed condition that the optimal allocation is symmetric:

$$p^* = \frac{1}{1 - \kappa} \left[\bar{p} + g(y^*)\right] \tag{3.7}$$

Given the assumptions on u, h, and g, it is pretty straightforward to show that equation (3.6) implies a negative relationship between p^* and y^*, while equation (3.7) implies a positive relationship between p^* and y^*. The optimal allocation is plotted in the left panel of Figure 3. One characteristic of the optimal allocation are immediate from Figure 3. First, if κ is larger, the optimal level of economy activity in each state is smaller. An increase in g' will have a similar effect. This is summarized in the proposition below:

Proposition 3.1. *Consider an economy with a given externality κ and function g. Let the corresponding optimal allocation that solves (3.6)-(3.7) be denoted with $(p^*(\kappa, g), y^*(\kappa, g))$. Then:*

- *If $\kappa_1 < \kappa_2$ then $y^*(\kappa_1, g) > y^*(\kappa_2, g)$*



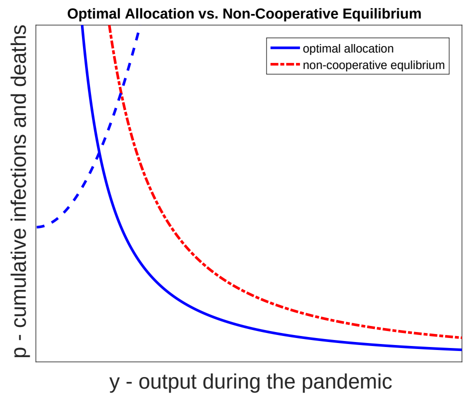


Figure 3: Allocations in the model

NOTES: $g(y) = \frac{1}{2}y^2$, $h(p) = \frac{1}{2}p^2$, $u(c) = \frac{1-\sigma}{c^{1-\sigma}}$ with $\sigma = 2$; benchmark $\kappa = 0.75$; high $\kappa = 0.825$.

- *If* $g_1(0) = g_2(0)$ *and* $g_1'(y) < g_2'(y)$ *for all* y*, then* $p^*(\kappa, g_1) < p^*(\kappa, g_2)$ *and* $y^*(\kappa, g_1) > y^*(\kappa, g_2)$

Proof. The proof of the first property follows directly from total differentiation of equations (3.6) and (3.7). See Appendix B. The second property is quite immediate so the proof is omitted (but available upon request).

□

3.3 Non-cooperative equilibrium with strategic transfers

Now consider the equilibrium outcome in the world, where the management of the outbreak is left to individual regions (e.g. states), but states can receive federal transfers. The transfers are designed to close the gap between the state's income and the country's average. Each state governor solves:

$$\max_{c_s, y_s, p_s} u(c_s) - h(p_s)$$

subject to:

$$p_s \geq \bar{p} + g(y_s) + \kappa \frac{1}{S} \sum_{s'} p_{s'} \tag{3.8}$$

$$c_s \leq y_s + T_s \tag{3.9}$$

$$T_s \leq \tau \cdot \left(\frac{1}{S} \sum_{s'=1}^{S} y_{s'} - y_s \right), \tag{3.10}$$

where T_s is the net transfer to state s. Again, let λ_s denote the Lagrange multiplier on and let μ be the multiplier on (3.10). Since the governor now takes p_{-s} as given, we get the following necessary first order conditions w.r.t. p_s, y_s, and T_s:

$$\begin{aligned}
\lambda_s &= h'(p_s) \\
u'(y_s + T_s) &= \lambda_s g'(y_s) - \mu\tau\left(\frac{1}{S} - 1\right) \\
u'(y_s + T_s) &= \mu
\end{aligned}$$

Combining and re-arranging those, we get:

$$h'(p_s) = \frac{u'(y_s + T_s)}{g'(y_s)}\left(1 - \tau + \frac{\tau}{S}\right) \tag{3.11}$$

Notice that when S equals 1 (there is only one state so there are no externalities (which is equivalent to $\kappa = 0$) and no strategic race to the bottom), then $T_s = 0$ and equations (3.6) and (3.11) are identical.

Proposition 3.2. *Let the degree of externality equal κ. If all states are ex-ante identical, then there exists a degree of income redistribution $\tau = \tau^*$ across states that implements the optimal allocation. That degree is given by $\tau^* = \frac{S}{S-1} \cdot \kappa$. In the non-cooperative equilibrium with $\tau = \tau^*$, the net transfer for each state s is null.*

Proof. Obvious, by comparing (3.11) and (3.6). □

The proposition above implies that the degree of optimal income redistribution will be larger when the externality κ is larger. It is also larger when the number of states, S, is smaller. Importantly, in the non-cooperative equilibrium, no state receives a transfer. This is because they all limit their economic activity, so no state is below or above the nation's average. In that sense, the policy can be considered a strategic Pigouvian taxation.

The result in Proposition 3.2 should be interpreted as one that highlights a potentially important mechanism present in a fiscal union. Typically, not all regions will be impacted to the same extent by the outbreak. Some will be forced to implement stricter lockdowns, e.g. due to a larger proportion of population living in densely populated, urban areas. People in those regions will then suffer greater income losses. Many fiscal relief programs are designed for people affected specifically by a particular event, so it is not a stretch to think of a post-pandemic (or a late-pandemic) system of transfers targeting individuals affected by the pandemic (to some extent, the federal boost to unemployment benefits in the United States served that purpose). In the context of inter-state transfers, the federal government could set aside a Covid relief fund to be distributed to state governments proportionally to the economic costs incurred. States that were hit harder would receive a larger transfer. At the end of the day, however, such program will have to be paid for by taxpayers, i.e. residents of the respective regions. The net transfer will then be different than the gross transfer. Proposition 3.2 then states that the mere presence of such federal relief programs makes the lives vs. livelihoods trade-off at the local level less severe. It also gives the local governments stronger incentives to battle the outbreak more aggressively.

4 Uncoordinated response and intra-state redistribution

The model in the previous section was designed to highlight the mechanism through which the externalities in the spread of a new disease between states lead to suboptimal responses of individual states who do not internalize these effects. Next, I want to consider an extension that captures an important aspect of our dealing with epidemic - the disproportionate burden of the lockdown on certain professions in the service industry, that often employs people who live hand-to-mouth, have no non-labor source of income, and for whom switching to working remotely may be more difficult.

4.1 Model - key features

I consider a framework similar to Michaud and Rothert (2018). Each state is inhabited by two types of households. For lack of better terminology, I will refer to one group as "poor", the other as "rich". The poor households' only source of income is labor. Rich households, in addition to labor income, also receive dividend income from the distributed profits of firms. The fraction of households that are poor is N^P, the fraction of households that are rich is N^R, so $N^P + N^R = 1$.

The first important difference between the two households is the budget constraint. With possible lump-sum transfers, the budget constraints for the rich and the poor take the following forms:

$$c_s^P \leq w_s \ell_s^P + T_s^P$$
$$c_s^R \leq w_s \ell_s^R + \frac{\pi_s}{N_s^R} + T_s^R$$

where w denotes wage, π are the total profits in the economy, T are lump-sum taxes/transfers. In general, all of the variables above can vary by state. The second important difference between the rich and the poor, which is also the key focus of this section, is that they work in different sectors of the economy, with different contribution to the spread of the disease. Equation (3.2) that describes how the economic activity affects the cumulative health outcomes in state s now takes the following form:

$$p_s = \bar{p} + N_s^P \cdot g_P\left(\ell_s^P\right) + N_s^R \cdot g_R\left(\ell_s^R\right) + \kappa \frac{1}{S} \sum_{s'=1}^{S} p_{s'}, \qquad s = 1, ..., S \tag{4.1}$$

As in Section 3, I will assume that $g'_P, g'_R > 0$ and that $g''_P, g''_R \geq 0$. Additionally, I am going to assume that $g'_P > g'_R$, always. In words, the poor households work in sectors that contribute more to the spread of the disease because they involve more frequent interactions between people. Hence, they are the ones that will be more affected by a potential lockdown policy.

While the assumption may seem quite strong, empirical studies provide strong evidence in favor of such heterogeneity between the lower and higher income households. For example, Adams-Prassl et al. (2020) document a positive correlation between a worker's earnings and the share of tasks that can be done from home by that worker, both in the United States and in the United Kingdom. For example, in the United States, the workers who fall in the annual income bracket of $40k - 49k$ can perform about 40% of their tasks from home. For those who fall in the annual income bracket of $80k - 89k$ that fraction exceeds 50%. Galasso (2020) showed that lower-income workers were most affected by the early lockdowns in Italy: they were more likely to lose a job, less likely to work from home, lost more of their income, and were more opposed to social distancing measures and other economic restrictions.[9] The empirical literature therefore suggests that the composition of the workforce and the increased need for income redistribution might play an important role in shaping the lives vs. livelihoods trade-offs faced by policymakers. Indeed, Lawrence and Rothert (2021) provide evidence that the elasticity of lockdown stringency w.r.t. outbreak severity is larger in countries with higher pre-pandemic degree of income redistribution.

Given the empirical evidence described above, I will consider two potential ways in which the states can differ. First, states potentially differ with respect to their abilities to redistribute resources between the poor and the rich households. The additional motivation for this assumption is that both within the United States, and in the European Union, there is a substantial heterogeneity in the fiscal situation of individual states and countries, and their ability to issue new bonds that would finance new social transfers.[10] Second,

[9] Other studies with similar (in spirit) results include Dingel and Neiman (2020), Palomino et al. (2020), or Basu et al. (2020).

[10] One of the problems discussed during the ongoing COVID-19 pandemic is the federal bailout of some U.S. states: https://www.politico.com/news/2020/04/28/trump-states-bailout-sanctuary-cities-215507;

states potentially differ with respect to the proportion of poor households N^P. The additional motivation for this assumption is the heterogeneity in net federal taxes paid and transfers received by U.S. states, and by the very existence of the European Regional Development Fund.[11]

A household of type $i = P, R$ has utility function of the form:

$$U\left(c^i, \ell^i\right) = u\left(c^i\right) - v\left(\ell^i\right) - h(p)$$

where p is the cumulative health outcome of the pandemic, as in the previous two sections. Each household takes it as given, so an individual household's action cannot affect it. As a result, competitive equilibrium during a pandemic will never be efficient, because households actions create externalities. The amount of effective labor in state s is then given by:

$$L_s \equiv N_s^P \ell_s^P + N_s^R \ell_s^R$$

The aggregate production function is given by:

$$Y_s = F\left(L_s\right), \qquad F' > 0, F'' < 0$$

The resource constraint in each state is:

$$N_s^P \cdot c_s^P + N_s^R \cdot c_s^R = Y_s$$

4.2 Optimal allocation

I start by characterizing the optimal allocation before and during the pandemic. The formal definition of the optimal allocation is quite intuitive.

Definition 4.1 (**Optimal Allocation**)**.** The optimal allocation is a tuple $\mathbf{z}^* \equiv \left({\ell^*}_s^P, {\ell^*}_s^R, {c^*}_s^P, {c^*}_s^R, p_s^*\right)_{s=1}^S$ that solves the following maximization problem of a Federal Social Planer (FSP):

$$\max_{\left(\ell_s^P, \ell_s^R, c_s^P, c_s^R, p_s\right)_{s=1}^S} \sum_{s=1}^S \left\{ \sum_{i=P,R} N_s^i \cdot \left[u\left(c_s^i\right) - v\left(\ell_s^i\right)\right] - h\left(p_s\right) \right\}$$

subject to (4.1) and:

$$\sum_{s=1} N_s^P c_s^P + N_s^R c_s^R \leq \sum_{s=1}^S F(L_s) \tag{4.2}$$

$$L_s \leq N_s^P \ell_s^P + N_s^R \ell_s^R \tag{4.3}$$

https://www.businessinsider.com/trump-bailing-out-blue-states-coronavirus-republicans-federal-help-2020-5.

[11]See https://ec.europa.eu/regional_policy/en/funding/erdf/.

Constraint (4.2) in the definition above states that the federal social planner can redistribute resources between states. Constraint (4.3) in the definition above is simply the constraint for aggregate hours worked in each state. Notice that the planner internalizes both the effect of ℓ_s^P and ℓ_s^R on infections in state s, as well as the effect of infections in one state on the infections in the other states which is captured by the term $\kappa \frac{1}{S} \sum_{s'=1}^{S} p_{s'}$ on the right hand side of (4.1).

4.2.1 Characterization

Let λ_s denote the Lagrange multipliers on the s^{th} constraint (4.1). It is pretty straightforward to show that the optimal allocation satisfies the following necessary and sufficient first-order conditions:

$$c_s^i = c = \frac{1}{S} \sum_s F(L_s), \qquad i = P, R; \quad s = 1, ..., S \tag{4.4}$$

$$h'(p_s^*) + \kappa \frac{1}{S} \sum_{s'=1}^{S} \lambda_{s'}^* = \lambda_s^*, \qquad s = 1, ..., S \tag{4.5}$$

$$v'(\ell^{*i}_s) = u'(c^*) F'(L_s^*) - g_i' \left(\ell^{*i}_s \right) \cdot \lambda_s^*, \qquad i = P, R; \quad s = 1, ..., S \tag{4.6}$$

The last equation must hold for both rich and poor households.

4.2.2 Optimal allocation before and during the pandemic

In order to compare the optimal allocations (as well as competitive and non-cooperative allocation in later sections) during the pandemic to those before the pandemic, I start with the formal definition of the world before and during the pandemic.

Definition 4.2. The world before the pandemic is defined as one where $h(\cdot) \equiv \mathbf{0}$. The world during the pandemic is defined as one where $h(\cdot) \geq 0$, $h'(\cdot) > 0$, and $h''(\cdot) > 0$.

The following proposition summarizes some of the properties of the optimal allocation before and during the pandemic.

Proposition 4.3. *Let $\mathbf{z}^*(\mathbf{h})$ denote the optimal allocation during the pandemic, and let $\mathbf{z}^*(\mathbf{0})$ denote the optimal allocation before the pandemic, with a similar notation for individual components of $\mathbf{z}^*$. Then:*

1. $\ell^{*i}_s(\mathbf{0}) = \ell^*(\mathbf{0})$*, for all s, and for $i = P, R$*

2. $\ell^{*P}_s(\mathbf{0}) > \ell^{*P}_s(\mathbf{h})$*, for all s*

3. $\ell^{*R}_s(\mathbf{h}) > \ell^{*P}_s(\mathbf{h})$, *for all* s

4. *If* $g'_R \equiv \mathbf{0}$ *then* $\ell^{*R}_s(\mathbf{h}) > \ell^*(\mathbf{0})$, *for all* s

Proof. First notice that before pandemic we have $\lambda^*_s = 0$, for all s, so (4.6) before the pandemic becomes $v'(\ell^{*P}_s) = u'(c^*)F'(L^*_s) = v'(\ell^{*R}_s)$. Since for each s we have $\ell^{*P}_s = \ell^{*R}_s = \ell^*{}_s$, and $N^P_s + N^R_s = 1$, it then follows that $L^*_s = \ell^*$ for each s, so $\ell^*{}_s = \ell^*$. During the pandemic, we have $\lambda^*_s > 0$. Subtracting the version of equation (4.6) for the poor households from the version for the rich households, we get:

$$v'\left(\ell^{*R}_s\right) - v'\left(\ell^{*P}_s\right) = \lambda^*_s \cdot \left[g'_P\left(\ell^{*P}_s\right) - g'_R\left(\ell^{*R}_s\right)\right] \tag{4.7}$$

which can only be satisfied if $\ell^{*R}_s > \ell^{*P}_s$, otherwise the right hand-side would be positive, while the left-hand side would be negative. Equation (4.6) for the poor households then implies that $\ell^{*P}_s(\mathbf{h}) < \ell^{*P}_s(\mathbf{0})$, because when evaluated at the optimal allocation before the pandemic, the right hand-side is smaller. Finally, if $g'_R \equiv \mathbf{0}$ then $v'(\ell^{*R}_s(\mathbf{h})) = u'(c^*(\mathbf{h}))F'(L^*_s(\mathbf{h}))$. Suppose that $\ell^{*R}_s(\mathbf{h}) \leq \ell^{*R}_s(\mathbf{0})$. Since $\ell^{*P}_s(\mathbf{h}) < \ell^{*P}_s(\mathbf{0})$, we have that $L^*_s(\mathbf{h}) < L_s(\mathbf{0})$ and $c^*(\mathbf{h}) < c^*(\mathbf{0})$. We then have $v'(\ell^{*R}_s(\mathbf{0})) \geq v'(\ell^{*R}_s(\mathbf{h})) = u'(c^*(\mathbf{h}))F'(L^*_s(\mathbf{h})) > u'(c^*(\mathbf{0}))F'(L^*_s(\mathbf{0}))$. The combination of the first and last inequality yields a contradiction. $\square$

In words, the optimal allocation during the pandemic, relatively to the allocation before the pandemic, features lower employment of the poor households, but it may or may not feature higher employment of the rich households. In the optimal allocation during the pandemic the poor households will always work less then the rich households.

4.3 Competitive equilibrium with federal transfers

Next, consider the characterization of the allocation in a competitive equilibrium with lump-sum transfers. Each household $i = P, R$ in state s receives a federal transfer T^i_s (regardless of the state in which it resides), with the federal constraint being:

$$\sum_s \left(N^P_s T^P_s + N^R_s T^R_s\right) \equiv \sum_s T_s = 0 \tag{4.8}$$

where $T_s \equiv N^P_s T^P_s + N^R_s T^R_s$ is the net federal transfer received by state s. The following proposition characterizes the competitive equilibrium allocation with federal transfers.

Proposition 4.4 (**Competitive Equilibrium allocation with federal transfers**). *Let* $\hat{\mathbf{z}}(\mathbf{T})$ *denote the competitive allocation with a vector of federal transfers* $\mathbf{T}$, *where* $\hat{\mathbf{z}} \equiv \left(\hat{\ell}^P_s, \hat{\ell}^R_s, \hat{c}^P_s, \hat{c}^R_s, \hat{p}_s\right)^S_{s=1}$ *and* $\mathbf{T} \equiv \left(T^P_s, T^R_s\right)^S_{s=1}$ *is a vector of federal transfers satisfying* (4.8). *Then:*

1. $\hat{\mathbf{z}}$ *is characterized by the following equations:*

$$v'(\hat{\ell}_s^i) = u'(\hat{c}_s^i)F'(\hat{L}_s), \qquad i = P, R \tag{4.9}$$

$$\hat{c}_s^P = F'(\hat{L}_s)\hat{\ell}_s^P + T_s^P \tag{4.10}$$

$$\hat{c}_s^R = \left[F(\hat{L}_s) - N_s^P \hat{c}_s^P + N_s^R T_s^R\right] \Big/ N_s^R \tag{4.11}$$

2. *Optimal allocation* $\mathbf{z}^*(\mathbf{0})$ *before the pandemic can be implemented with the vector of transfers given by:*

$$T^{*P} = -\frac{\sum_s N_s^R}{\sum_s N_s^P} T^{*R} = c^*(\mathbf{0}) - F'(L^*(\mathbf{0}))\ell^*(\mathbf{0}), \qquad \text{for all } s. \tag{4.12}$$

If $N_s^P = N_{s'}^P$ *for all* $s \neq s'$ *then* $T_s^* = 0$.

Proof. Omitted. Available upon request. □

In words, in the pre-pandemic world, the federal social planner will want to redistribute resources from the rich to the poor households, within and potentially across states. The transfer that an indivitual poor household receives does not depend on the state the household lives in. Moreover, if all states are homogenous w.r.t. the proportion of the poor households, the net transfer received by any state 0 is zero. In general, states with $N_s^P > \frac{1}{S} N_s^P$ will receive positive net transfers, while states with $N_s^P < \frac{1}{S} N_s^P$ will be net contributors.

4.4 Non-cooperative allocations

Now consider the uncoordinated response of state governors to the outbreak. Each governor takes as given the actions of other governors, and the resulting peaks of the infections curves in other states, so equation (3.2) in state s takes the following form:

$$p_s = \bar{p} + N_s^P \cdot g_P\left(\ell_s^P\right) + N_s^R \cdot g_R\left(\ell_s^R\right) + \kappa p_{-s} \tag{4.13}$$

The maximization problem and the resulting optimal lockdown policy of the governor in state s will depend on the degree to which that governor can redistribute resources within his/her own state, and on the degree to which governors in other states can redistribute resources within their states.

Definition 4.5 (Non-cooperative allocation). Let $S_R \subseteq S$ be the set of states where governors can costlessly redistribute resources between the rich and the poor households, and let $S_{NR} \subseteq S$ be the set of states where governors cannot redistribute resources between the rich and the poor households, with

$S_R \cup S_{NR} = S$ and $S_R \cap S_{NR} = \emptyset$. Given the vector of federal transfers $\mathbf{T} \equiv \left(T_s^P, T_s^R\right)_{s=1}^S$, the non-cooperative allocation is a tuple $\tilde{\mathbf{z}}(\mathbf{T}) \equiv \left(\tilde{\ell}_s^P, \tilde{\ell}_s^R, \tilde{c}_s^P, \tilde{c}_s^R, \tilde{p}_s\right)_{s=1}^S$, such that, for each $s \in S$, $\left(\tilde{\ell}_s^P, \tilde{\ell}_s^R, \tilde{c}_s^P, \tilde{c}_s^R, \tilde{p}_s\right)$ solves:

$$\max_{\left(\ell_s^P, \ell_s^R, c_s^P, c_s^R, p_s\right)} \sum_{i=P,R} N_s^i \cdot \left[u\left(c_s^i\right) - v\left(\ell_s^i\right)\right] - h\left(p_s\right)$$

subject to:

$$L_s \leq N_s^P \ell_s^P + N_s^R \ell_s^R, \qquad \text{for all } s \in S \tag{4.14}$$

$$p_s \geq \bar{p} + N_s^P \cdot g_P\left(\ell_s^P\right) + N_s^R \cdot g_R\left(\ell_s^R\right) + \kappa p_{-s}, \qquad \text{for all } s \in S \tag{4.15}$$

$$N_s^P c_s^P + N_s^R c_s^R \leq F(L_s) + T_s^P + T_s^R, \qquad \text{for all } s \in S_R \tag{4.16}$$

$$c_s^P \leq F'(L_s)\ell_s^P + T_s^P \qquad \text{for all } s \in S_{NR} \tag{4.17}$$

$$N_s^R c_s^R \leq F(L_s) - N^P c_s^P + N_s^R T_s^R \qquad \text{for all } s \in S_{NR} \tag{4.18}$$

and such that, for each s, the following condition holds:

$$p_{-s} = \frac{1}{S} \sum_{s'=1}^{S} p_{s'}. \tag{4.19}$$

The difference between the maximization problems of the governors that can and those that cannot easily redistribute resources within their states lies in the constraints. Governors in states where redistribution is costless face the resource constraint (4.16). Governors that cannot locally redistribute resources, face constraints (4.17)-(4.18). Constraint (4.17) is simply the budget constraint for the poor household, where $F'(L_s)$ is the real wage. Constraint (4.18) then becomes the resource constraint for that state.

4.4.1 Characterization

Let $\tilde{\lambda}_s = h'(\tilde{p}_s)$ denote the Lagrange multiplier on the constraint (4.15) evaluated at the non-cooperative allocation. Basic algebra yields that the non-cooperative allocation must satisfy (4.14)-(4.19) and, additionally, the following optimality conditions:

$$c_s^i = c_s = F(L_s) + T_s^R + T_s^P, \qquad i = P, R \qquad \text{all } s \in S_R \tag{4.20}$$

$$v'(\ell_s^i) = F'(L_s)u'(c_s) - \tilde{\lambda}_s \cdot g_i'(\ell_s^i), \qquad i = P, R \qquad \text{all } s \in S_R \tag{4.21}$$

$$v'\left(\ell_s^P\right) = F'(L_s)u'\left(c_s^P\right) - \tilde{\lambda}_s \cdot g_P'\left(\ell_s^P\right) + \left[u'\left(c_s^P\right) - u'\left(c_s^R\right)\right] N_s^P \ell_s^P F''(L_s), \qquad \text{all } s \in S_{NR} \tag{4.22}$$

$$v'\left(\ell_s^R\right) = F'(L_s)u'\left(c_s^R\right) - \tilde{\lambda}_s \cdot g_R'\left(\ell_s^R\right) + \left[u'\left(c_s^P\right) - u'\left(c_s^R\right)\right] N_s^P \ell_s^P F''(L_s), \qquad \text{all } s \in S_{NR} \tag{4.23}$$

Taking the difference between the consumpetion-leisure trade-offs for the rich and the poor households we obtain:

$$v'(\tilde{\ell}_s^R) - v'(\tilde{\ell}_s^P) = \tilde{\lambda}_s \cdot \left[g_P'(\tilde{\ell}_s^P) - g_R'(\tilde{\ell}_s^R)\right], \qquad \text{for all } s \in S_R \tag{4.24}$$

$$v'(\tilde{\ell}_s^R) - v'(\tilde{\ell}_s^P) = \tilde{\lambda}_s \cdot \left[g_P'(\tilde{\ell}_s^P) - g_R'(\tilde{\ell}_s^R)\right] + \underbrace{F'(\tilde{L}_s) \cdot \left[u'\left(\tilde{c}_s^R\right) - u'\left(\tilde{c}_s^P\right)\right]}_{<0}, \qquad \text{for all } s \in S_{NR} \tag{4.25}$$

Those two equations provide key intuition behind the two main propositions that will be stated next.

4.4.2 Inter-state externalities, lack of local redistribution, and uncoordinated local response

In order to abstract from the redistributive motive that seeks to equate consumptions across all households types in all states, I will consider a partially symmetric world in the sense that $N_s^P = N^P$ in every state s. That way, the only heterogeneity between states is in the ability to redistribute resources between different households. Propositions 4.6 and 4.7 provide two key results of this section.

Proposition 4.6 (The impact of inter-state externalities and lack of redistribution). *Consider the world during the pandemic without federal transfers and with $N_s^P = N^P$ for all s. Let $s \in S_R$ and let $s' \in S_{NR}$. Then:*

1. *If $\kappa > 0$ then $\tilde{\ell}_s^P > \ell_s^{*P}$.*
2. *For any $\kappa \geq 0$ we have:*
   - *(a) $\tilde{\ell}_{s'}^P > \tilde{\ell}_s^P \geq \ell_s^{*P}$*
   - *(b) If $g_R' \equiv 0$ then $\tilde{p}_{s'} > \tilde{p}_s \geq p_s^*$*

Proof. The complete proof is in Appendix C.1. The first part of the proposition follows from the difference between the Lagrange multipliers in the optimal allocation $\left(\lambda_s^* = h'\left(p_s^*\right) + \kappa \frac{1}{S} \sum_{s'} \lambda_{s'}^*\right)$ vs. non-cooperative allocation $\left(\tilde{\lambda}_s = h'\left(\tilde{p}_s\right)\right)$. Part a) of the second part follows from the fact that if $s' \in S_{NR}$ then $\tilde{c}_{s'}^P < \tilde{c}_{s'}^R$. Part b) is then an immediate implication of part a). □

Other things being equal, states that cannot easily redistribute resources, will implement less strict lockdown policies for poor households. Under certain conditions (e.g. when rich households can completely switch to remote form of employment), such states will have higher levels of total infections and deaths than states where the redistribution by the local authorities is costless.[12] This is true even if there are no inter-state infection externalities across the states. In the presence of externalities, the lack of redistribution by some states, will incur additional costs on other states, as stated in the next proposition.

Proposition 4.7 (The lack of redistribution exacerbates the effect of inter-state externalities). *Consider the world during the pandemic without federal transfers. Let $\tilde{z}(S_R, S_{NR})$ be the non-cooperative allocation during the pandemic, given the sets of states with and without the ability to redistribute resources by the local authorities (S_R, S_{NR}), with the similar notation for the individual components of $\tilde{z}$. Suppose $g_R' \equiv 0$. If $S_R \subsetneq S_R'$ and $S_{NR}' \supsetneq S_{NR}$, and if $\kappa > 0$, then*

1. $\tilde{p}_s(S_R', S_{NR}') > \tilde{p}_s(S_R, S_{NR})$ *for any* $s \in S_R'$, *and*

2. $\tilde{\ell}_s^P(S_R', S_{NR}') < \tilde{\ell}_s^P(S_R, S_{NR})$ *for any* $s \in S_R'$.

Proof. If the set of states where local authorities cannot issue new bonds to redistribute resources expands, than in all those states $\ell_{s'}^P$ will be larger, so $p_{s'}$ will be larger for any $s' \in S_{NR}'$ (following the result in Proposition 4.6). Because of externalities p_s, will also be larger for any $s \in S_R'$, and so $h'(p_s)$ will be larger. The consumption-leisure trade-off then implies that ℓ_s^P will be lower. □

Proposition 4.7 implies that if the number of states that cannot redistribute resources is larger, the cumulative infections and deaths in the remaining states that can will be larger. In that sense, inter-state externalities are amplified by the lack of income redistribution at the local level.

4.5 Federal redistribution and the lives vs. livelihoods trade-off

When it is too costly for states to redistribute resources, the federal government can step in. In normal times, federal redistribution can provide additional safety net for lower income households in states where such redistribution is more costly. The analysis in the previous section suggests that such federal redistribution can ease the lives vs. livelihoods trade-off faced by the states during the pandemic.

[12] In general, if the functions $g_P(\cdot)$ and $g_R(\cdot)$ are very similar, it is possible that states in S_{NR} will end up with lower infections peaks, depending on the relative curvature of the production function and the difference between $g_P(\cdot)$ and $g_R(\cdot)$. Section 4.5 provides results from the numerical simulations where federal redistribution towards the poor may actually increase the infection peaks if $g_P(\cdot) \equiv g_R(\cdot)$.

An immediate result (essentially, a corollary to Proposition 4.6) is that if $N_s^P = N_{s'}^P$ then a complete federal redistribution of income that sets $c^R = c^P$ will make the local government implement the optimal lockdown policy, as long as there are no epidemiological externalities between states.

In this section I will consider a more general environment where states differ with respect to the fraction of rich and poor households. Of course, combining appropriate federal transfers to specific households with the inter-state transfers, will implement the optimal allocation. Such redistributive scheme, with state-specific income-based transfers, would be very complex. We can, however consider the impact of a much simpler, second-best policy - the transfer towards the poor which ensures that the average consumption level between the rich and the poor households is equalized.

For simplicity, assume that $S_R = \emptyset$ - no state can quickly issue new debt in order to provide a safety net for the poorest household (beyond already existing transfers that had implemented the optimal pre-pandemic allocation). Each household of type $i = P, R$ in the country will receive a total federal transfer (a stimulus check) in the amount of T^i such that:

$$\sum_s \left(N_s^P T^P + N_s^R T^R\right) = 0 \qquad \Longleftrightarrow \qquad T^R = -T^P \frac{\sum_s N_s^P}{\sum_s N_s^R}$$

A positive transfer to the poor implies a negative transfer to the rich which simply means that the rich will buy the bonds the government will have to issue to finance that policy. The size of the transfer T^P is chosen to ensure that the non-cooperative allocation satisfies:

$$\sum_s N_s^P c_s^P = \sum_s N_s^R c_s^P,$$

i.e. the average consumption of the poor and rich households is the same (in the world with symmetric states this would boil down to the perfect within-state redistribution).

4.5.1 Numerical simulations

Figures 4 and 5 show the outcome of such policy, when states differ w.r.t the proportion of poor households. Both figures were generate from the model with the following assumptions on the functional forms and on parameter values:

- $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ with $\sigma = 2$
- $v(\ell) = \frac{\ell^{1+\theta}}{1+\theta}$ with $\theta = 1$

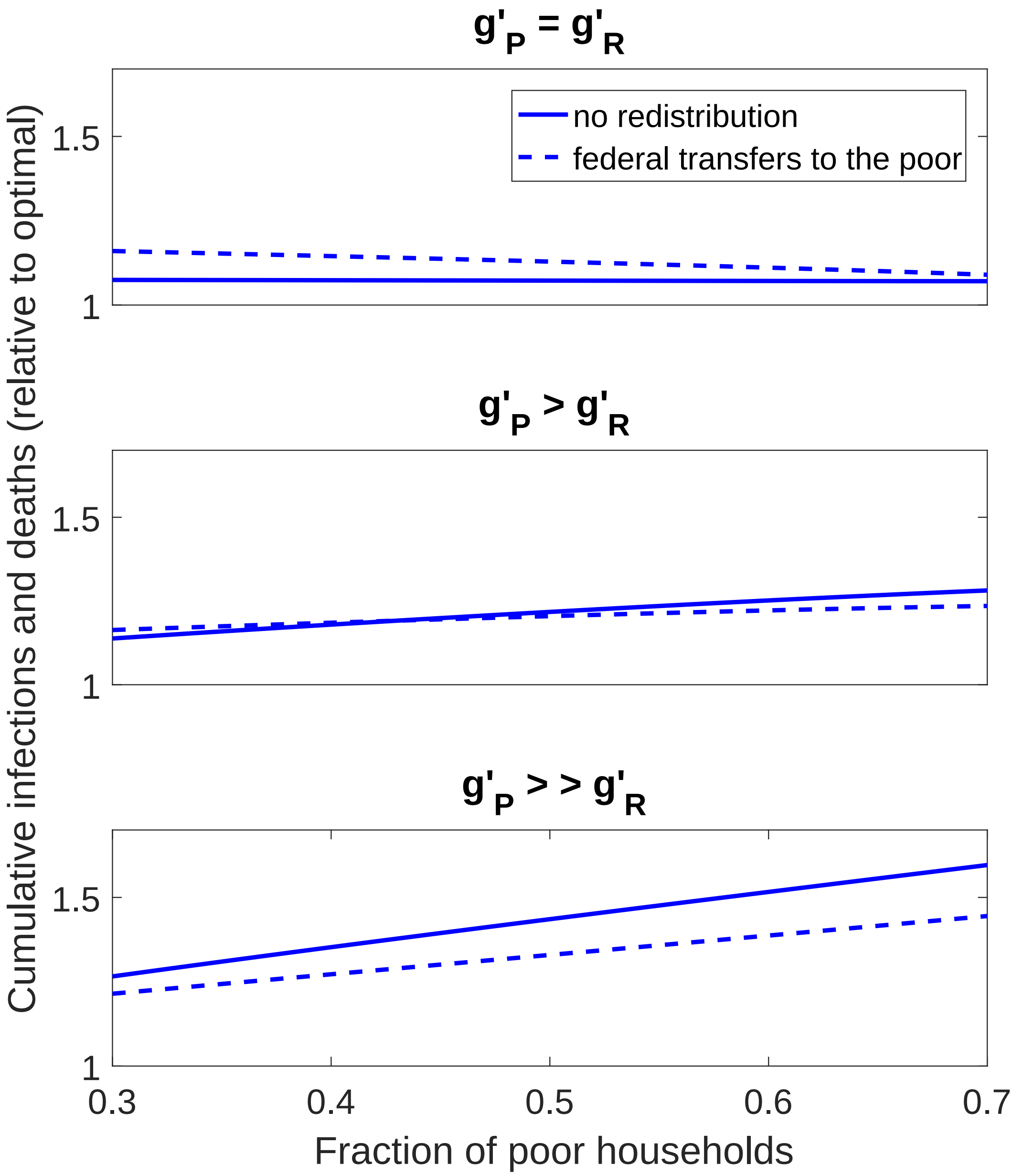


Figure 4: Impact of federal transfers on total infections and deaths by state

NOTES: $g_R(\ell) = \ell^2$ always; The three cases correspond to:

(top) — $g_P = g_R$; (middle) — $g_P(\ell) = 3 \cdot \ell^2$; (bottom) — $g_P(\ell) = 10 \cdot \ell^2$

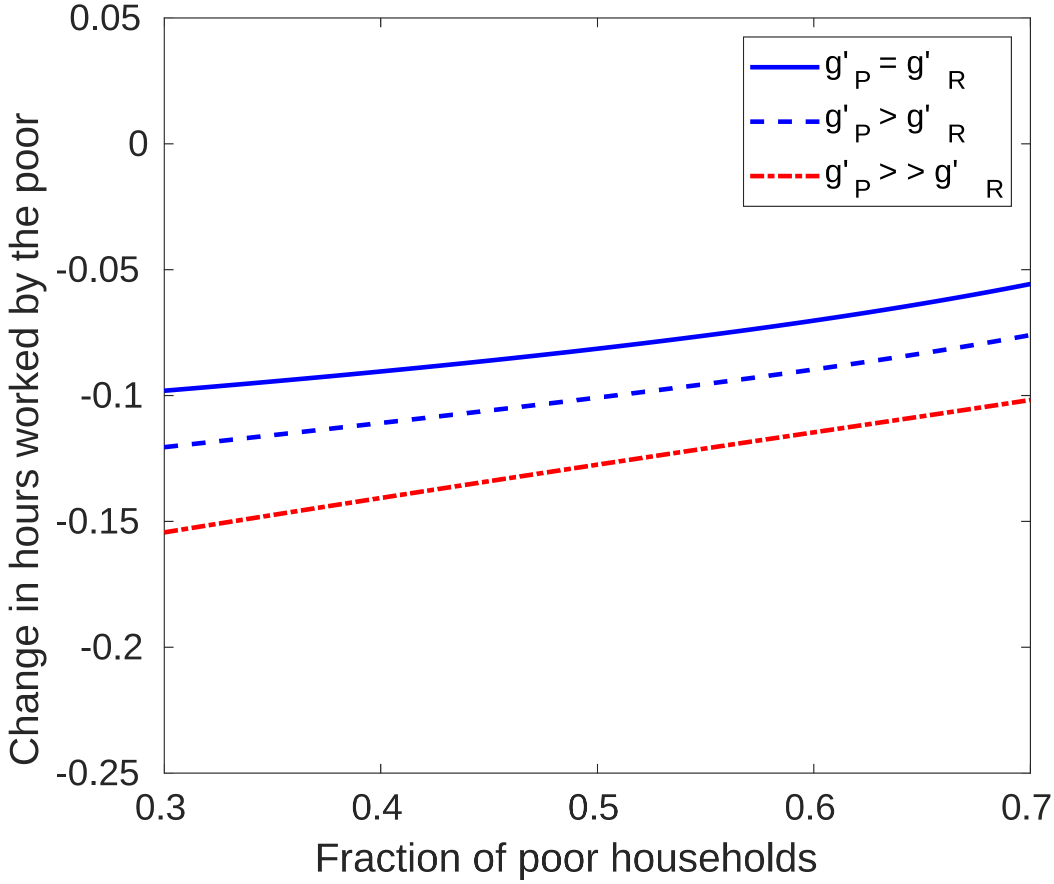


Figure 5: State-by-state impact of federal transfers on the lockdown of the poor

NOTES: See Figure 4.

- $h(p) = \frac{p^{1+\omega}}{1+\omega}$ with $\omega = 1$
- $F(L) = L^{\alpha}$ with $\alpha = 0.67$
- $g_i(\ell) = \gamma^i \cdot {\ell^i}^2$ with $\gamma^R = 1$ and $\gamma^P = 1, 3,$ or 10
- $\kappa = 0.25$, $S = 48$
- The 48 states differ w.r.t. N^P that ranges from $N_1^P = 0.3$ to $N_{48}^P = 0.7$.

The simulations consider three different cases of the difference between the functions g_P and g_R - the impact of the regular employment of the poor and rich on the spread of disease, or, equivalently, the ability of the poor and the rich to switch to a remote work. The greater is the difference between g'_P and g'_R, the more the regular employment of the poor contributes to the spread of the disease.

The main message from the two figures is that when the difference between g'_P and g'_R is large, the federal transfers towards the poor provides a greater incentive for the state governors to implement a stricter lockdown policy that results in a larger drop of the employment by the poor. That results in a bigger decline in cumulative infections and deaths, especially within states that have a large proportion of poor households.

5 Conclusions

The management of the ongoing COVID-19 outbreak in the United States has been, to a large extent, done at the state level (including decisions on shelter-at-home orders or closures of certain businesses). Given that travel between states cannot be easily restricted, the lack of coordination between states' in their responses to the outbreak, can result in a larger than optimal peak of the infection curves within each state, because governors are unwilling to sufficiently limit the economic activity in their states. In this paper I highlighted the mechanism through which it occurs and showed how a simple inter-state transfers policy can induce the governors to depress their local economies more then they otherwise would.

The general logic of the policy is that the federal government would subsidize states which depress their economies more and tax those that depress their economies less. In a symmetric equilibrium, no state receives any transfers, but the presence of the policy alone generates a race-to-the-bottom type of response by states' governors, implementing the optimal allocation. The result holds in both the endowment economy and in the economy with elastic labor supply and heterogenous agents, in which different households contribute differently to the spread of the disease, due to the nature of their jobs.

The paper lays out a tractable theoretical framework to study the coordination problem between governors who need to limit the outbreak in their states, in a situation with a very clear inter-state externality. There are still many questions that can and need to be addressed in this theoretical environment. How does the optimal federal policy change when the total death-toll of the whole pandemic impacts future size of the population, tax base, and the ability of each state to honor their debt obligations? How does it change when different people have different access to healthcare services, so that the utility cost of the steep infection curve may be different for different households? Those are very important questions that are left for further research.

Combined with empirical literature that documents substantial inter-state spillovers of Covid-19 (Rothert et al., 2020; Brinkman and Mangum, 2020; Dave et al., 2020; Renne et al., 2020), and recent work on policy coordination (Rothert, 2021; Beck and Wagner, 2020), the results in this paper emphasize the important role that the federal government can and should play in battling current and future disease outbreaks.

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A Derivations and proofs for Section 2

Proof of Proposition 2.1

The second derivative is:

$$C''(\beta) = \frac{d\frac{dCI}{d\beta}}{d\beta} = \frac{-\frac{dS_\infty}{d\beta}\left(\frac{\alpha}{S_\infty} - \beta\right) + (N - S_\infty)\left(1 + \frac{\alpha}{S_\infty^2}\frac{dS_\infty}{d\beta}\right)}{\left(\frac{\alpha}{S_\infty} - \beta\right)^2}$$

Since $\frac{dS_\infty}{d\beta} = \frac{S_\infty - N}{\frac{\alpha}{S_\infty} - \beta}$, we get:

$$C''(\beta) = \frac{N - S_\infty + (N - S_\infty)\left(1 - \frac{\alpha}{S_\infty^2}\frac{N - S_\infty}{\frac{\alpha}{S_\infty} - \beta}\right)}{\left(\frac{\alpha}{S_\infty} - \beta\right)^2} = \frac{N - S_\infty}{\left(\frac{\alpha}{S_\infty} - \beta\right)^2}\left(2 - \frac{\alpha}{S_\infty^2}\frac{N - S_\infty}{\frac{\alpha}{S_\infty} - \beta}\right)$$

We then get that $\lim_{S_0 \nearrow N} \lim_{\beta \searrow \frac{\alpha}{S_0}} C''(\beta) > 0 \iff \lim_{S_0 \nearrow N} \lim_{\beta \searrow \frac{\alpha}{S_0}} \frac{\alpha}{S_\infty^2}\frac{N - S_\infty}{\frac{\alpha}{S_\infty} - \beta} < 2$. Setting $\beta = \frac{\alpha}{S_0}$ we get:

$$\frac{\alpha}{S_\infty^2}\frac{N - S_\infty}{\frac{\alpha}{S_\infty} - \beta} = \frac{\alpha}{S_\infty^2}\frac{N - S_\infty}{\frac{\alpha}{S_\infty} - \frac{\alpha}{S_0}} = \frac{1}{S_\infty^2}\frac{N - S_\infty}{\frac{S_0 - S_\infty}{S_\infty S_0}} = \frac{S_0}{S_\infty}\frac{N - S_\infty}{S_0 - S_\infty}$$

Next, we need to show that $\lim_{S_0 \to N} \frac{S_0}{S_\infty}\frac{N - S_\infty}{S_0 - S_\infty} < 2$. Setting $S_0 = N$, we get:

$$\frac{S_0}{S_\infty}\frac{N - S_\infty}{S_0 - S_\infty} < 2 \iff \frac{S_\infty}{N} > \frac{1}{2}$$

To see that this is the case, recall that S_∞ solves: $S_\infty = S_0 e^{\frac{\beta(S_\infty - N)}{\alpha}}$. When $\beta = \frac{\alpha}{S_0}$ and $S_0 = N$, this reduces to $\frac{S_\infty}{N} = e^{\frac{S_\infty}{N} - 1}$. But since $\frac{1}{2} < e^{-\frac{1}{2}}$, and left hand side is an increasing while the right hand side is a decreasing function, it then follows that $\frac{S_\infty}{N} > \frac{1}{2}$.

B Derivations and proofs for Section 3

Proof of Proposition 3.1

Plugging $p^* = \frac{1}{1-\kappa}\left[\bar{p} + g(y^*)\right]$ into (3.6) we get:

$$h'\left(\frac{1}{1-\kappa}\bar{p} + \frac{1}{1-\kappa}g(y^*)\right) - (1-\kappa)f(y^*) = 0$$

where I define $f \equiv \frac{u'}{g'} > 0$. Then $f' < 0$ and $h'' > 0$. Total differentiation w.r.t. y^* and κ yields:

$$d\kappa \cdot \left[f(y^*) + h''(y^*) \cdot \frac{\bar{p} + g(y^*)}{(1-\kappa)^2}\right] - dy^* \cdot \left[(1-\kappa)f'(y^*) - h''(y^*) \cdot \frac{g'}{1-\kappa}\right] = 0$$

which yields

$$\frac{dy^*}{d\kappa} = \frac{f(y^*) + h''(y^*) \cdot \frac{\bar{p}+g(y^*)}{(1-\kappa)^2}}{(1-\kappa)f'(y^*) - h''(y^*) \cdot \frac{g'(y^*)}{1-\kappa}} = \frac{(1-\kappa)f(y^*) + h''(y^*) \cdot \frac{\bar{p}+g(y^*)}{(1-\kappa)}}{(1-\kappa)^2 f'(y^*) - h''(y^*) \cdot g'(y^*)} < 0$$

because numerator is always positive and the denominator is always negative.

C Derivations and proofs for Section 4

C.1 Proof of Proposition 4.6

1. **Proof that $\tilde{\ell}^P_s > {\ell^*}^P_s$ if $\kappa > 0$**

   Fix $s \in S_R$. Suppose $\tilde{\ell}^P \leq {\ell^*}^P$. Then $v'(\tilde{\ell}^P) \leq v'({\ell^*}^P)$ and $g'_P(\tilde{\ell}^P) \leq g'_P({\ell^*}^P)$.

   Case 1: $\tilde{\ell}^R \leq {\ell^*}^R$. Then $\tilde{L} \leq L^*$ and $\tilde{c} \leq c^*$, so $F'(\tilde{L})u'(\tilde{c}) \geq F'(L^*)u'(c^*)$. Moreover, $\tilde{p} \leq p^*$, so $\tilde{\lambda} = h'(\tilde{p}) \leq h'(p^*) < \lambda^*$, where the last inequality is implied by $\kappa > 0$. Since $\tilde{\ell}^P \leq {\ell^*}^P$, we also have $g'_P(\tilde{\ell}^P) \leq g'_P({\ell^*}^P)$, and therefore we have $\tilde{\lambda} g'_P(\tilde{\ell}^P) < \lambda^* g'_P({\ell^*}^P)$. Hence we end up with:

   $$v'(\tilde{\ell}^P) \leq v'({\ell^*}^P) = F'(L^*)u'(c^*) - \lambda^* g'_P({\ell^*}^P) < F'(\tilde{L})u'(\tilde{c}) - \tilde{\lambda} g'_P(\tilde{\ell}^P)$$

   which contradicts (4.21).

   Case 2: $\tilde{\ell}^R > {\ell^*}^R$. Then $v'(\tilde{\ell}^R) > v'({\ell^*}^R)$ and $g'_R(\tilde{\ell}^R) > g'_R({\ell^*}^R)$, and therefore:

   $$v'(\tilde{\ell}^R) - v'(\tilde{\ell}^P) > v'({\ell^*}^R) - v'({\ell^*}^P) = \lambda^* \cdot \left[g'_P({\ell^*}^P) - g'_R({\ell^*}^R)\right] > \lambda^* \cdot \left[g'_P(\tilde{\ell}^P) - g'_R(\tilde{\ell}^R)\right]$$

   The last thing that needs to be shown is that it is impossible for $\tilde{\lambda} > \lambda^*$, which is not yet obvious, because $\tilde{\ell}^R$ may be sufficiently large to yield $\tilde{p} > p^*$. Suppose then that $\tilde{\lambda} > \lambda^*$. For that to be the case we need $\tilde{p} > p^*$. In order to get that, we need $\tilde{L} > L^*$ and hence $\tilde{c} > c^*$, because $g'_P > g'_R$, which implies that $F'(L^*)u'(c^*) > F'(\tilde{L})u'(\tilde{c})$. We then get:

   $$v'(\tilde{\ell}^R) > v'({\ell^*}^R) = F'(L^*)u'(c^*) - \lambda^* g'_R({\ell^*}^R) > F'(\tilde{L})u'(\tilde{c}) - \tilde{\lambda} g'_R(\tilde{\ell}^R)$$

   which again contradicts (4.21).

2. Proofs of part 2)

   (a) **Proof that $\tilde{\ell}^P_{s'} > \tilde{\ell}^P_s \geq {\ell^*}^P_s$ for any $\kappa \geq 0$, $s' \in S_{NR}$, $s \in S_R$.**

   First notice that equation 4.24 implies that $\tilde{\ell}^R_s > \tilde{\ell}^P_s$ for all $s \in S_R$. Notice also, that without federal transfers, we will have $\tilde{c}^R_{s'} > \tilde{c}^P_{s'}$. If not, we would have $F'(\tilde{L}_{s'})\tilde{\ell}^P_{s'} \geq F'(\tilde{L}_{s'})\tilde{\ell}^R_{s'} + \frac{\tilde{\pi}}{N^R}$, which would require that $\tilde{\ell}^P_{s'} > \ell^R_{s'}$, which would yield:

$$\underbrace{v'(\tilde{\ell}^R_{s'}) - v'(\tilde{\ell}^P_{s'})}_{<0} = \tilde{\lambda}_s \cdot \underbrace{\left[g'_P(\tilde{\ell}^P_{s'}) - g'_R(\tilde{\ell}^R_{s'})\right]}_{>0} + \underbrace{F'(\tilde{L}_{s'}) \cdot \left[u'\left(\tilde{c}^R_{s'}\right) - u'\left(\tilde{c}^P_{s'}\right)\right]}_{\geq 0 \text{ if } \tilde{c}^R_{s'} \leq \tilde{c}^P_{s'}},$$

which is a contradiction. Hence, we must have: $\tilde{c}^R_{s'} > \tilde{c}^P_{s'}$ and $u'\left(\tilde{c}^R_{s'}\right) - u'\left(\tilde{c}^P_{s'}\right) < 0$.

Case 1: $\tilde{\ell}^R_{s'} \leq \tilde{\ell}^R_s$. Then $\tilde{L}_{s'} \leq \tilde{L}_s$, and since $u'(\tilde{c}^P_{s'}) > u'(\tilde{c}^P_s)$ it then follows that overall welfare can be improved by increasing $\tilde{\ell}^P_{s'}$, because:

$$F'(\tilde{L}_{s'})u'(\tilde{c}^P_{s'}) - v'(\tilde{\ell}^P_{s'}) > h'(\tilde{p}_{s'}) \cdot g'_P(\tilde{\ell}^P_{s'}).$$

Case 2 a): $\tilde{\ell}^R_{s'} > \tilde{\ell}^R_s$ **such that** $p_{s'} \leq p_s$. If that is the case, then $\tilde{\lambda}_{s'} \leq \tilde{\lambda}_s$ and (4.24)-(4.25) imply that:

$$\begin{aligned} v'(\tilde{\ell}^R_{s'}) - v'(\tilde{\ell}^P_{s'}) > v'(\tilde{\ell}^R_s) - v'(\tilde{\ell}^P_s) = \tilde{\lambda}_s \left[g'_P(\tilde{\ell}^P_s) - g'_R(\tilde{\ell}^R_s)\right] > \\ > \tilde{\lambda}_{s'} \left[g'_P(\tilde{\ell}^P_{s'}) - g'_R(\tilde{\ell}^R_{s'})\right] + F'(\tilde{L}_{s'}) \underbrace{\left[u'\left(\tilde{c}^R_{s'}\right) - u'\left(\tilde{c}^P_{s'}\right)\right]}_{<0} \end{aligned}$$

which violates (4.25).

Case 2 b): $\tilde{\ell}^R_{s'} > \tilde{\ell}^R_s$ **such that** $p_{s'} > p_s$. If that is the case then $\tilde{\lambda}_{s'} > \tilde{\lambda}_s$ and $\tilde{L}_{s'} > \tilde{L}_s$ implying that $F'(\tilde{L}_{s'})u'(\tilde{c}^R_{s'}) < F'(\tilde{L}_s)u'(\tilde{c}_s)$. We then have:

$$\begin{aligned} v'(\tilde{\ell}^R_{s'}) > v'(\tilde{\ell}^R_s) = F'(\tilde{L}_s)u'(\tilde{c}_s) - \tilde{\lambda}_s g'_R(\tilde{\ell}^R_s) > \\ > F'(\tilde{L}_{s'})u'(\tilde{c}_{s'}) - \tilde{\lambda}_{s'} g'_R(\tilde{\ell}^R_{s'}) + \underbrace{\left[u'\left(\tilde{c}^P_{s'}\right) - u'\left(\tilde{c}^R_{s'}\right)\right] N^P \tilde{\ell}^P_{s'} F''(L_{s'})}_{<0}, \end{aligned}$$

which violates (4.23).

(b) **Proof that** $p_{s'} > p_s$ **if** $g'_R \equiv 0$**.**

The first part follows immediately from part 2a) - $\tilde{\ell}^P_{s'} > \tilde{\ell}^P_s$. For the second part, consider

C.2 Proof of Proposition 4.7

Optimal allocation during the pandemic

The first order condition w.r.t. c_s^i, ℓ_s^i $(i = P, R)$, and p_s are:

$$\frac{\partial \mathcal{L}}{\partial c_s^i} = N_s^i u'\left(c_s^i\right) - N_s^i \mu = 0 \Rightarrow \qquad \mu = u'\left(c_s^i\right) = u'\left(c^*\right), \qquad i = P, R; \quad s = 1, ..., S \tag{C.1}$$

$$\frac{\partial \mathcal{L}}{\partial \ell_s^i} = -N_s^i \cdot v'\left(\ell_s^i\right) + N_s^i \cdot \eta_s - N_s^i \cdot \lambda_s g'_P\left(\ell_s^i\right) = 0 \tag{C.2}$$

$$\frac{\partial \mathcal{L}}{\partial L_s} = \mu F'\left(L_s\right) - \eta_s = 0 \tag{C.3}$$

$$\frac{\partial \mathcal{L}}{\partial p_s} = -h'\left(p_s\right) + \lambda_s - \kappa \frac{1}{S} \sum_{s'} \lambda_{s'} = 0 \tag{C.4}$$

Last equation implies that

$$\lambda_s = h'\left(p_s\right) + \kappa \frac{1}{S} \sum_{s'} \lambda_{s'}$$

Equations (C.3) and (C.1) together imply that:

$$\eta_s = u'\left(c^*\right) F'\left(L_s\right)$$

Plugging those two into (C.2) we get so:

$$v'\left(\ell_s^i\right) = u'\left(c^*\right) F'\left(L_s\right) - g'_i\left(\ell_s^i\right) \cdot \left[h'\left(p_s\right) + \kappa \frac{1}{S} \sum_{s'} \lambda_{s'}\right]$$

which is Equation (4.6) in Section 4.