Incomplete rationality and old-age savings

Background paper

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Contents

1 Introduction

Overlapping generations models predict life-cycle assets accumulation path which is at odds with observational data. Rational agents accumulate assets in the working ages and de-accumulate assets after retirement in the models. Meanwhile, there is evidence for savings behavior after retirement (De Nardi et al., 2016) and persistent lack of savings in the working ages (Weil, 1992; Kaplan et al., 2014). This study provides an overlapping generations model with agents characterized by incomplete rationality. We study the patterns of asset accumulation by such agents and evaluate the effects of introducing a government-subsidized old-age saving instrument with voluntary participation.

Prior literature attempted to bring the model closer to the data by increasing impatience, but these attempts were not satisfactory as demonstrated in a review by Frederick et al. (2002). They also cannot reconcile predictions from structural overlapping generations models with econometric evidence on introducing government-subsidized old-age saving instruments (e.g. Hubbard and Skinner, 1996; Chetty et al., 2014). We contribute to the literature by studying agents with incomplete rationality. We introduce a government-subsidized old-age saving instrument with voluntary participation in this setup and study its effects. We also characterize the features of incomplete rationality which are conducive to increasing welfare ensuing an introduction of such instrument. Specifically, we add four types of incomplete rationality to an otherwise standard overlapping generations model.

In economics, rationality means that the agents are characterized by three basic features. First, agents have perfect foresight and their preferences are stable over time. Second, agents have unconstrained ability to process all this information to absorb it in current and future choices. Third, agents have unconstrained ability to transfer assets between periods (i.e. accumulate for the old-age or store precautionary savings for the periods of adverse shocks to earned income). These three premises guarantee that agents in models construct optimal life-cycle profiles, execute optimal plans without any deviations, and use all resources efficiently. It does not mean that all agents are wealthy, all the less so that agents are necessarily equal. It may still hold that one is endowed with lower productivity (and so earns less), higher leisure preference (and so works less) or higher impatience (and therefore saves less). This is referred to as intra-cohort inequality. Moreover, an agent of a given type has less wealth at the young age than an agent of the same type at older age. This is referred to as inter-cohort inequality. However, these differences (inequalities) in wealth and lifetime profiles reflect merely differences in preferences.

Our study provides several findings. First, government-subsidized old-age saving instrument improves welfare for those types of agents, for whom incomplete rationality restricts the choice set. Second, we find that agents with limited access to savings technology benefit from being able to participate in the instrument. Third, we find that agents whose low savings stem from quasi-hyperbolic time preferences experience welfare deterioration ensuing the introduction of the instrument and we identify premises of this result. Fourth, we show that agents without foresight, while their assets accumulation patterns differ from those of the fully rational agents, experience similarly negative welfare effects. Finally, we show in detail the mechanisms which prevent fully rational agents to gain welfare from the instrument, despite participation. In essence, the fiscal cost of the tax incentives is not outweighed by individual gains from those incentives. Our study delivers both the microeconomic and the macroeconomic evaluation of the crowd-out from government-subsidized old-age saving instruments, thus shedding light on the discrepancies between econometric evaluations from observational data and predictions from structural macroeconomic models such as the overlapping generations models. These results were derived in a fully annuitized economy, hence we abstract from a potential annuity value of the government-subsidized old-age saving instruments.

This study is structured as follows. The following section discusses related literature. Section 3 describes the model. In particular, in section 3.1 we present incompletely rational agents and study the differences between fully rational agents and agents with incomplete rationality. Section 4 discusses the calibration of our model. The results are presented in section 5, where both microeconomic and macroeconomic implications of a government-subsidized old-age saving instrument are studied. Finally, in the concluding sections, we summarize our findings and discuss policy implications.

Related Work

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The literature on old-age savings starts from the premises that rational agents adjust consumption in the working period such that income generated during the working ages can be partially put aside to subsidize consumption after retirement. Overlapping generations models have become a conventional tool to study the problem of optimal consumption smoothing over life-cycle (Auerbach and Kotlikoff, 1987). Fully rational agents in these models optimize savings based on their time preference and (as price takers) the interest rate in the economy. If interest is taxed or subject to preferential tax treatment, the effective interest rate relevant for the consumer optimization is different from the market interest rate by this wedge. In this setup, agents accumulate assets during the working ages, reaching peak wealth prior to the retirement and subsequently de-accumulate assets financing old-age consumption. Bequest motive may result in non-zero wealth at death. Similarly, lack of access to annuity markets may result in unintended bequests, i.e. do not benefit from their prior precaution to save for the old age.

Empirical evidence from observational data is in stark contradiction to those theoretically previewed patterns (Gale and Scholz, 1994; Bernheim, 2002). While it is the case that typically the peak of asset holdings occurs during the working ages, it is typically not the case that agents leave zero wealth at death. It is also not the case that wealth is accumulated with the objective of financing old-age consumption. Strong empirical evidence about savings regret (Boersch-Supan et al., 2018) suggests that it is only in the old age that the agents realize how much they should have saved for this stage of life-cycle or even that the savings for this stage of life-cycle are imperative. Another important departure from the optimal consumption smoothing over life-cycle concerns households who systematically have no accumulated assets (Xu and Zia, 2012; Klapper et al., 2015).

These two strong empirical regularities have given rise to multiple government-subsidized old-age savings instruments. These instruments are intended to encourage and incentivize specifically saving for the old-age consumption. On the one hand, they provide a variety of tax exemptions. On the other hand, participants are not allowed to withdraw the funds prior to reaching retirement eligibility. Across countries, typically more than one instrument exists (OECD, 2018). Multiplicity of instruments is typically associated with different caps on contributions, varying extent of tax preferences and effectively uneven rate of return on those contributions. In addition, eligibility differs as some of the instruments are offered exclusively to selected groups, e.g. based on occupation.

The differences between these instruments have been an important policy concern in many countries. For example, in Germany, in 2003, the Supreme Court ruled that the benefits across differentiated government-subsidized old-age saving instrument should be equalized. In a broad overview of the government-subsidized old-age savings instruments in Germany, Boersch-Supan and Quinn (2015) study the extent to which one unit of savings accumulated in those instruments translates to comparable pension wealth at retirement, as a consequence of this court ruling, whereas Boersch-Supan and Luehrmann (2000) study the extent of tax benefits across the instruments. We are not aware of similar comparisons for other countries (see also Gruber and Wise, 2009; OECD, 2018)

The economic literature studying the macroeconomic and welfare effects of those instruments in an overlapping generations setup is scarce. Fehr and Jess (2007) use a structural overlapping generations model to study the between cohort redistribution effects from the reforms ensuing the ruling of the German Supreme Court. Indeed, majority of the literature uses observational data and econometric techniques for policy evaluation (e.g. Engen et al., 1994; Poterba et al., 1995; Hubbard and Skinner, 1996; Engen and Gale, 2000; Chetty et al., 2014; Lachowska and Myck, 2018, admittedly, most of this literature concerns the case of the USA). While these studies are often ingenious in identifying the causal effects of government-subsidized old-age savings instruments on intended and realized pension wealth, the main limitation of this approach is that the analyses may only concern the part of the adjustments in wealth which has already occurred. Meanwhile, total effects of such instruments take typically decades to materialize. Against these shortcomings of the empirical literature, structural macroeconomic models of overlapping generations offer an interesting opportunity to obtain ex ante evaluation of the effects. However, in order for these models to deliver reliable insights, the behavioral response of the agents in the model should be in line with the behavioral patterns from the observational data. This implies a necessity to depart from fully rational agents.

To the best of our knowledge, there is no literature studying these instruments in the context of incomplete rationality. A recent study by Moser and Silva (2019) proposes an alternative approach to the question of government-subsidized old-age savings instruments. Recognizing that incentives aiming at raising savings for the old-age consumption are in fact purely paternalistic and as such ineffective, the authors propose a setup with heterogeneity in time preferences (specifically: present bias) and heterogeneity in abilities. In such setup, providing incentives to old-age consumption may generate dead-weigh loss, if some agents are effectively disincentivized to save on their own due to the instrument. Moser and Silva (2019) identify efficiency gains along the mechanisms known from the New Dynamic Public Finance literature (Golosov et al., 2006; Kocherlakota, 2010). Namely, properly designed incentives for the old-age savings make it optimal for the agents to reveal their type in terms of time preference and abilities, thus eliminating the distortions along the labor supply margin (intra-temporal choice) and consumption-savings margin (inter-temporal choice) also with reference to other taxes. In their setup, agents with high ability have effectively undistorted intra-temporal choice, whereas agents with low abilities are effectively incentivized to increase labor supply through government-subsidized old-age savings instruments. In a calibrated application to the 401(k) program in the US, the Moser and Silva (2019) show that welfare and efficiency could be raised in the US if the incentives (notably the caps and the tax exemptions) were improved in line with the proposed model.

Our study builds on the overlapping generations literature, and explores the differences in behavioral patterns across several well identified forms of incomplete rationality in agents life-cycle optimization. Having established how these patterns differ, we then introduce a government-subsidized old-age savings instrument and study how agents with incomplete rationality react to such instrument. We also study the macroeconomic and welfare effects of a policy reform consisting of unexpected introduction of the government-subsidized old-age savings instrument.

3 The model

This chapter describes the model structure. Our setup features fully rational agents as well as agents with various forms of incomplete rationality, which we describe in detail in section 3.1. We discuss intuitions concerning the lifetime optimization for these various types of agents in section 3.1.6. These agents populate an economy with a perfectly competitive production sector described in section 3.2. In our economy, the government collects taxes and provides for the balance of the pension system. The pension system and the government sector are described in sections 3.4 and 3.3, respectively.

Every period t, $N_{21,m,t}$ agents of type m at age $j = 1$ arrive in the economy in year t and live for $j = 21, 22, 23, ..., J$ periods with $\pi_{j,t}$ denoting unconditional survival probability and $\pi_{J,t} = 0$ (hence in every period t there is $N_{j,m,t}$ agents of age j and type m in the economy). Denote by $\mu_{j,t} = N_{j-1,m,t-1}/N_{j,m,t}$ the inverse of mortality risk (or annuity premium), i.e. the share of population of age j in time t that did not survive to $j+1$ in period $t+1$. Entry at $j = 21$ corresponds to age of labor market entry and permits abstracting from modeling human capital investment. Longevity is operationalized by gradually increasing $\pi_{j,t}$ and is common across the types of the agents (homogeneous within a birth cohort). See section 4 for details on calibration.

The agents optimize in a deterministic environment and have no bequest motive. Despite life-span uncertainty, the economy provides annuity on all assets: private voluntary savings and savings in the government-subsidized scheme. This setup allows to abstract from the value of annuity insurance in analyzing the welfare effects of government-subsidized scheme. Consequently, the unintended bequests are accounted for in the effective rate of return on assets, rather than entering the budget constraint on the income side.

3.1 Households

Agents share preferences in a sense that they derive lifetime utility from leisure $(1 - l_{j,m,t})$ and consumption $(c_{j,m,t})$ with the logarithmic instantaneous utility function of

$$U(c_{j,m,t}, l_{j,m,t}) = \log\left[c_{j,m,t}^{\phi}(1 - l_{j,m,t})^{1-\phi}\right], \tag{3.1}$$

where ϕ denotes leisure preference, identical across and within cohorts, and $l_{j,m,t}$ denotes endogenous and perfectly elastic labor supply. We denote by $\bar{J}$ retirement age, common across types. For brevity, we denote by $\mathcal{C}_{j,m,t} = (1+\tau_t^c)c_{j,m,t}$ gross consumption, where τ_t^c reflects consumption taxation and $c_{j,m,t}$ enters the agents' utility function. We also denote $\mathcal{I}_{j,m,t} = (1-\tau^l-\tau)w_t l_{j,m,t}$ earned labor income for agents with $j < \bar{J}$ and $\mathcal{I}_{j,m,t} = (1-\tau^l)b_{j,m,t}$ pension income for agents with $j \geq \bar{J}$. In this notation, w_t denotes wages and $b_{j,m,t}$ denotes pension benefits, τ^l denotes labor income taxation and τ denotes contribution rate to the universal defined contribution pension system. Likewise, we denote by $\mathcal{K}_{j,m,t} = (\mu_{j,t} + \bar{r}_{j,t}(1-\tau^k)) \cdot a_{j-1,m,t-1}$ the capital income of the household, with $\bar{r}_{j,t}$ denoting annuitized interest earned on accumulated assets $a_{j,m,t}$. The economy is thus fully annuitized, i.e. agents receive the annuity premium on all financial assets. The annuity premium consists of two parts: the interest earned by the assets previously held by the deceased: $((1-\tau^k)\bar{r}_{j,t})a_{j-1,m,t-1}$; and the assets of the deceased $(\mu_{j,t}-1)a_{j-1,m,t-1}$. Finally, throughout the paper we denote by Υ the lump-sum taxes/transfers.

In light of the research question at hand, we impose a no borrowing constraint, i.e. $\forall_{j,m,t}\, a_{j,m,t} \geq 0$. Agents with low desired levels of private savings would prefer to borrow against the future stream of pension benefits. With the constraint on non-negative asset holding, agents may dissave only once they accumulate assets. The agents may have zero or negative savings rate, but only up to a limit of previously accumulated assets. The no borrowing constraint, if binding, limits the set of possible solutions in the Euler condition. Agents may find that a solution consistent with their marginal rate of substitution is not feasible, i.e. it lies outside the choice set. We discuss this issue at length in section 3.1.6.

We introduce behavioral heterogeneity in terms of preferences and in terms of the budget constraint. Namely, in addition to conventional fully rational agents, we populate each birth cohort with agents following with alternative decision rules. First, we consider agents who, unlike fully rational agents, do not have perfect foresight abilities. We name these agents *adaptive learners*, as they adjust their expectations concerning the macroeconomic aggregates every period. Second, we consider agents with *time inconsistent* preferences. Third, we consider agents who consume their entire income instantaneously, typically referred to as *hand-to-mouth* agents in the literature. Finally, we also consider *financially illiterate* agents who do store wealth to smooth consumption into the old ages, but have no access to financial markets, hence their savings earn no interest.

3.1.1 Fully rational agents

Fully rational agents are the workhorse of the current macroeconomic literature. They are characterized by perfect foresight, rational expectations and internally consistent optimizing. Rational expectation assumption is a standard in contemporary economics mainly because rational expectations are model-consistent. This way of modeling expectations was introduced by Muth (1961) and later popularized by Lucas (1972), and others. Under rational expectations, the agents use all the available information, thus their perception of prices and tax rates in the future does not differ systematically from the equilibrium outcomes.

The fully rational agents find optimum consumption and leisure path solving the following problem:

$$\max_{\{c_{j,m,t},l_{j,m,t},a_{j,m,t}\}_{j=1}^{J}} U_{j,m,t} = u(c_{j,m,t}, l_{j,m,t}) + \sum_{s=1}^{J-j} \delta^s \frac{\pi_{j+s,t+s}}{\pi_{j,t}} u(c_{j+s,m,t+s}, l_{j+s,m,t+s}) \tag{3.2}$$

subject to:

$$a_{j,m,t} - a_{j-1,m,t-1} = \mathcal{I}_{j,m,t} + \mathcal{K}_{j,m,t} + \Upsilon - \mathcal{C}_{j,m,t}. \tag{3.3}$$

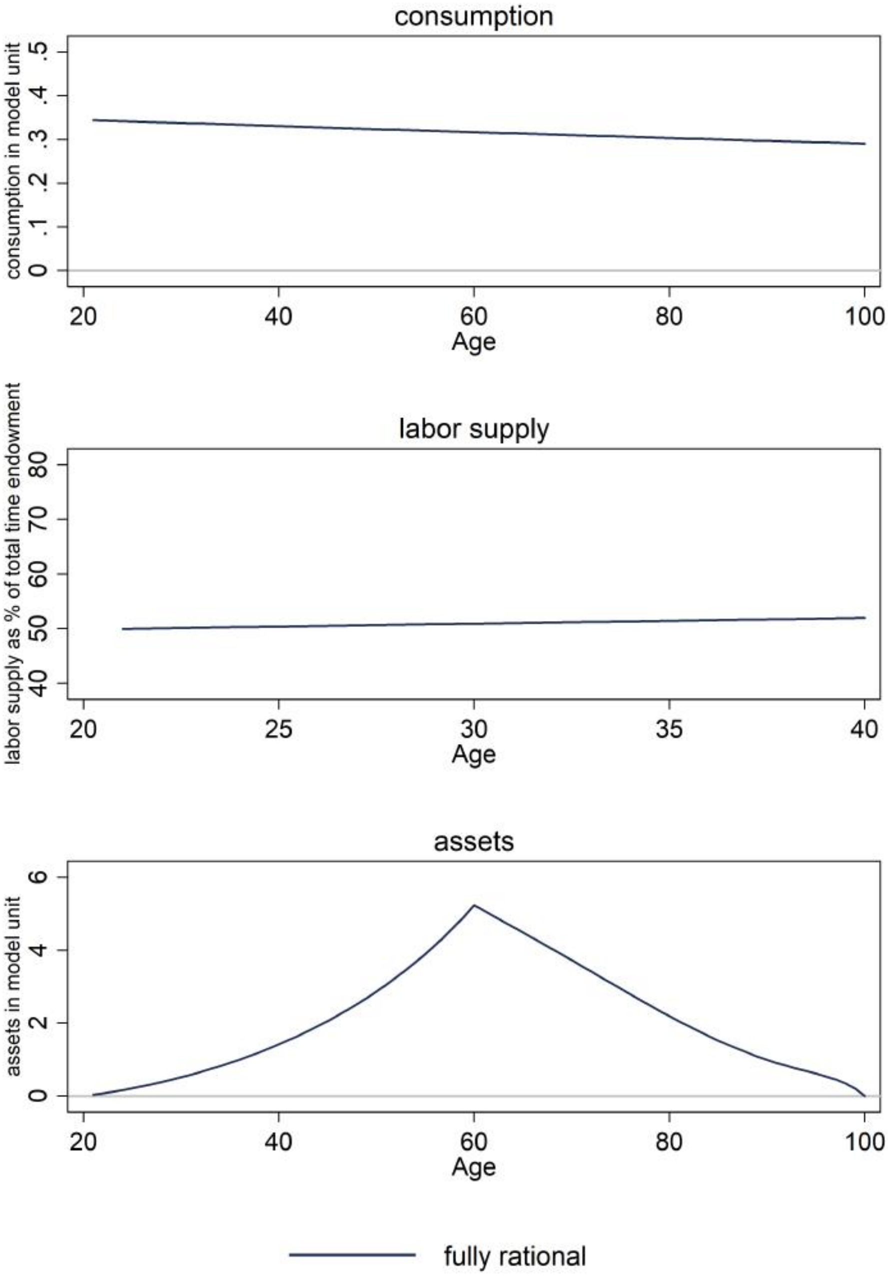
where index $m = \{\text{fully rational agents}\}$ and δ signifies exponential time discounting parameter, and with instantaneous utility function $u(c_{j,m,t}, l_{j,m,t})$ given by (3.1).

Fully rational agents, make labor supply (intra-temporal) and savings (inter-temporal) choices such that their lifetime consumption profile is smooth and possibly equal in present value terms (i.e. adjusting not only for time preference, but also for life expectancy), see Figure 3.1. Agents expecting to have zero earned income during retirement, will accumulate assets in order to supplement old-age consumption financed through pension benefits. In an extreme case, where pension benefits are equal in present value terms to the earned income (a 100% replacement rate in net present terms), there will be virtually no need for assets accumulation, as consumption would be smooth.

With preference for flat consumption profile in present value terms, the instantaneous savings flows are a consequence of the lifetime plan for accumulating assets up to a maximum just prior to retirement. These premises yield a bell-shaped assets accumulation lifetime pattern, with agents reaching maximum wealth in the year of retirement and gradually de-accumulating in the subsequent years. The exact shape of the bell – its curvature – depends on the relationship between the interest rate in the economy, the time preference and the life expectancy. Agents expecting higher survival probabilities after retirement – a process referred to as longevity – will accumulate higher wealth at retirement, *ceteris paribus*. Likewise, agents expecting pension benefits to decline, will increase instantaneous savings flows, *ceteris paribus*.

The main criticism towards modeling agents as fully rational is that this assumption implicitly requires the agents to gather enormous amount of information and to actually posses the ability to process it. Naturally, in reality people have only limited access to information (or gathering information is costly) as well as only limited ability to derive conclusions. We build on these

Figure 3.1: Fully rational agents: lifetime profiles for consumption, labor supply and assets



Note: The above lifetime patterns for consumption, labor supply and assets are obtained with the following parametrization: $\alpha = 0.33$, $\delta = 0.9717$, $\phi = 0.456$, $r = 0.065$, $\tau^c = 0.2291$, $\tau^k = 0.19$, $\tau^l = 0.06725$, $\tau = 0.07$, and mortality according to the current mortality rates published by the Central Statistical Office. The details of the final calibration for the purposes of this study are described in section 4.

intuitions about limitations to complete rationality in subsequent sections. Specifically, fully rational agents exhibit very strong reaction to longevity and immediate reaction to future changes in pension and tax system parameters. These strong reactions tend to be at odds with empirical evidence. For example, studying the reaction of the Polish population to a substantial decline in pension wealth, Lachowska and Myck (2015) show that roughly 13% of the sample adjusted consumption in the directions and magnitudes predicted by the theoretical optimization of the fully rational agents. This discrepancy between actual reactions and theory premises is our primary motivation to study incomplete rationality behavioral patterns in a general equilibrium framework.

3.1.2 Adaptive learners

The empirical macroeconomic literature identifies many phenomena, which are inconsistent with complete rationality. One of the most dominant explanations for these phenomena relies on the cost of updating expectations for the agents: it is rational to only update lifetime choices if reality deviates from the previous situation by a sufficient amount. This phenomenon is referred to as rational inattention (see for example Mankiw and Reis, 2002; Sims, 2003; Mackowiak and Wiederholt, 2009; Mackowiak and Wiederholt, 2015; Caplin and Dean, 2015). Agents with such preferences update their lifetime patterns, when reality "surprises" them (enough), but continue to uphold their expectations until that occurs. In a world with uncertainty, the agents may form their expectations through observing past e.g. regressing on the past data, as the changes in prices and taxes are driven by longevity and unanticipated policy change (see Stahl, 1996; Evans and Honkapohja, 1999; Milani, 2007, for a discussion of different treatment of adaptive learning). In a deterministic setup, such as ours, knowing model parameters and *current* exogenous macroeconomic aggregates, the agents may form expectations about future evolution of prices and taxes, choosing lifetime paths for labor supply, assets and consumption accordingly, by simply assuming that each period is actually a steady state. With a change in macroeconomic aggregates, their previous choices may be suboptimal, necessitating adjustment in lifetime paths.

In our setup, the adaptive learners are rational and thus they solve an identical problem as fully rational agents. However, every period they expect tax rates, wages, interest rates and survival probabilities (and hence also pension benefits) to remain unchanged relative to the current period. Their solution to the consumer problem is identical in the steady state to the solution of the fully rational agents, but along the transition path, they readjust every period, when observing changed prices, tax rates and survival probabilities. Note that since macroeconomy is not uncertain per se, there would be no benefit from e.g. Bayesian learning. Moreover, in the steady states, adaptive learners behave the same way as fully rational agents, hence lifetime profiles for consumption, labor supply and assets are identical as in Figure 3.1. The main difference between fully rational agents and adaptive learners arises when the economy enters a transition, altering exogenous variables and/or macroeconomic aggregates. In such case, fully rational agents adjust immediately, whereas adaptive learners learn about the new equilibrium gradually observing new prices and taxes. This operationalization has also been adopted by Cottle Hunt (2019).[1]

3.1.3 Time inconsistent agents

Time inconsistency has been at the core of behavioral economics since Strotz (1955). In colloquial terms, this phenomenon has been used to model procrastination (belief that in the future one will improve on ones behavior) or myopia (excessively strong discounting of the future) and may be related to a variety of behavioral and cognitive failures of human brains (for review see Ainslie, 1992; Wilson and Gilbert, 2003; Wilson and Gilbert, 2005). Time inconsistency is typically modeled in economics as quasi-hyperbolic discounting, following the formalization by Laibson (1997) and Laibson (1998).

The role of pension systems if agents display time inconsistent preferences seems particularly appealing: *when old* the agents would like to have their consumption smoothed, but do not do so when *optimizing at young age* due to discounting the future too strongly. This intuition was formalized by Feldstein (1985) in a two-period overlapping generations economy with inelastic labor supply and no uncertainty: myopic agents under-save for old age, hence a mandatory pension system can improve welfare, because it provides a commitment device.

However, in the setup by Feldstein (1985), the agents do not decide about labor supply, nor assets. Hence, they cannot react to the features of the pension system at all. In a full-sized computational application with income uncertainty and perfectly elastic labor, İmrohoroğlu et al. (2003) show, that agents with sufficient degree of time inconsistency do not find it optimal to participate in the pension system at all, even if it provides actuarial reward for survivors (i.e. an additional return on top of the equilibrium interest rate). They also show that in the world populated by time inconsistent agents, pension systems provide replacement for a perfect commitment device, but with strong negative externalities in the form of reduced capital stock (and thus smaller economy). In fact, the general equilibrium effects coming from reduced capital stock outweigh the individual gains from more smooth consumption over lifetime.

The time inconsistent agents solve the following problem:

$$\max_{\{c_{j,m,t},l_{j,m,t},a_{j,m,t}\}_{j=1}^{J}} U_{j,m,t} = u(c_{j,m,t},l_{j,m,t}) + \beta \sum_{s=1}^{J-j} \delta^s \frac{\pi_{j+s,t+s}}{\pi_{j,t}} u(c_{j+s,m,t+s},l_{j+s,m,t+s}) \tag{3.4}$$

[1] Adaptive learners presented in this work are equivalent to the Life-cycle Horizon Learning model in the taxonomy of (Cottle Hunt, 2019) with the constant gain parameter $\gamma = 1$.

subject to equation (3.3), where index $m = \{$time inconsistent agents$\}$, and with instantaneous utility function $u(c_{j,m,t}, l_{j,m,t})$ given by (3.1). In this notation β denotes additional discounting that the agents apply to all future periods. Time inconsistency stems from the fact that agents revise their plans every period. Namely, the agents plan their future as if from in the next period onward they would behave as time-consistent fully rational agents. However, when deciding in the next period, they still apply β discounting to all future periods.

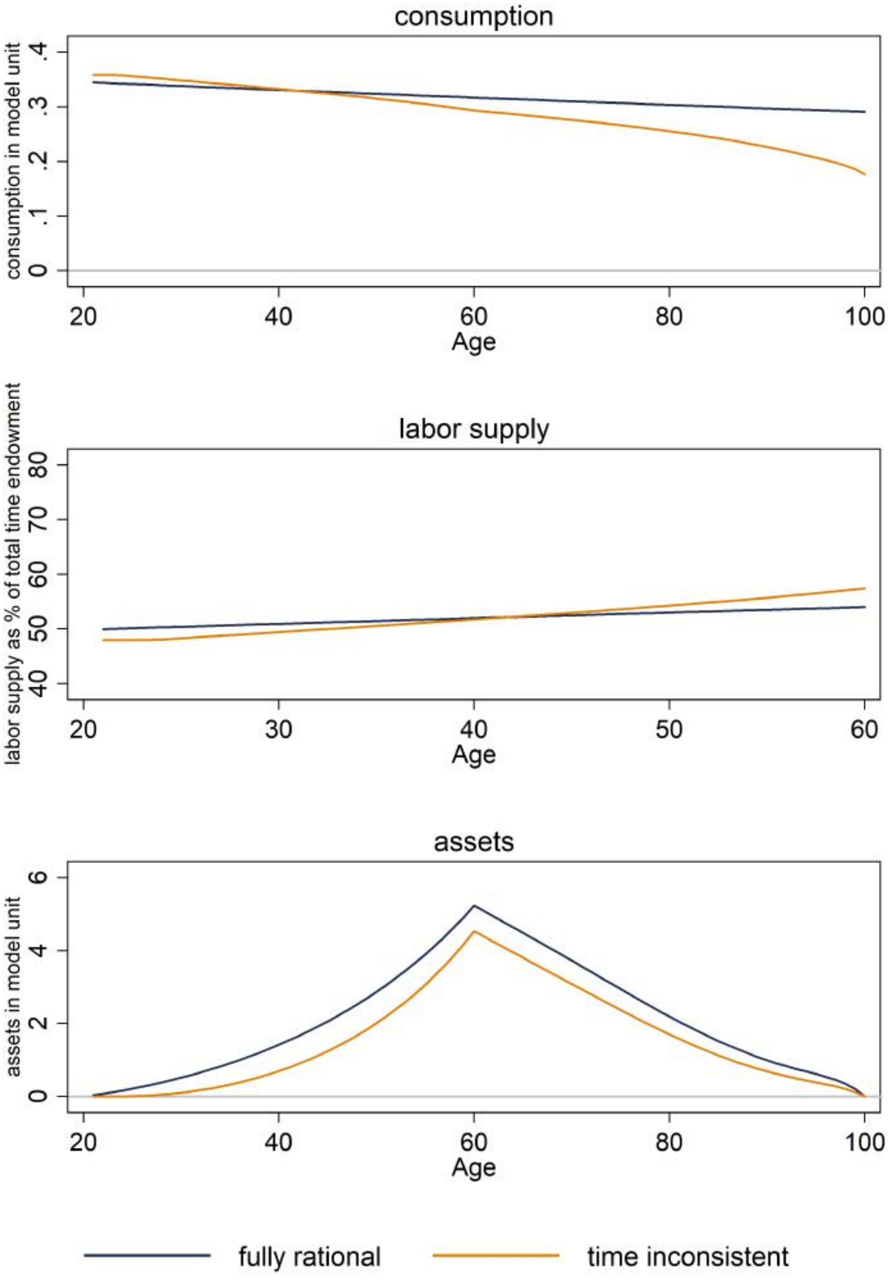
Figure 3.2 portrays the life-time profiles of consumption, labor supply and assets for time inconsistent agents. In the interest of comparability with fully rational agents portrayed in Figure 3.1, we recalibrate the δ parameter. We set $\beta = 0.95$ and adjust δ such that $\beta * \delta = 0.97$ as in other calibrations. This illustration of life-time choices applies only in the current section, in the actual simulations in section 5, we maintain $\beta = 0.95$, but calibrate aggregate δ common to all types of agents to match macroeconomic aggregates, as described in section 4. Thanks to this recalibration of time-inconsistent agents for the illustrative purposes, we maintain the same discounting between t and $t+1$, and thus time-inconsistent agents differ from fully rational agents only in how they discount in all the future periods ($t + s$ to $t + s + 1$).

Time inconsistent agents generally chose higher level of consumption when young, which forces them to raise labor supply with age and despite this adjustment in earned income, implies lower consumption at retirement. Lower stock of accumulated wealth due to slower accumulation is the main mechanism behind substantially lower old-age consumption. Note also, unlike fully rational agents, the time inconsistent agents are at the no-borrowing constraint in the early working periods of their life. In practice, they start accumulating assets much later than fully rational agents and do it at a slower pace. Without a no-borrowing constraint, time-inconsistent agents would incur debt when young, further raising consumption and reducing labor supply in the young ages. Time inconsistency manifests itself the strongest at $j = 41$, i.e. upon retirement: life-cycle consumption is inflecting as the agents incomes are reduced due to switching from earned labor income to pension benefit. Due to lower assets and higher effective impatience, the time-inconsistent agents reduce assets (and thus consumption) at a faster rate than fully rational agents.

3.1.4 Hand to mouth agents

It is well established in empirical literature that a substantial fraction of each generation holds no assets: be it financial or illiquid (e.g. real estate, see Christelis et al., 2013; Mian et al., 2013, for Europe and the US, respectively). The phenomenon of households who consume total current income and do not smooth consumption over shocks or into the old ages has been introduced into macroeconomic modeling by Weil (1992). Subsequent contributions have studied the role of

Figure 3.2: Time inconsistent agents: lifetime profiles for consumption, labor supply and assets



Note: The above lifetime patterns for consumption, labor supply and assets are obtained with the following parametrization: $\alpha = 0.33$, $\delta = 0.9717$, $\beta = 0.95$, $\phi = 0.456$, $r = 0.065$, $\tau^c = 0.2291$, $\tau^k = 0.19$, $\tau^l = 0.06725$, $\tau = 0.07$, and mortality according to the current mortality rates published by the Central Statistical Office. The details of the final calibration for the purposes of this study are described in section 4.

short-term credit (e.g. Parker, 2017), instantaneous wealth (e.g. Kaplan et al., 2014; Olafsson and Pagel, 2018; Heathcote and Perri, 2018) as well as the context of financial literacy (e.g. Lusardi et al., 2017).[2] The literature has firmly established that some individuals tend to immediately dispose of income, even if their current stream of e.g. labor revenue permits sufficient liquidity for regular instantaneous consumption and robust savings rate. This result holds even after adjusting for observable health and labor market shocks.

We introduce HTM features through the budget constraint, since our setup already features agents with high discounting of the future. Hence, the HTM agents solve the following problem:

$$\max_{\{c_{m,j,t},l_{m,j,t},a_{m,j,t}\}_{j=1}^{J}} U_{j,m,t} = u(c_{j,m,t}, l_{j,m,t}) + \sum_{s=1}^{J-j} \delta^s \frac{\pi_{j+s,t+s}}{\pi_{j,t}} u(c_{j+s,m,t+s}, l_{j+s,m,t+s}) \tag{3.5}$$

subject to:

$$0 = \mathcal{I}_{j,m,t} + \Upsilon - \mathcal{C}_{j,m,t}. \tag{3.6}$$

where index $m = \{\text{hand to mouth agents}\}$, and with instantaneous utility function $u(c_{j,m,t}, l_{j,m,t})$ given by (3.1). HTM agents do not store assets.

Hand to mouth agents consume their whole disposable income both when working and when retired, see Figure 3.3. When retired they consume only to the level of pension benefits. HTM agents have a simplified problem to solve during the working period as well: absent inter-temporal choice, they essentially solve intra-temporal problem only, i.e. they match the marginal utility from leisure with the the marginal utility from consumption. Solution to this intra-temporal problem yields a flat labor supply profile throughout the whole life-cycle. On the transition path, with varying relative price of leisure, the labor supply of HTM agents stops being flat.

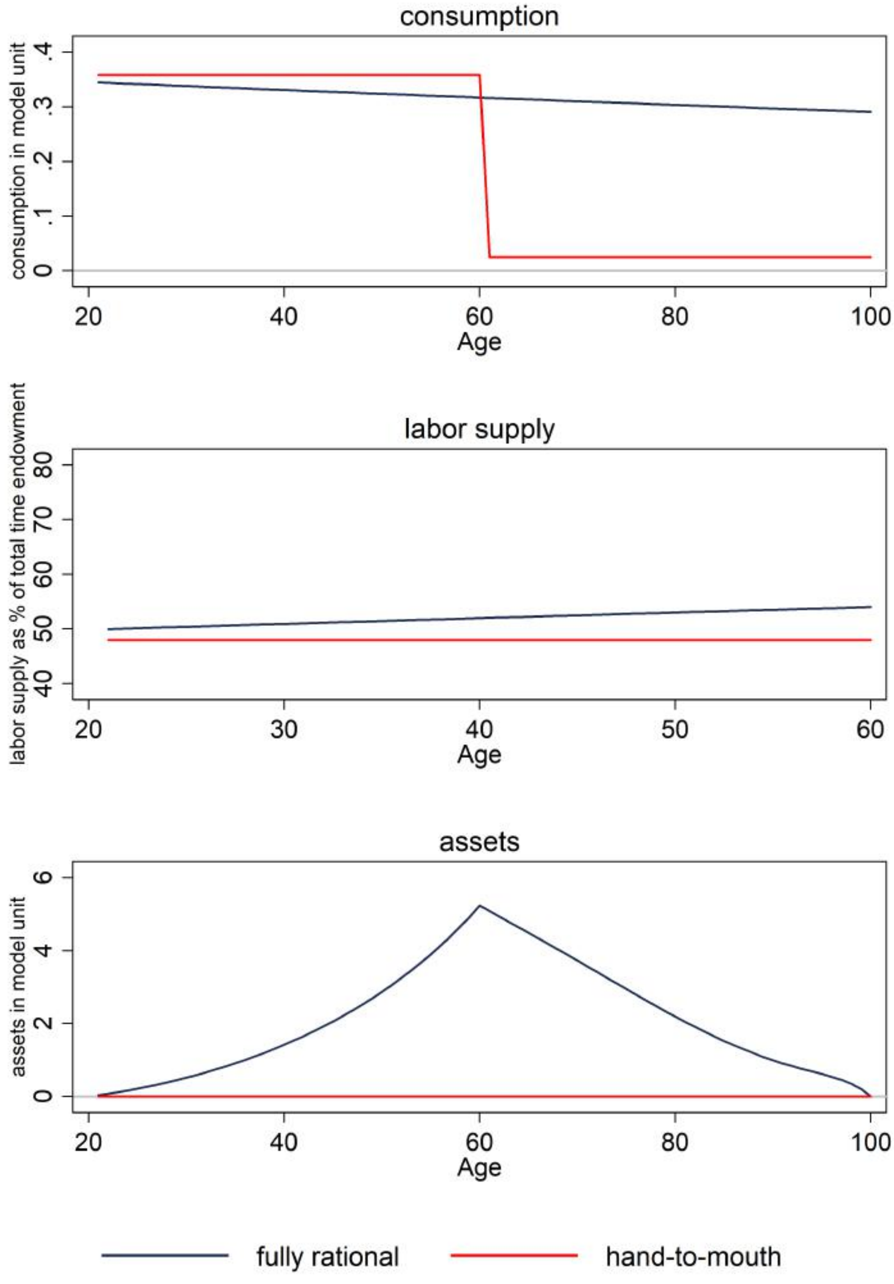
3.1.5 Financially illiterate agents

Hand-to-mouth or rule-of-thumb behavior is not the only one to be consistent with negligible consumption smoothing. Extant literature documents on the role of insufficient financial literacy around the world (Xu and Zia, 2012; Klapper et al., 2015). Typically, financial literacy is diagnosed through a set of relatively simple questions, testing an ability to compound interest and turn nominal values into real terms.[3] The subjects are not even expected to give actual figures: they are expected to identify the ball park of the correct answer (Lusardi, 2012; Lusardi and Mitchell,

[2] In macroeconomic literature, hand-to-mouth agents (or rule-of-thumb) agents have been used to study for example the monetary transmission channels (Colciago, 2011; Auclert, 2019), fiscal policy (Kaplan and Violante, 2014; Rossi, 2014; House et al., 2019) and business cycle (De Giorgi and Gambetti, 2017).

[3] The questionnaire has been proposed by Olivia Mitchell and Annamaria Lusardi, it is used in longitudinal and cross-country context. For a recent overview, see Lusardi and Mitchell (2014) and Lusardi (2019).

Figure 3.3: Hand-to-mouth agents: lifetime profiles for consumption, labor supply and assets



Note: The above lifetime patterns for consumption, labor supply and assets are obtained with the following parametrization: $\alpha = 0.33$, $\delta = 0.97$, $\phi = 0.454$, $r = 0.0675$, $\tau^c = 0.229$, $\tau^k = 0.19$, $\tau^l = 0.06725$, $\tau = 0.07$, and mortality according to the current mortality rates published by the Central Statistical Office. The details of the final calibration for the purposes of this study are described in section 4.

2014). Despite this admittedly low bar for qualifying into the financially literate group, roughly 30% of adults in the advanced economies reach the bar.

While the simple arithmetic of financial literacy underlies decisions about repaying credit card debt or leasing a car to the same extent as they underlie the life-time consumption smoothing, the former choices are driven by preferences which are much more heterogeneous and depend on various factors such as tastes (e.g. about the car or banking institutions). Meanwhile, the hypothesis of life-time consumption smoothing yields straight forward prediction: an agent who is not knowledgeable enough to acquire interest on income withheld from immediate consumption, cannot achieve equilibrium between giving up contemporaneous for future consumption (as captured by time preference) and the return on this trade off (as captured by the interest rate in the economy). In fact, inability to achieve interest on saving implies that the saving is solely motivated by the smoothing concerns.

Financially illiterate agents solve the following problem:

$$\max_{\{c_{j,m,t},l_{j,m,t},a_{j,m,t}\}_{j=1}^{J}} U_{j,m,t} = u(c_{j,m,t}, l_{j,m,t}) + \sum_{s=1}^{J-j} \delta^s \frac{\pi_{j+s,t+s}}{\pi_{j,t}} u(c_{j+s,m,t+s}, l_{j+s,m,t+s}) \tag{3.7}$$

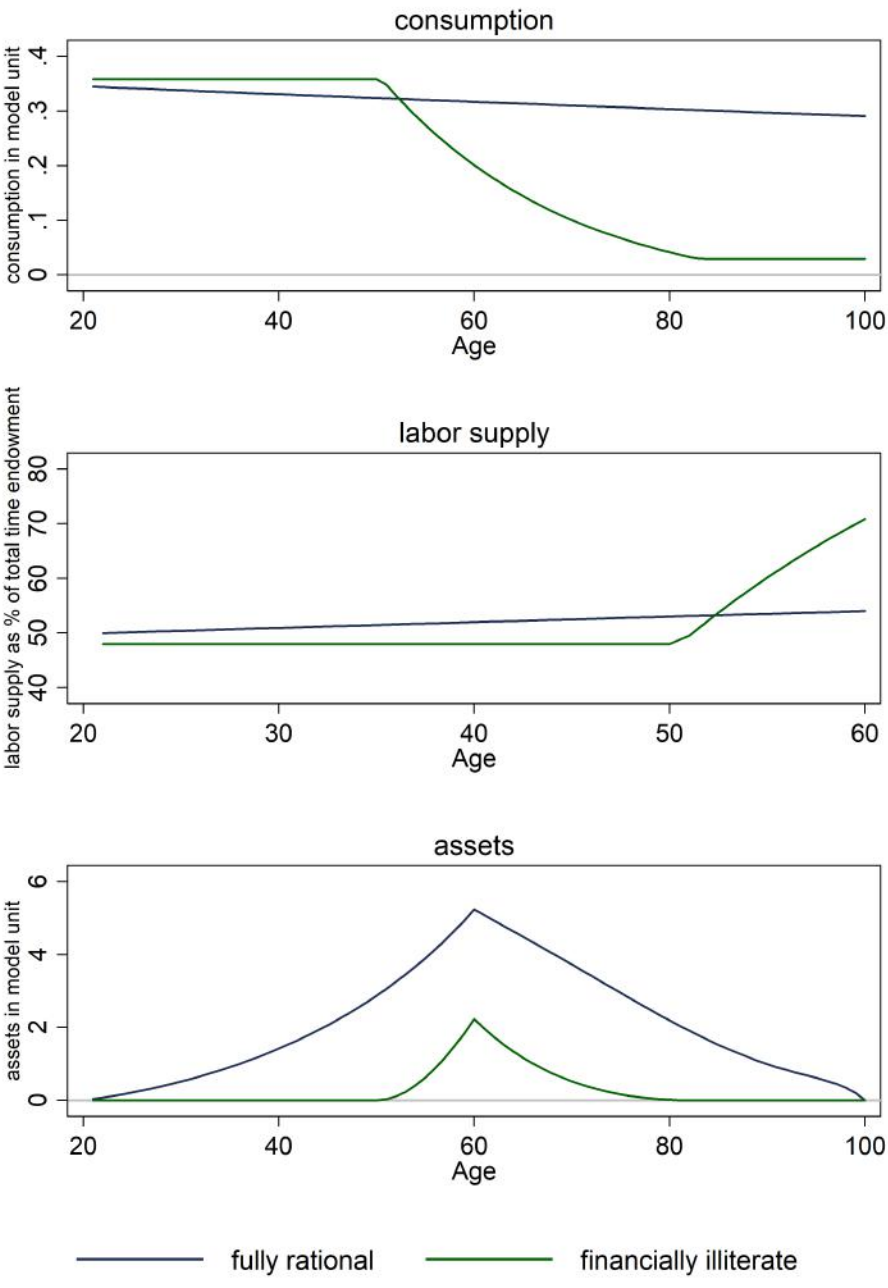
subject to:

$$a_{j,m,t} - a_{j-1,m,t-1} = \mathcal{I}_{j,m,t} + \Upsilon - \mathcal{C}_{j,m,t}. \tag{3.8}$$

where index $m = \{\text{financially illiterate agents}\}$, and with instantaneous utility function $u(c_{j,m,t}, l_{j,m,t})$ given by (3.1). HTM agents do not store assets. The financially illiterate agents can put funds aside (store assets), but do not receive capital income gains. Financially illiterate agents miss out on both parts of the interest rate: the part associated with capital productivity and the part associated with the annuity. This means that they are not insured against life-time uncertainty. As survival probability drops with age, financially illiterate agents simply leave aside less assets for the future in order to mitigate the risk of leaving large unintended bequest. In void of bequest motive, leaving any bequest is simply consumption lost.

Comparing financially illiterate agents to fully rational agents, shows that interest rate is a powerful driver of asset accumulation decisions. Financially illiterate agents postpone asset accumulation to the last few years of the working period and deaccumulate these assets relatively fast, effectively achieving only little consumption smoothing. In essence, they are able to smooth consumption more than HTM agents, but less than time-inconsistent agents (Figures 3.3 and 3.2, respectively). Lack of the consumption smoothing motive translates to relatively higher consumption in the young ages and sharp increase in labor supply when the agents start accumulating assets prior to the retirement. The pattern of labor supply adjustment is indeed similar between time-inconsistent

Figure 3.4: Financially illiterate agents: lifetime profiles for consumption, labor supply and assets



Note: The above lifetime patterns for consumption, labor supply and assets are obtained with the following parametrization: $\alpha = 0.33$, $\delta = 0.97$, $\phi = 0.454$, $r = 0.0675$, $\tau^c = 0.229$, $\tau^k = 0.19$, $\tau^l = 0.06725$, $\tau = 0.07$, and mortality according to the current mortality rates published by the Central Statistical Office. The details of the final calibration for the purposes of this study are described in section 4.

agents and financially illiterate agents, but the adjustment is stronger for the latter. The key reason behind this stronger reaction is the fact that financially illiterate agents delay asset accumulation much further in life, hence they need to raise incomes by much more in the working years prior to the retirement to achieve much lower extent of consumption smoothing.

3.1.6 Commonalities and differences across behavioral patterns

To sum up fully rational agents aim to have a slightly decreasing consumption throughout their life. The slope is set explicitly by their time discounting factor, δ. The adaptive learners do not anticipate any changes in the economy, e.g. they do not anticipate that they will "live longer then their grandparents", hence they accumulate assets slower than the fully rational agents. Time inconsistent agents discount the future more and in an internally inconsistent way: different discounting is applied between t and $t+1$, and between $t+1$ and any $t+1+s$. Hand to mouth are left to the generosity of the pension system in the old ages, as they accumulate no wealth. Financially illiterate agents store assets, but do not earn interest: they have no reward for postponing consumption, despite non-unitary time preference.

Our treatment of behavioral heterogeneity is of dual nature. First, we modify the preferences, introducing greater discounting of future with time inconsistent agents. Second, we modify the budget constraint, with inability to store assets (with hand to mouth agents) and with inability to achieve interest on accumulated assets (with financially illiterate agents). These departures from fully rational agents permit to identify specific components of incomplete rationality and their role for life-time optimization. In addition to this duality, we also introduce agents who act as fully rational agents, but are unable to acquire and process new data about the economy, assuming each next period is the same as a current one (i.e. permanent steady state). Thus, overall, we introduce three behavioral mechanisms, each of which addresses the concerns raised towards rationality assumption.

The departures from rationality assumption have been introduced into macroeconomic modeling in order to reconcile evidence from observational data with economic models. For example, assumptions such as liquidity constraints or informational frictions were not sufficient to generate in the models certain important features of the economy. Matching the share of households with negligible assets requires the macroeconomic models to include one or more features of incomplete rationality.

The observed improvement in model match in terms of assets behavior, comes at the expense, however. The labor supply life-time profiles growing by nearly a 100% in the final decade prior to the retirement (as is the case with time inconsistent agents and financially illiterate agents, recall Figures 3.2 and 3.4, respectively) is at odds with the data. In a similar spirit, there is ample empirical evidence that consumption declines upon retirement (e.g. Aguiar and Hurst, 2005; Aguila et al., 2011; Aguiar and Hurst, 2013, even adjusting for work-related expenses). Nonetheless, the declines from observational data rarely exceed 15%, whereas our profiles for HTM agents would show substantially larger declines under plausible replacement rates. This disparity between incompletely rational agents and the observational data does not need to be worrisome in terms of model properties, but should be taken into account when interpreting the model results with policy instruments.

An important commonality across the types of the agents is that the intra-temporal choice (consumption vs leisure) is governed for each type of agents by the same instantaneous utility function with the same set of parameters. This means that if there was no tomorrow agents of all types would make exactly the same choices. All the differences between the agents stem from the different preferences or abilities to smooth consumption over life-time.

Consequently, marginal rate of inter-temporal substitution provides a particularly useful and informative way to think about the commonalities and differences in the optimization problem of our agents. Denote by u_c the derivative of the utility function with respect to instantaneous consumption $u_{c,j,m,t} = \frac{\partial u(c_{j,m,t}, l_{j,m,t})}{\partial c_{j,m,t}}$. Recall, that the interest rate in our economy is fully annuitized, i.e. on accumulated assets in the current period period $a_{j,m,t}$ the agents receive in net terms:

$$\bar{r}_{j,t} = \mu_{j,t} \cdot r_t \tag{3.9}$$

where r_t follows from financial markets equilibrium and is defined in equations (3.10) and (3.17). Then, we obtain the summary of the Euler conditions as displayed in Table 3.1.

Another commonality across the agents is that a no borrowing constraint is imposed on all agents in all periods: $\forall_{j,t}\ a_{j,m,t} \geq 0$. For the HTM agents actually $\forall_{j,t}\ a_{j,m,t} = 0$ strictly holds, but for the other agents, all non-negative values are allowed. The no borrowing constraint is a standard assumption in the OLG literature, but in our setup it is particularly relevant. Consider a fully rational agents, who is offered to participate in an old-age saving scheme. If the scheme delivers the same interest rate as the economy overall, this instrument offers no particular gains and thus the fully rational agent is indifferent between this instrument and private voluntary saving in our economy. Note that the economy is fully annuitized. Now consider a government-subsidized old-age saving scheme. The subsidy makes the scheme more valuable than private voluntary savings, causing the fully rational agent to participate in the scheme and replace the latter with the former (crowding out). Now consider that in the government-subsidized scheme the contribution

Table 3.1: The Euler condition for agents in the model

Type of agent	MRS
Fully rational	$\frac{u_{c,j,m,t}}{u_{c,j+1,m,t+1}} \cdot \frac{1}{\delta} \cdot \frac{\pi_{j,t}}{\pi_{j+1,t+1}} = \mu_{j+1,t+1} + (1-\tau^k)\bar{r}_{j+1,t+1}$
Adaptive learner	$\frac{u_{c,j,m,t}}{u_{c,j+1,m,t+1}} \cdot \frac{1}{\delta} \cdot \frac{\pi_{j,t}}{\pi_{j+1,t}} = \mu_{j+1,t+1} + (1-\tau^k)\bar{r}_{j+1,t}$
Time inconsistent	$\frac{u_{c,j,m,t}}{u_{c,j+1,m,t+1}} \cdot \frac{1}{\beta\delta} \cdot \frac{\pi_{j,t}}{\pi_{j+1,t+1}} = \mu_{j+1,t+1} + (1-\tau^k)\bar{r}_{j+1,t+1}$
Hand-to-mouth	irrelevant, $\forall_j a_{j,m,t} = 0$
Financially illiterate	$\frac{u_{c,j,m,t}}{u_{c,j+1,m,t+1}} \cdot \frac{1}{\delta} \cdot \frac{\pi_{j,t}}{\pi_{j+1,t+1}} = 1$

Note: For time inconsistent agents this MRS holds only to describe optimal choice between t and $t+1$ at time t. The optimum for subsequent periods $t+s$ when evaluated at t is given by $\frac{u_{c,j,m,t}}{u_{c,j+s,m,t+s}} \cdot \frac{1}{\beta\delta^s} \frac{\pi_{j,t}}{\pi_{j+s,t+s}} = \prod_{i=1}^{s}(\mu_{j+s,t+s} + (1-\tau^k)\bar{r}_{j+s,t+s})$ and $s \in (1, J-j)$. For HTM agents, the standard Euler conditions holds, as it applies to their preferences for inter-temporal choice, whereas HTM is introduced through the budget constraint (no access to storing technology of any kind). Therefore, by construction HTM agents cannot act accordingly to the Euler condition, even though they would like to.

rate is above the net savings rate in some years of the fully rational agent's life-time (typically: the first few years of the life-time). If the agent could borrow funds in these years, at the rate equivalent to the interest rate in the economy, the agent would (a) certainly participate in the scheme for the whole life and (b) arbitrage borrowing at a lower rate than she may invest. Now consider an analogous case, but with a no borrowing constraint. The agent has to weigh lower consumption in the young ages against potential benefits of saving in the government-subsidized scheme. This second case is at the core of old-age poverty debate, whereas the first case is not likely to constitute a frequent situation in reality. For example, in most advanced economies, even if young households have mortgages against their real estate, it seldom ever happens that their net worth is actually negative.

The no borrowing constraint may impose restriction on preferred life-time consumption allocations, as portrayed in Table 3.1. The choices consistent with respective MRS conditions for each type of the agent may be outside the choice set with $\forall_{j,t}\ a_{j,m,t} \geq 0$ condition.

3.2 Production

Using capital and labor, the economy produces a composite consumption good. Production function takes a standard Cobb-Douglas form $Y_t = K_t^{\alpha}(z_t L_t)^{1-\alpha}$ with labor augmenting exogenous technological progress, $z_{t+1}/z_t = \gamma_t$. Capital depreciates at rate d. Standard maximization problem of the firm yields the return on capital and real wage:

$$w_t = (1-\alpha)K_t^{\alpha} z_t^{1-\alpha} L_t^{-\alpha} \quad \text{and} \quad r_t = \alpha K_t^{\alpha-1}(z_t L_t)^{1-\alpha} - d. \tag{3.10}$$

3.3 The pension system

There is a universal pay-as-you-go defined contribution pension system with a contribution rate τ. The contributions are used to finance the contemporaneous benefits. The contributions paid to the pension system are accumulated on individual accounts ($f_{j,m,t}$). Before retiring the accumulation follows the rule:

$$\forall_{j<\bar{J}} : f_{j,m,t} = (\mu_{j,t} + g_t) f_{j-1,m,t-1} + \tau w_t l_{j,m,t}, \tag{3.11}$$

where $g_t = \frac{w_t L_t}{w_{t-1} L_{t-1}}$ denotes the growth rate of the payroll in the economy. The notional value of the contributions is converted to an annuity at retirement ($j = \bar{J}$). The actual value of the old age pension benefit for a cohort retiring in period t is given by:

$$b_{\bar{J},t} = \frac{f_{\bar{J},t}}{LE_{\bar{J},t}} \quad \text{and} \quad \forall_{j>\bar{J}} \; b_{j,t} = (1+g_t) b_{j-1,t-1}, \tag{3.12}$$

where $LE_{\bar{J},t} = \sum_{s=0}^{J-\bar{J}} \frac{\pi_{\bar{J}+s,t+s}}{\pi_{\bar{J},t}}$ denotes life expectancy at retirement of cohort reaching $\bar{J}$ in year t. The budget constraint of the pension system is given by

$$\sum_{j=\bar{J}}^{J} N_{j,t} b_{j,t} = \tau_t w_t L_t + subsidy_t, \tag{3.13}$$

where $subsidy_t$ denotes the pension system deficit (negative in the case of actual surplus) which, if necessary, is financed by the government. The economy continues with this this system in the baseline and in the reform scenarios.

3.4 The government

There are four taxes: tax on labor income (τ^l), tax on capital income (τ^k), tax on consumption (τ_t^c) and a lump sum tax (Υ). Tax revenue jointly with a change in public debt D_t is used to finance spending on public goods and services (G_t), balance the pension system ($subsidy_t$), and service debt ($r_t D_{t-1}$), with $\Delta D_t \equiv D_t - D_{t-1}$. We assume that *per capita* public spending is growing at the rate of labor augmenting exogenous technological progress z_t.

$$G_t + subsidy_t + r_t D_{t-1} = T_t + \Delta D_t, \tag{3.14}$$

$$T_t = \tau^l(1-\tau)\mathcal{I}_t + \tau^k \mathcal{K}_t + \tau_t^c \mathcal{C}_t + \Upsilon \sum_{j=1}^{J} \sum_{m=1}^{M} N_{j,m,t}, \tag{3.15}$$

where $\mathcal{C}_t$, $\mathcal{K}_t$ and $\mathcal{I}_t$ denote, respectively, aggregate consumption, capital income and labor income. We set the initial debt D_t at par with the data to 55% of GDP. The final steady state debt to GDP ratio is the same as the initial one, to avoid welfare effects stemming from permanent change in public debt ratio.

In the initial steady state, we close the government budget with lump-sum tax (Υ_1) and set G_1, D_1 to match the government expenditures and debt to GDP ratios, as reflected by the national accounts. On the transition path we keep constant the debt/GDP ratio. The values of Υ_1 and G_1 set in the initial steady state are held fixed in *per capita* terms throughout the transition path in the baseline scenario. In order to keep government budget balanced every period on the transition path and in the final steady state we allow for consumption tax (τ_t^c) adjustments.

In the baseline scenario, the debt/GDP ratio is calibrated in the initial steady state to reflect the macro aggregates (more on the calibration process in section 4). This ratio is then held fixed across the transition path and in the final steady state. This means that in the baseline scenario the consumption tax in the final steady state may differ from the initial steady state, but public debt to GDP ratio is identical.

In the reform scenario, the public debt is held fixed in terms of amount of the debt, rather then in terms of debt/GDP ratio. In the reform scenario, the public debt amount for each period from baseline scenario is used. As a result to this in the reform scenario both the consumption tax and the debt/GDP ratio in the final steady state may differ from the initial steady state. This procedure allows us to isolate the impact of the government-subsidized old-age saving scheme.

3.5 Equilibrium and model solving

As is standard in the literature, we employ the notion of a competitive equilibrium.

Definition 1 *A competitive equilibrium is a sequence of:*
allocations for households $\{c_{j,m,t}, a_{j,m,t}, f_{j,m,t}, l_{j,m,t})_{j\in(21,\dots,J),m\in(1,\dots,M)}\}_{t=1}^{\infty}$, *prices* $\{r_t, w_t\}_{t=1}^{\infty}$, *government policies* $\{\tau_t^c, \tau^l, \tau^k, \Upsilon_t, D_t\}_{t=1}^{\infty}$, *pension system characteristics* $\{\tau, subsidy_t,\}_{t=1}^{\infty}$, *aggregate quantities* $\{L_t, A_t, K_t, C_t, Y_t\}_{t=1}^{\infty}$ *such that:*

- ***consumer problem:*** *for each* m, j *and* t *the allocation for households* $\{c_{j,m,t}, a_{j,m,t}, f_{j,m,t}, l_{j,m,t}\}$ *solve the consumer problem described in equations 3.1 through 3.8, given set of prices;*
- ***firm problem:*** *for each* t*, given prices* (r_t, w_t)*, the aggregates* (K_t, L_t, Y_t) *solve the representative firm problem, satisfying equation* (3.10)*;*
- ***government sector:*** *the government budget and the PAYG pension system are balanced, i.e. equations* (3.13)*,*(3.14) *and equation* (3.15) *are satisfied;*
- ***markets clear:***

$$labor : L_t = \sum_{j=1}^{\bar{J}} \sum_{m=1}^{M} l_{j,m,t} N_{j,m,t} \tag{3.16}$$

$$capital : A_t = \sum_{j=1}^{\bar{J}} \sum_{m=1}^{M} a_{j,m,t} N_{j,m,t} \quad \textit{and} \quad K_{t+1} = A_t + D_t \tag{3.17}$$

$$goods : C_t = \sum_{j=1}^{\bar{J}} \sum_{m=1}^{M} c_{j,m,t} N_{j,m,t} \quad \textit{and} \quad Y_t = C_t + K_{t+1} - (1-d)K_t + G_t \tag{3.18}$$

We solve the consumer problem with value functions iterations. Once the consumer problem is solved for a given set of prices and taxes, we apply the Gauss-Seidel algorithm to obtain the general equilibrium. Using the outcome of the consumer problem, the value of aggregate capital is updated. The procedure is repeated until the difference between the aggregate capital from subsequent iterations is negligible, i.e. l_1-norm of the difference between capital vector in subsequent iterations falls below 10^{-12}. Once the algorithm converges, utilities at $j = 1$ for all generations are computed.

Agents participate in the instrument endogenously, i.e. they participate in the instrument if their utility rises. Agents are free to join the instrument at any age $j \in [1, \bar{J} - 1]$, but once they join, participation is constant until retiring at $\bar{J}$. Hence, at the moment of decision, the agents compare $\bar{J} - j$ subsequent life-cycle paths: one for baseline of not joining and the remaining paths for joining at subsequent ages from the current age to $\bar{J} - 1$.

This process is iterative in a sense that depending on the share of agents participating in the instrument, the general equilibrium effects are different, which could affect the welfare maximization problem for the agents. For each iteration (participation choices), agents re-optimize their lifetime choices. We use a fixed point method, i.e. we assume the economy is in equilibrium if agents do not change decision about participation between subsequent iterations.

3.6 Policy reforms

Among the OECD members, all have introduced some form of government-subsidized old-age saving scheme (OECD, 2018). The subsidy typically concerns an exempt on capital income gains taxation. These instruments are capped, i.e. legislation imposes a limit on assets that can be saved (flow, e.g. annually) or held (stock, e.g. a life-time maximum) and be subject to tax exemption. The other savings, even if held with the sole purpose of subsidizing old-age consumption, are not exempt from capital income taxation. Typically, these schemes are voluntary.

We replicate these features introducing a government-subsidized old-age savings instrument in the reform scenario. It has three main features. First, the instrument provides full exemption from capital gains tax (τ^k). Second, the participation is voluntary: the agents endogenously choose the age at which they choose to participate and in principle they may prefer not to enter the instrument until $\bar{J}$. Third, the instrument is capped and the participation is binary: when deciding about participation, the agents consider only the contribution rate of $\tau^{ref} = 0\%$ (no participation) and $\tau^{ref} = 3.5\%$ (full participation). The agents are not allowed to participate partially at any age, but may choose at which age to join.

This instrument is introduced unexpectedly as of period 2, i.e. the first period of transition path. At retirement, the assets accumulated in this government-subsidized old-age saving scheme are converted to an annuity (and subject to general taxation). Hence, participating in the instrument will result in lower labor income when working ($j < \bar{J}$): $\mathcal{I}_{j,m,t} = (1 - \tau^l - \tau - \tau^{ref})w_t l_{j,m,t}$ and a higher pension income for retired agents ($j \geq \bar{J}$): $\mathcal{I}_{j,m,t} = (1 - \tau^l)(b_{j,m,t} + b^{ref}_{j,m,t})$.

The government-subsidized old-age saving scheme is a defined contribution one. The contributions are set aside to a funded account ($f^{ref}_{j,m,t}$) which earns the tax exempt market interest rate:

$$\forall_{j<\bar{J}} : f^{ref}_{j,m,t} = (\mu_{j,t} + \bar{r}_{j,t}) f^{ref}_{j-1,m,t-1} + \tau^{ref} w_t l_{j,m,t}, \tag{3.19}$$

and upon retiring the assets accumulated in the accounts are annuitized:

$$b^{ref}_{\bar{J},m,t} = \frac{f^{ref}_{\bar{J},m,t}}{LE_{\bar{J},t}} \quad \text{and} \quad \forall_{j>\bar{J}} \;\; b^{ref}_{j,m,t} = (1 + r_t) b^{ref}_{j-1,t-1}, \tag{3.20}$$

Observe that equations (3.19) and (3.20) are analogous to equations (3.11) and (3.12), respectively, as both the mandatory pay-as-you-go system and the voluntary capital system are of defined contribution nature. There are two main differences between them. The first difference concerns the contribution rates (τ in the pay-as-you-go system and τ^{ref} in the government-subsidized old-age saving scheme). Second, the pay-as-you-go system only provides indexation of "funds" (at

g_t), whereas the capital system provides interest at effectively the market interest rate. Both are fully annuitized, i.e. survivors inherit contributions of those agents, who did not survive until the next period. When working ($j < \bar{J}$) as well as when retired ($j > \bar{J}$) the scheme accrues the gross interest rate, as it is exempt from capital income taxation. The agents who may not participate in the government-subsidized old-age savings instrument at all, because they retire at the moment of the reform were born in year $t - \bar{J} - 21$. The first birth cohort of agents who can participate in the government-subsidized old-age savings instrument for the entire working period was born in year $t - 21$. The birth cohorts between these two boundaries are transition cohorts, i.e. they may endogenously choose to participate, but their working period during the reform treatment is below $\bar{J}$.

3.7 Measuring welfare effects

In the model, government subsidized savings instruments are introduced as of period 2, i.e. on a transition path, unexpectedly. We assume that the current pension benefit recipients cannot participate in the instrument, but agents of all other birth cohorts have endogenous choice of participation. In order to make this decision, the agents compute welfare of the status quo and welfare of the scenario when they participate. In a nutshell, they obtain U from equation (3.1) with and without instrument. For illustrative purposes, we present the the welfare gains in terms of consumption equivalent of lifetime consumption of the agent is measured as:

$$W_{m,t} = 1 - \exp\left(\frac{V_{m,t} - V_{m,t}^r}{\sum_{s=0}^{J} \delta^s \frac{\pi_{j+s,t}}{\pi_{j,t}}}\right), \tag{3.21}$$

where $V_{j,m,t}$ optimized value function (i.e. maximum utility) in status quo scenario and $V_{j,m,t}^r$ is the analog in case the change in question was implemented. Note that $V_{j,m,t}^r$ optimizes not only over the life-path for consumption, leisure and wealth, but also on the decision to participate in the government-subsidized old-age saving instrument and at which age. Note that $W_{m,t}$ does not require to be indexed by j, as this measure is summed over lifetime and discounted to $j = 1$ for each birth cohort (indexed by t in this notation).

4 Calibration

This chapter describes the calibration of our model. While overlapping generations models are not estimated from the data, the deep parameters of the model are calibrated in order to replicate the features of an economy, as reported in observational data. We report on the available demographic data, which provides the inputs for the population structure in our model. We then move to describing which macroeconomic aggregates our model structure describes. Finally, we report systematically on the literature identifying households behaving consistently with various types of incompletely rational preferences.

4.1 Demographic processes

The early empirical literature using overlapping generations models relied on simplified assumptions about population structure and size. Typically, the models assumed uniform survival probability and subsequent cohorts of equal size (Samuelson, 1958; Diamond, 1965) even in the large scale computational models (Auerbach and Kotlikoff, 1987; Galor, 1992). Further developments featured a growing but still stationary population (Willis, 1979; Michel and Pestieau, 1993),[1] but it was only when longevity became a relevant policy problems that the full demographic projections entered the overlapping generations model (Zhang et al., 2001; Razin et al., 2002; Hassler et al., 2003; Ignacio Conde-Ruiz and Profeta, 2007; Gonzalez-Eiras and Niepelt, 2008). On the one hand, assuming uniform survival probability allows the problem to be analytically tractable and thus permits broader scope of the theoretical intuitions (i.e. without a large scale computational model). On the other hand, however, this assumption becomes increasingly at odds with the data with mortality dynamics changing with subsequent cohorts: survival probabilities are nearly 100% until the end of the prime age and continue to be high until retirement across developed countries. Moreover, longevity implies that the age of increasing mortality occurs at later ages. Given this discrepancy between the simplified and actual mortality patterns, one cannot expect the model to replicate the observed economic patterns if demographics is not modeled in line with the observational data.[2]

[1] Modelling non-stationary populations, i.e. population structures with changing shares of cohorts remains rare until today.

[2] Whether or not an average person accurately estimates survival probabilities and longevity remains an open debate. In general, people appear to be excessively optimistic about their survival before reaching the retirement age (Weinstein, 1980), but expect to live shorter than statistically probable after retirement (O'Dea and Sturrock, 2018; O'Dea and Sturrock, 2019).

We use the detailed demographic projection released by the Aging Work Group (AWG) of the European Commission Commission (2018) to reproduce the arrival of new cohorts to the economy as well as annual survival probabilities for each cohort. The projection is available until 2080. Subsequently, we assume the mortality rates and births constant and equal to each other. Demography projection does not change between baseline and reform scenario, therefore those basic dynamics are present in both scenarios.

Figure 4.1 visualizes the demographic processes in our model economy. Longevity implemented in the model is depicted on the two top panels of Figure 4.1. According to the demographic projection used the mortality curve shifts every year to the right. On 2080 the demographic projection is over and we take a conservative that the longevity remains with no further changes. Projected changes in fertility are pictured on the bottom left panel of Figure 4.1. Fertility is defined as a number of 21 year old agents ($j = 1$) entering the model. Same as with mortality we make a conservative assumption that from 2080 forward the fertility will experience no further changes.

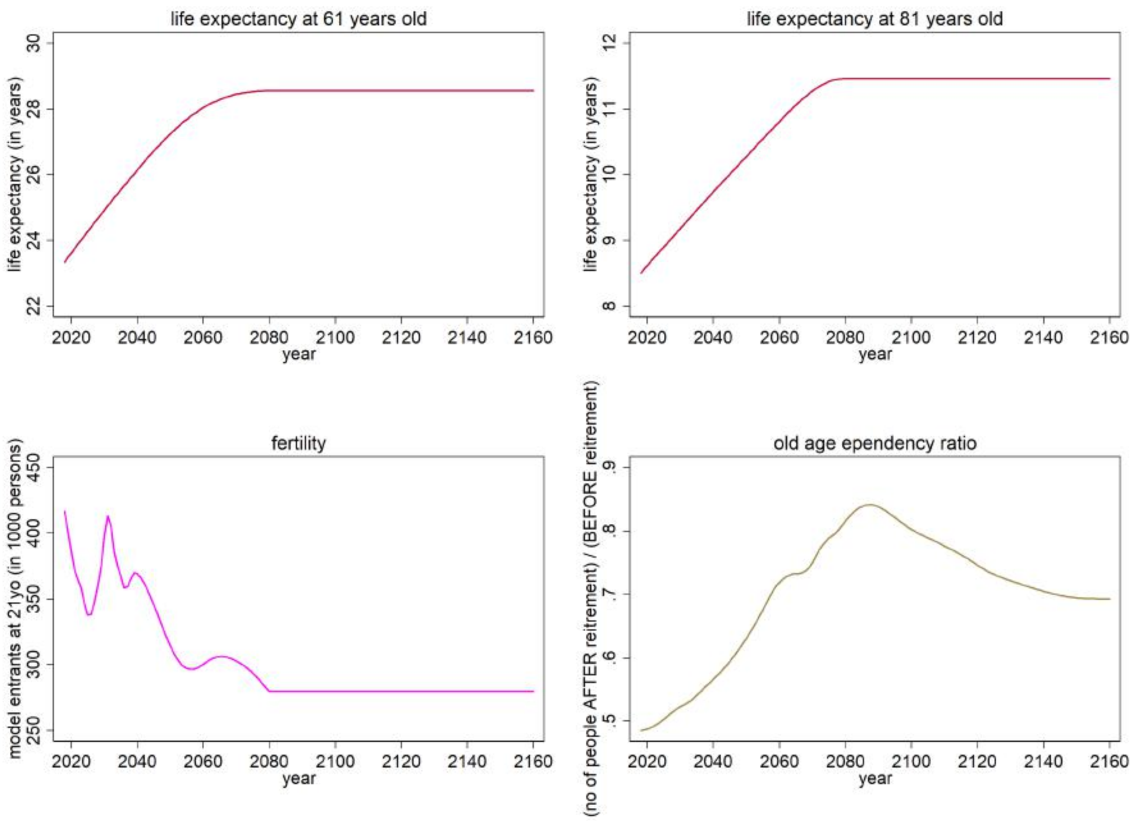
The joint dynamics of decreasing fertility and increasing longevity result in growing old-age dependency ratio. In this work we define old-age dependency ratio as a ratio of retired to working age population ($\frac{\sum_{j=\bar{J}}^{J}\sum_{m=1}^{M}N_{j,m,t}}{\sum_{j=1}^{\bar{J}-1}\sum_{m=1}^{M}N_{j,m,t}}$). Old-age dependency ratio will double around year 2090 in comparison with year 2018 (initial steady state). Note that the demographic projection of longevity is available only until 2080. We assume that $\pi_{j,t}$ stabilize henceforth, with steady inflow of new generations assuring population replacement. Hence, the longevity eventually fades out in our population structure with stationary population (equal share for every age group) in subsequent years. The stationary population is a necessary assumption for our derivation of the steady state conditions, and is a technical assumption, i.e. it does not reflect our beliefs about future evolution of the population. It is also irrelevant for the transition path analyses, as it only affects choices in the final steady state. In the final steady state, the old-age dependency ratio is substantially higher than in the initial steady state even with our conservative assumption about population structure: it rises by 50%.

4.2 Macroeconomic aggregates

4.2.1 Technological progress

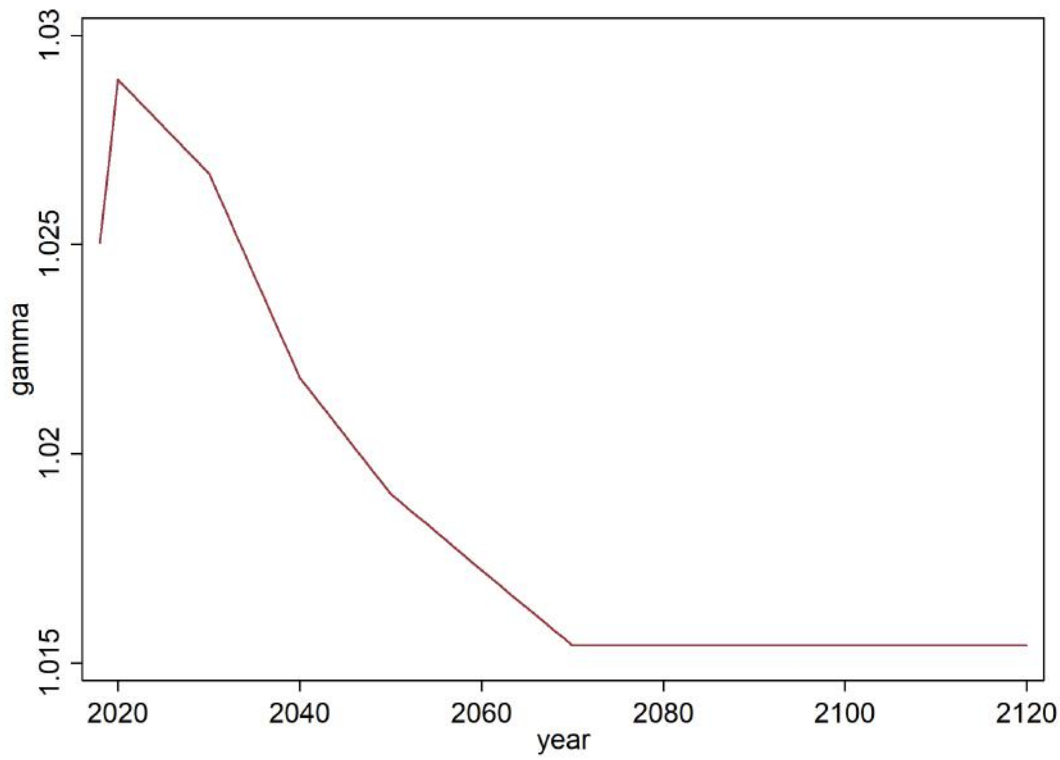
Our model features exogenous technological progress. Such setup allows capital accumulation to translate to higher output, but does not permit a feedback between capital accumulation and economic efficiency. Models with endogenous technological progress in an overlapping generations framework exist (Bertola, 1996; Yakita, 2003). It has been demonstrated that the assumption

Figure 4.1: Population structure and dynamics



Note: Data come from Aging Work Group (AWG) of the European Commission and are available online at [this link↑].

Figure 4.2: Technological progress



Note: Data come from Aging Work Group (AWG) of the European Commission and are available online at [this link↑].

about the exogenous technological progress is effectively innocuous (Bouzahzah et al., 2002; Buyse et al., 2013; Choi and Shin, 2015). In fact, unless the reform in question was to vastly alter the link between capital accumulation and e.g. growth of variety or quality improvements, the economy can be studied in stationary terms, i.e. adjusting for the efficiency growth.

We use the projection for the exogenous technological progress from Aging Work Group (AWG) of the European Commission Commission (2018) as of 2018, that is the first year of our simulations. The AWG scenario for productivity assumes gradual convergence to the average EU level of 1.54% *per annum* until 2070 and a stable growth at this rate thereafter (see Figure 4.2)

4.2.2 Production sector

It is conventional in overlapping generations models to adopt the capital share $\alpha \in (0.3, 0.4)$ range. These values reflect long-run aggregate capital share, even if the two recent decades have seen a trend of declining labor share common across developing and developed economies alike (Karabarbounis and Neiman, 2013; Mućk et al., 2018). Note that the measurement of labor share in observational data is challenged by self-employment and entrepreneurship, absent in our theoretical framework. Hence, the conventional calibration of $\alpha = 0.33$ was chosen.

Given the value of capital share, we calibrate the depreciation rate to replicate the investment rate observed in the economy. Over the past 20 years, on average, this target is about 20.6%, which is low by the OECD standards, but seems to reflect quite reliably also the modal value in the distribution of the quarterly national accounts data.

4.2.3 Aggregate preferences

With demographic evolution and production sector established, we calibrate the preferences parameters. For the leisure parameter ϕ we choose the conventional target rate of employment rate. In Poland, over the previous two decades, employment rate (i.e. the number of working individuals over the number of working age individuals, regardless of whether they are in the labor force or not) reached roughly 64% in late 2010s, but it was as low as 54% in mid 1990s, according to the Labor Force Survey. This indicator is based on extensive hours adjustment. An alternative calibration target to consider would be the number of hours worked, i.e. the intensive hours adjustment. OECD (2014) reports that in Poland the number of hours worked by an average worker was stable over time, in excess of 1840 hours per year, or 44% of the available non-rest time for the adults. Note that OECD statistics are based on Labor Force Survey as well, and are obtained through multiplying the average number of hours worked reported by the working individuals by 52 weeks. Thus, the OECD measure based on hours worked excludes extensive margin adjustment.

Depending on the selected target value, the meaning of the labor aggregate in the model – as well as adjustments in the labor supply – differs. Our model assumes labor supply to be perfectly elastic and throughout lifetime all agents work at some point. If we choose the ϕ parameter to match the employment rate, the adjustment in the model would signify extensive margin adjustments (joining or leaving the labor force without the intensive margins of changing the number of hours worked). If we choose the ϕ parameter to match the working hours, the adjustment in the model would signify the intensive margin adjustment (raising or increasing working time, without the corner solutions of extensive margins).

It is an empirical regularity in Poland, that part-time employment is rare, among the lowest in the European Union and on average short of 4% of the labor force over the past two decades, according to the Labor Force Survey data. Moreover, the unit of analysis in our model is the household rather than an individual. Meanwhile, the share of households where no adult in the working age is working is even lower according to the Household Budget Survey data. Given these two premises, we calibrate our model to replicate the extensive margin adjustments, as they appear to be more relevant for the Polish economy. Hence, we calibrate the ϕ parameter to match the employment ratio of 52% (as the mid-point between the high and the low value over the past two decades).

With the labor supply and labor demand matched (through the calibration of the production sector) we calibrate the time preference parameter δ, which is crucial for the inter-temporal choice of the agents. The time preference, jointly with the depreciation rate and capital productivity stemming from capital share determine the equilibrium interest rate, which we match to replicate 6.5%. This target value is the average rate of return by the Open Pension Funds over 1999-2019 period, net of fees and adjusted for inflation, i.e. expressed in the real terms. Advanced market economies calibrate time preference δ matching the interest rates of 3-4% typically (Krueger and Kubler, 2006; McGrattan and Prescott, 2017). However, the Polish economy is still catching up and thus is characterized by higher technological progress accruing to capital productivity. Moreover, the literature even for the US calibrates the interest rates in excess of 5% (Nishiyama and Smetters, 2005; Nishiyama and Smetters, 2007).

Table 4.1 summarizes the calibration of the macroeconomic aggregates.

Table 4.1: Calibrated parameters for the initial steady state

	Macroeconomic parameters	Calibration	Target	Value
α	output elasticity w.r.t. capital	0.33	conventional level	33%
d	depreciation rate of capital	0.0412	investment rate	20.6%
δ	discount factor	0.9717	interest rate	6.5%
ϕ	preference for consumption	0.456	average hours	52%
τ^c	consumption tax	0.2291	revenue (% of GDP)	12.1%
τ^k	capital tax	0.19	nominal rate	19%
τ^l	labor tax	0.06725	revenue (% of GDP)	4.82%
τ	pension system contribution	0.07	benefits (% of GDP)	5%
G	govt. expenditures (% of GDP)	0.2656	G/Y	26.56%
	debt to GDP ratio	0.55	Debt/GDP	55%

Notes: Data on tax revenues from the OECD Tax Database, the rest of the macroeconomic aggregates following the National Accounts. The target values have been averaged from the data over 1995-2018 (or longest available time series). The target for the pension system following Komada et al. (2017). Consumption tax τ^c is calibrated in the initial steady state in order to match the effective tax rate. On the transition path and in the final steady state τ^c is used to balance the government budget so it varies.

4.2.4 Taxes and the government

The government in our setup collects consumption taxes τ^c, labor taxes τ^l, capital income gains taxes τ^k and the lump sum tax Υ in order to finance G government expenditure and service public debt. We calibrate government expenditure to 26.56% of GDP, following the national accounts average for 1995-2018. Since government expenditure does not bring utility, nor output in our model, we take the following procedure. Once we know the value of government expenditure in the initial steady state (from the calibration), we calculate the government spending *per capita*. We then continue with this *per capita* spending and keep it constant on the transition path in baseline as well as reform scenario. The demographic projection in baseline and reform scenario are the same, hence the values of government spending will be the same as well. GDP differs between baseline and reform scenario. Therefore share of government spending in GDP will vary between baseline and reform scenario.

We use OECD tax data in order to obtain revenues from each of the three tax sources as a share of GDP, averaged over 1995-2018, i.e. the available period. We calibrate the respective tax rates such that the tax revenues as a share of GDP matched the data reported by the OECD. We compute the average shares in the data as an average of annual shares. Table 4.1 reports the matched tax rates.

4.2.5 The pension system

The pension system is set to replicate the features of the Polish economy. We assume all agents participate in the notionally defined contribution (NDC) system with no capital pillar. This assumption is the same for baseline and reform, so it does not affect the comparison between the two paths. Naturally, a small fraction of individuals still holds assets in the reduced, previously mandatory capital pillar, but (a) it is already substantially reduced both in terms of the contribution rate and the share of the contributing cohorts; (b) it is effectively eradicated during the ten years before retirement due to gradual transformation of accumulated assets in to NDC drawing rights; (c) before within a couple of months as of when this study is concluded, the existing capital pillar with seize to exist in its current form and the new form has not yet been determined.

We use the OECD data to calibrate the retirement age $\bar{J}$. While this statistic has some variation over the past decade, as the OECD data are available, it varies between 60 and 61 years of age. Given the narrow range, we chose 61 years of age, as plausibly more relevant for the cohorts born after the second world war. Note that the retirement age does not change between baseline and reform scenarios, hence it does not affect the measured effect of the reforms we study.

The size of the contribution rate τ is typically calibrated to match the share of pension expenditure in GDP. However, currently in the data on pension expenditure cover both old cohorts receiving pensions from the previous, defined benefit system and the few cohorts receiving pensions already from a defined contribution system. We cannot thus use raw data and thus we follow Komada et al. (2017) who report final steady state values for Poland, once the transition from defined benefit to defined contribution pension system is complete. Their calibration of Polish economy is analogous to ours, with the exception of behavioral heterogeneity.

4.3 Heterogeneity of preferences

Finding observational data on prevalence of incomplete rationality is a challenge, as it essentially requires large scale experiments to diagnose preferences of agents in a given economy, at least on a large representative sample. An alternative approach to calibrating heterogeneity could consist of matching the distribution of the wealth moment across age groups. The challenge here lies in the fact that data on wealth are scarce and rarely available in a panel, thus permitting observation of the same birth cohort as it ages rather than a cross-section of different birth cohorts at a one point in time. In Poland, there was no data on wealth across households until 2013 wave of Polish edition of the Household Finance and Consumption Network survey. However, for the time being this data is a cross-section and thus cannot be used to infer data about age distribution of wealth. The available long selection of repeated cross-sections from Household Budget Survey (HBS) cannot be used as it informs about instantaneous savings flow rather than stocks. Moreover, the flows in the observational data may refer to any consumption smoothing, including e.g. putting money aside in the winter months for the holidays, housing renovation or a new vehicle. Hence, roughly 80% of households save in HBS data (Liberda, 2000; Liberda, 2013), but it is not plausible to assume that all of those households save for the old-ages (or that all of those savings are for the old-ages).

Matching the shares of incompletely rational agents to the moments of the wealth distribution faces also another challenge. The mere fact that many individuals fail to accumulate wealth does not have to imply any particular form of incomplete rationality. For one, the functional forms we chose for the incompletely rational agents cannot be differentiated one from another based just on moments of the wealth distribution. For example, one could think of HTM agents as perfectly identifiable, but their asset holdings will be observationally equivalent to the asset holdings of fully rational agents with very high preference for presence as well as time inconsistent agents with conventional levels of δ but very low value of β. As Bernheim et al. (2001) note, "if one takes the view that saving reflects rational, farsighted optimization, then low savers are simply expressing their preferences for current consumption over future consumption" (p. 832). They contrast this view with one in which "households are shortsighted, irrational, prone to regret, or heavily influenced by psychological motives". Distinguishing between these two approaches is not possible with conventional observational data in cross-section. of view requires more detailed analysis of the data.

Given these data and conceptual limitations, in our model the fully rational agents comprise 99.6% of the population in each birth cohort. Meanwhile, for each of the type of agents with incomplete rationality, we set their share to 0.1%. Consequently, our setup will be informative of the introduction of the government-subsidized old-age saving schemes in an economy with complete rationality in the aggregate terms. However, we are able to discuss the behavioral adjustment patterns for each type of incomplete rationality.

Results

5

This chapter describes the reaction of all the modeled types of agents to the policy reform. Namely, to the introduction of voluntary old-age saving scheme introduction. Section 5.1 describes in details effects the policy causes for each type of agents. In section 5.2, we discuss the effects of the instrument in aggregate terms. Since our model features very low shares of incompletely rational agents, the aggregate analysis concerns effectively a fully rational economy.

5.1 Effects of policy reform across types of agents

Our key research question concern the reaction of fully rational and incompletely rational agents to government-subsidized old-age saving instrument. We first demonstrate the overall welfare effects of the instrument in the final steady state across the types of agents. We then study adjustments for each type of agents. In partial equilibrium, we observe how the agents adjust their individual plans to a world with instrument (conditional on optimizing the participation choice). In general equilibrium, we study the effects as the economy adjusts to the participation choices and thus changes prices and tax rates. In order to understand the behavioral changes for each type of agents, we compare the final steady states with and without instruments (reform and baseline). However, our model captures the entire transition, which is why we may portray the welfare effects across subsequent birth cohorts, as described in section 3.7.

5.1.1 Welfare effects in the final steady state

Welfare effects in the final steady state are generally positive only for those agents, for whom the government-subsidized old-age saving instrument substantially expands the choice set. Specifically, contributions reduce the choice set for consumption for all types of agents: the net income $\mathcal{I}_{j,m,t}$ is reduced by $\tau^{ref} w_t l_{j,m,t}$. The trade off is the benefits that this instrument offers. For the fully rational agents, the only direct benefit is tax exemption from capital income taxation. The same applies to adaptive learners, though their adjustment to the new equilibrium in the economy occurs with delay, as they do not internalize future changes in prices and tax rates in their optimization problem. By contrast, hand-to-mouth agents and financially illiterate agents *obtain* a vehicle to receive interest at all, let alone the tax exemption on capital income gains. Hence, in relative terms, the choice set for HTM and financially illiterate agents is substantially expanded with the introduction of government-subsidized old-age saving instrument.

The case is more complex for the time-inconsistent agents. On the one hand a commitment device is introduced, which in principle expands their choice set. On the other hand, since all the agents share time preference δ in our economy, time inconsistent agents are far more impatient in this economy, due to $\beta \neq 1$ in equation (3.4). Two effects are thus at play: expansion of the choice set for agents, who value future substantially less than the other agents. Evaluating welfare effects for this type of agents, one should bear in mind the important difference in the effective discount factor.

Table 5.1 portrays welfare effects in the final steady state, decomposing the total effects into the consequences of introducing the (voluntary) contribution of $\tau^{ref} w_t l_{j,m,t}$, the consequences of tax exempt on the assets accumulated in this government-subsidized old-age saving instrument (both in partial equilibrium) and the general equilibrium effects. As expected, in the final steady state, the economy does not change, hence there are no differences between fully rational agents and adaptive learners. In the case of the time inconsistent agents, the effect of stronger discounting quantitatively dominates the effects of commitment device. Finally, the agents for whom the choice set is expanded – HTM and financially illiterate agents – observe welfare gains from both ability to smooth consumption and subsequently tax exempt on assets accumulated accumulated in this government-subsidized old-age saving instrument.

Table 5.1: Welfare effects in the final steady state

Agent type	Partial equilibrium		General equilibrium	
	Contributing	Contr. + tax exempt.	Optimal part.	Non-part.
fully rational	-0.30%	0.88%	-1.07%	-1.93%
adaptive learners	-0.30%	0.88%	-1.07%	-1.93%
time inconsistent	-0.32%	0.85%	-1.10%	-1.93%
hand to mouth	31.15%	37.8%	35.13%	-1.91%
financially illiterate	6.12%	9.21%	7.10%	-1.92%

Notes: Welfare change due to $\tau^{ref} w_t l_{j,m,t}$, and due to a combination of $\tau^{ref} w_t l_{j,m,t}$ and tax exemption from capital income tax on these assets obtained through partial equilibrium, i.e. *ceteris paribus*. The total welfare effects of policy reform were calculated once all prices have adjusted, i.e. in general equilibrium. Negative value denotes a welfare loss in comparison to baseline scenario, positive a gain. Welfare effects are calculated as per section 3.7. Optimal participation denotes the case where agents endogenously choose if and for how long to participate. Non-participation denotes welfare if the government-subsidized old-age saving instrument is introduced, but a given type of agents is not participating in it at all.

Welfare effects portrayed in Table 5.1 are a consequence of changes in utility, which is derived from adjustments in leisure and consumption by the agents. Below we discuss these adjustments for each type of the agents.

5.1.2 Fully rational agents

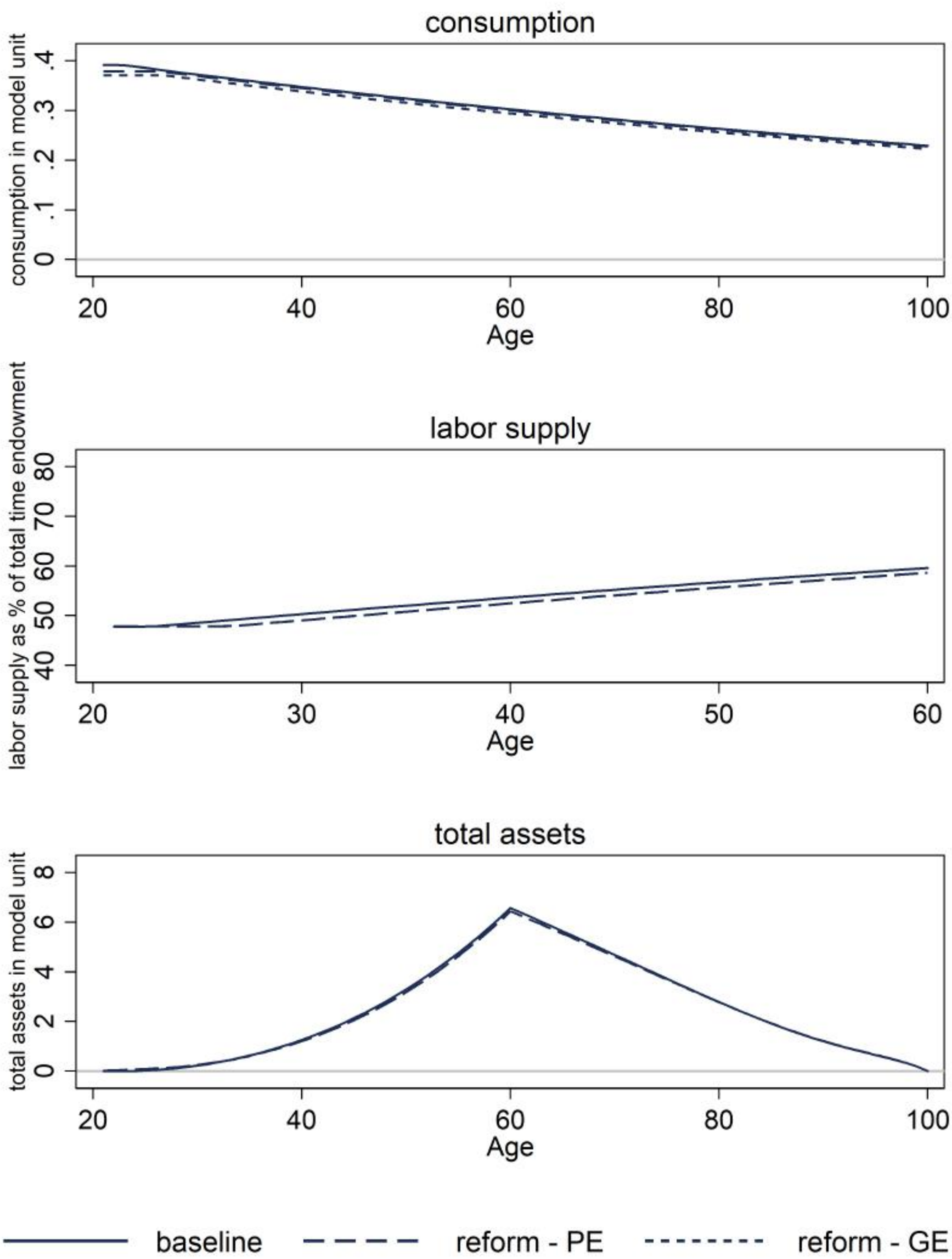
Recall that fully rational agents are on optimal consumption path even without government-subsidized old-age saving instrument. The instrument offers higher effective rate of return, hence they can reduce asset accumulation to achieve the same consumption plan given leisure or higher leisure given consumption. In early working ages the contributions forced by $\tau^{ref} w_t l_{j,m,t}$ substantially reduce their ability to accumulate voluntary assets outside the government-subsidized old-age saving instrument (hence lower consumption), but this effect concerns only years with the lowest (life-time) labor supply. Figure 5.1 portrays the adjustments in consumption and leisure plans as well as assets. Indeed adjustments are incremental in labor (reduced supply) and consumption. These small changes are consistent with overall minor welfare effects in the final steady state: fully rational agents are willing to give up 1.07% of their life-time consumption to avoid the reform. They choose to participate overall (participation yields higher utility than non-participation), but would rather chose the world without the government-subsidized old-age saving instrument than with it.

Accordingly, Figure 5.1 portrays the entire transition path, that lifetime welfare effects for subsequent birth cohorts (expressed in percentage points of lifetime consumption discounted to the age of $j = 21$). Note that demographic change and gradual decline in exogenous technological progress rate (as described in section 4) are the same in baseline and reform. Figure 5.1 informs in particular about the between-cohort distribution of the welfare effects. Indeed, each subsequent birth cohort of the fully rational agents experiences a welfare loss, notably due to reduced choice set and adverse general equilibrium effects. In the case of cohorts already retired at the moment of reform, the negative effects come from general equilibrium and are associated with generally lower labor supply (which reduces pension benefits indexation). The subsequent birth cohorts may chose whether or not to participate in the government-subsidized old-age saving instrument. These cohorts gradually experience all the effects we describe above, hence welfare effects become somewhat less negative. Once the economy reaches the new equilibrium, the effects are mainly driven by large, negative general equilibrium effects, which dominate fiscal gains in the capital income tax exemption (we discuss macroeconomic effects in section 5.2.

5.1.3 Adaptive learners

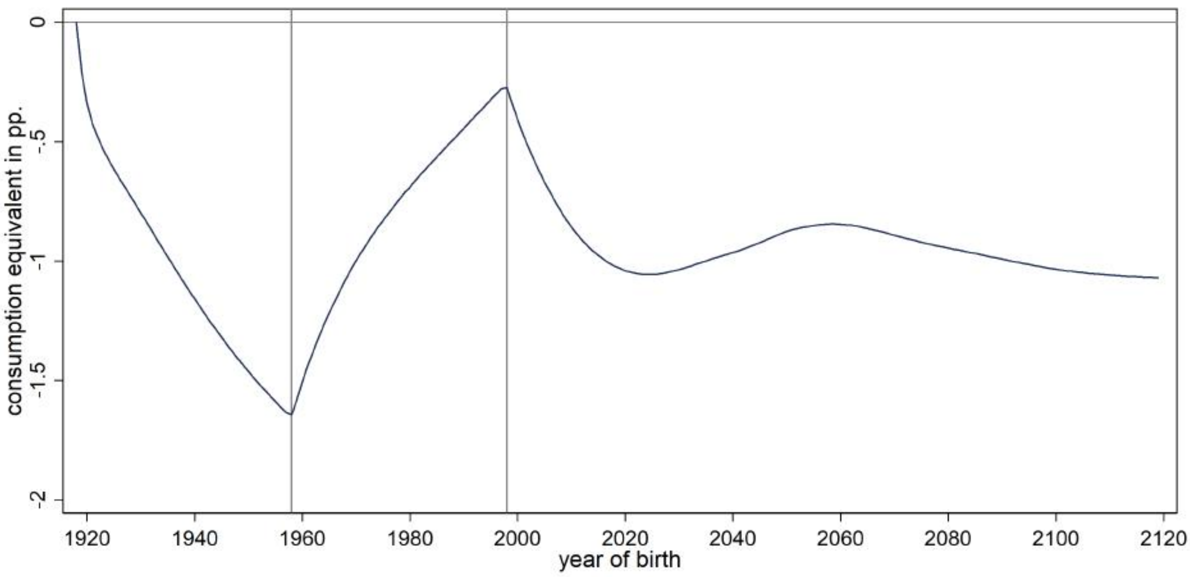
The final steady state for the adaptive learners is equivalent to fully rational agents. Studying adaptive learners across subsequent birth cohorts is an illustrative way to show how they differ from fully rational agents, when the economy is in transition. As portrayed in Figure 5.3, in cohorts retired already prior to introducing the government-subsidized old-age saving instrument, there no large differences between adaptive learners and fully rational agents. First, changes in

Figure 5.1: Fully rational agents: lifetime profiles for consumption, labor supply and assets



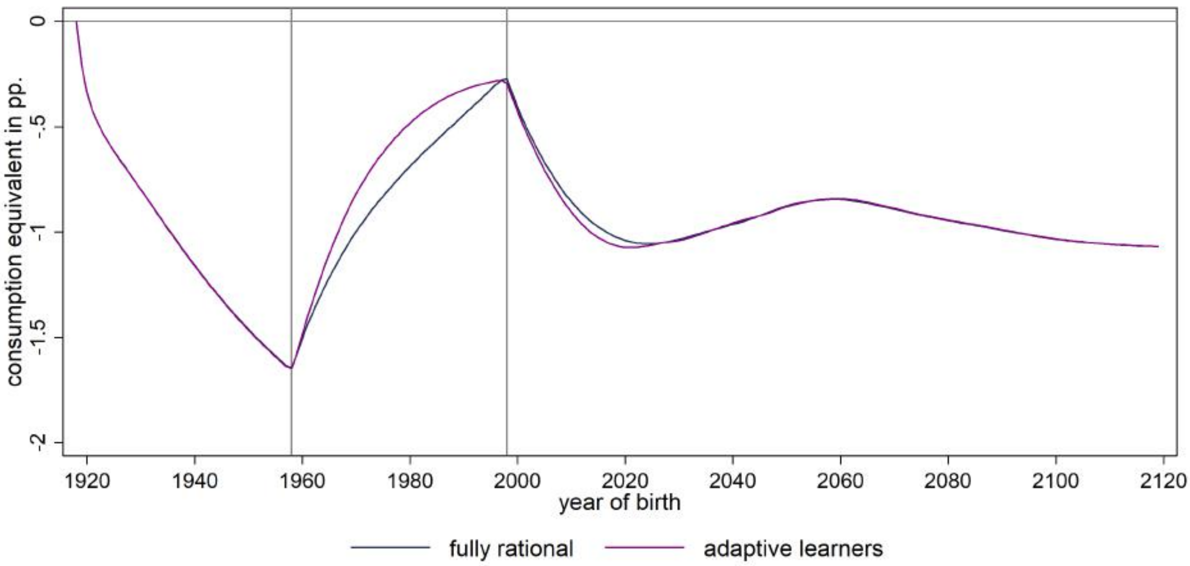
Notes: Figure portrays life-time pattern in the final steady state for consumption, labor supply and total assets (i.e. voluntary savings subjected to capital income taxation and government-subsidized old-age saving instrument).

Figure 5.2: Fully rational agents: Consumption equivalent by year of birth



Note: We report the consumption equivalents (% of permanent consumption in baseline scenario). Vertical lines mark the birth cohort which may join the government-subsidized old-age saving instrument (the first vertical line) and the first birth cohort which may participate in the government-subsidized old-age saving instrument for the entire lifetime (the second vertical line). Positive values signify welfare gains. Conversely, negative values signify that a given birth cohort is willing to give up certain fraction of their lifetime consumption to prevent government-subsidized old-age saving instrument from implementation.

Figure 5.3: Adaptive learners: Consumption equivalent by year of birth



Note: We report the consumption equivalents (% of permanent consumption in baseline scenario). Vertical lines mark the birth cohort which may join the government-subsidized old-age saving instrument (the first vertical line) and the first birth cohort which may participate in the government-subsidized old-age saving instrument for the entire lifetime (the second vertical line). Positive values signify welfare gains. Conversely, negative values signify that a given birth cohort is willing to give up certain fraction of their lifetime consumption to prevent government-subsidized old-age saving instrument from implementation.

the interest rates and the wage rates have negligible impact on re-optimizing lifetime when agents are already in the deaccumulation phase and cannot adjust labor supply. Second, the changes in the economy appear only gradually, in particular the tax rates adjust slowly. Among the birth cohorts of agents who may choose to participate in the government-subsidized old-age saving instrument for a span shorter than the working period, welfare adjustments trace the fully rational agents, but with a delay, because adaptive learners cannot fully internalize the consequences of *future* changes in prices and tax rates. Similar logic applies to the cohorts which may participate in the government-subsidized old-age saving instrument for the entire working period. However, since fully rational agents are the vast majority in this economy, the prices adjust swiftly enough for the adaptive learners to obtain welfare effects similar to fully rational agents. This result is a consequence of the fact that majority of the welfare effects stems from general equilibrium rather than partial equilibrium.

5.1.4 Time inconsistent agents

As discussed above, time inconsistent agents have stronger discounting. When faced with the choice of participating in the instrument, time inconsistent agents weigh current decline in consumption due to $\tau^{ref} w_t l_{j,m,t}$ against a smoother consumption path. Since assets accumulated in the government-subsidized old-age saving instrument yield a higher effective tax rate, the same level of savings allows them to reduce labor supply maintaining consumption unchanged, as is portrayed in Figure 5.4. Consequently, they choose to participate in the instrument due to fiscal gains, even though time inconsistent agents would rather live in the world without the government-subsidized old-age saving instrument.

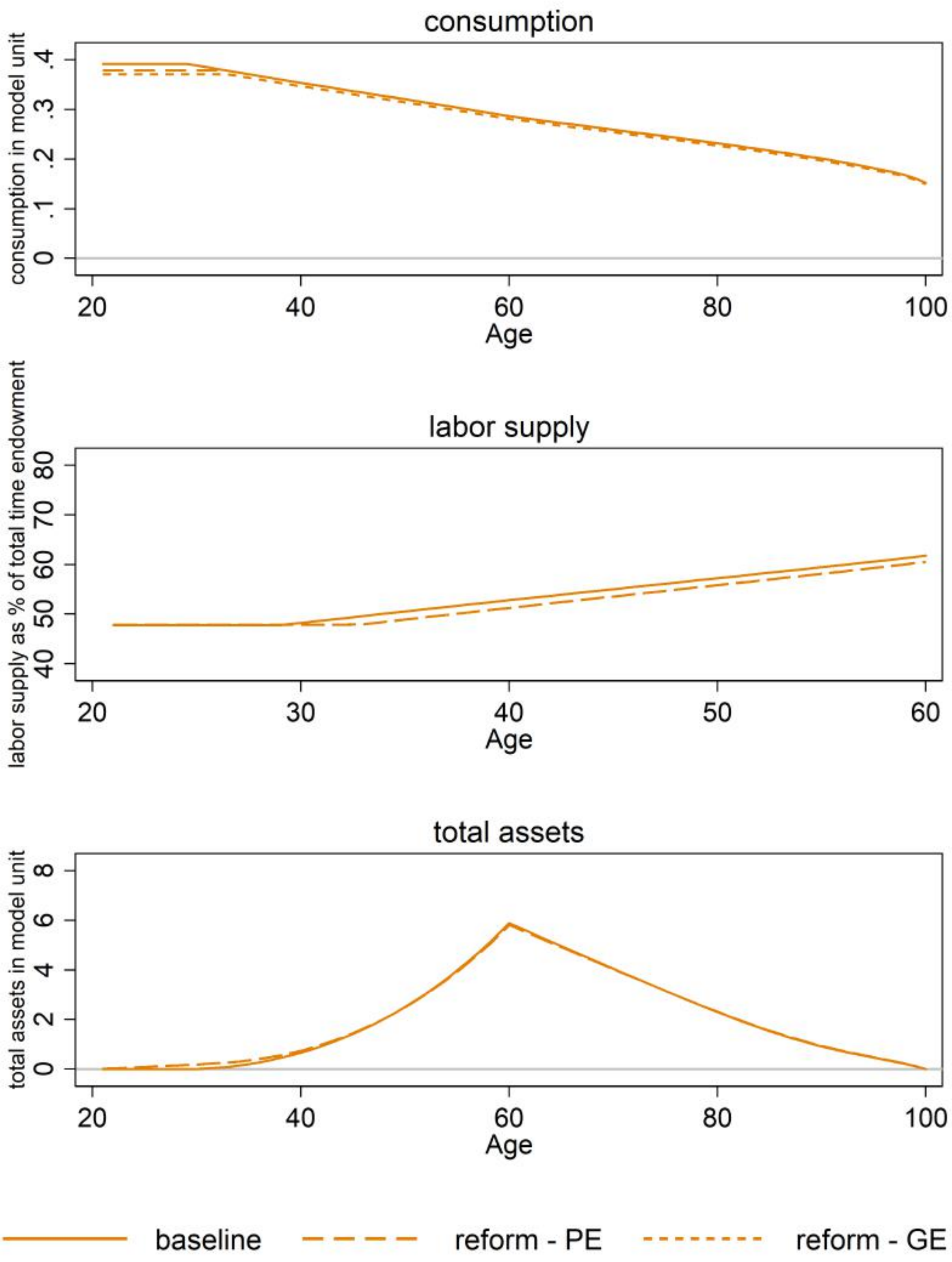
Across the birth cohorts, the same mechanisms apply as in the case of fully rational agents, with additional discounting of the future periods. Hence, welfare effects across birth cohorts for time inconsistent agents are similar, see Figure 5.4. Note that one should expect differences between the welfare for time inconsistent agents relative to fully rational agents only if there were substantial differences in participation. Some experiments with model calibration reveal that for lower values of β and for lower tax exemption, we observe much lower participation in the government-subsidized old-age saving instrument for time inconsistent agents. For example, with β of 0.6 and only 40% of capital income tax exemption, time inconsistent agents choose to participate for no longer than the last 6 years prior to the retirement. In such case, welfare effects are driven to a larger extent by general equilibrium effects than by individual (partial equilibrium) effects, which implies that the distribution of welfare across cohorts for the time inconsistent agents is not the same as for the fully rational agents.

5.1.5 Hand to mouth agents

The HTM agents experience fundamentally different consumption path in reform relative to baseline, because in the baseline scenario they accumulate no assets and they do accumulate $\tau^{ref} w_t l_{j,m,t}$ in the reform scenario. Note that HTM agents will not adjust labor supply to reform, because they still make the same intra-temporal choice. Consequently, two adjustments occur: reduced consumption in the working period and increased consumption in the old-age. These adjustments are portrayed in Figure 5.7. As we saw from Table 5.1, the tax exempt brings welfare gains of secondary relevance when compared to the very welfare gains from consumption smoothing for the HTM agents. This is because the rate at which the accumulated assets accrue is quantitatively less relevant than the ability to accrue assets at all.

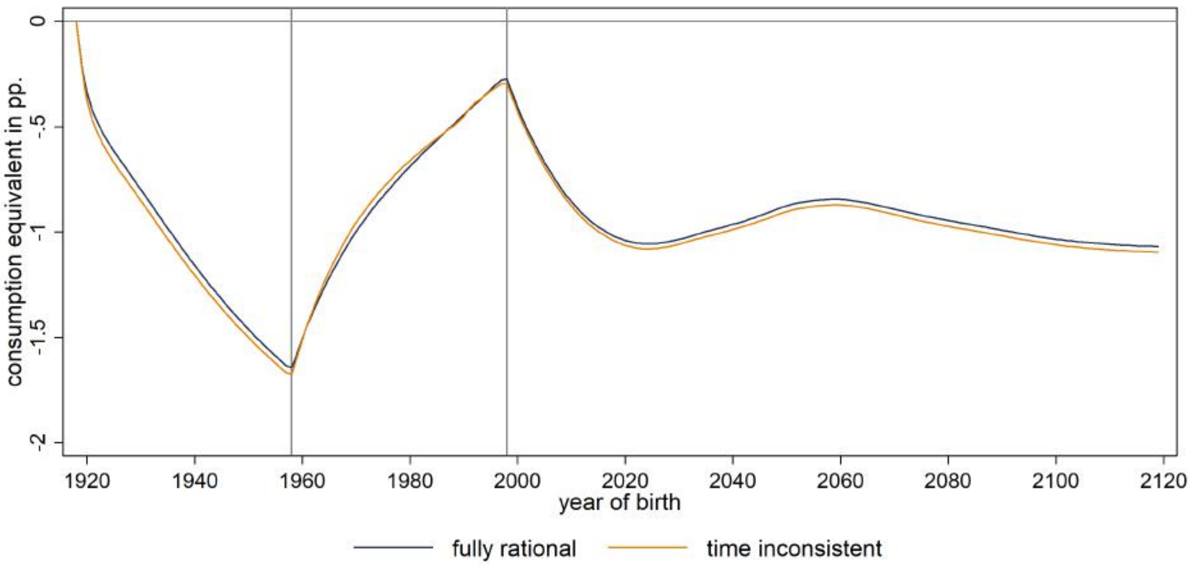
Given that the old age consumption is increased by many orders of magnitude for the HTM agents (on top of smoothing thanks to assets accumulation, they also obtain annuity premium), the reform generates enormous welfare gains. The measured 40 percentage points of lifetime consumption is misleading, however for two major reasons. First, we express welfare in lifetime consumption from the baseline scenario, which is substantially lower for the HTM agents than their reform lifetime consumption. As they put aside $\tau^{ref} w_t l_{j,m,t}$ in effectively every working period, they

Figure 5.4: Time inconsistent agents: lifetime profiles for consumption, labor supply and assets



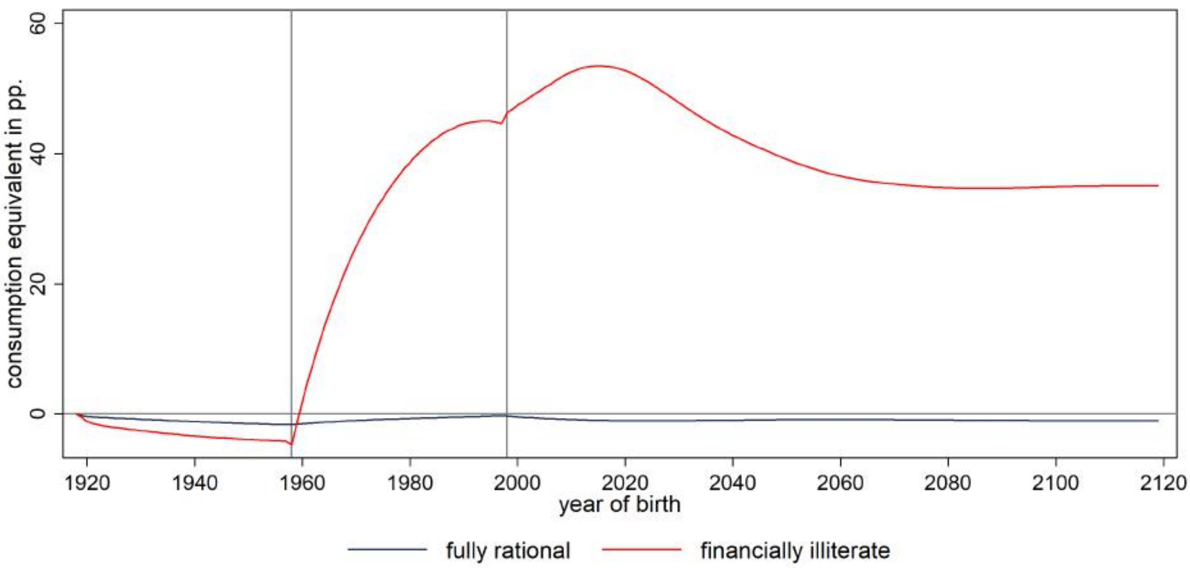
Notes: Figure portrays life-time pattern in the final steady state for consumption, labor supply and total assets (i.e. voluntary savings subjected to capital income taxation and government-subsidized old-age saving instrument).

Figure 5.5: Time inconsistent agents: Consumption equivalent by year of birth



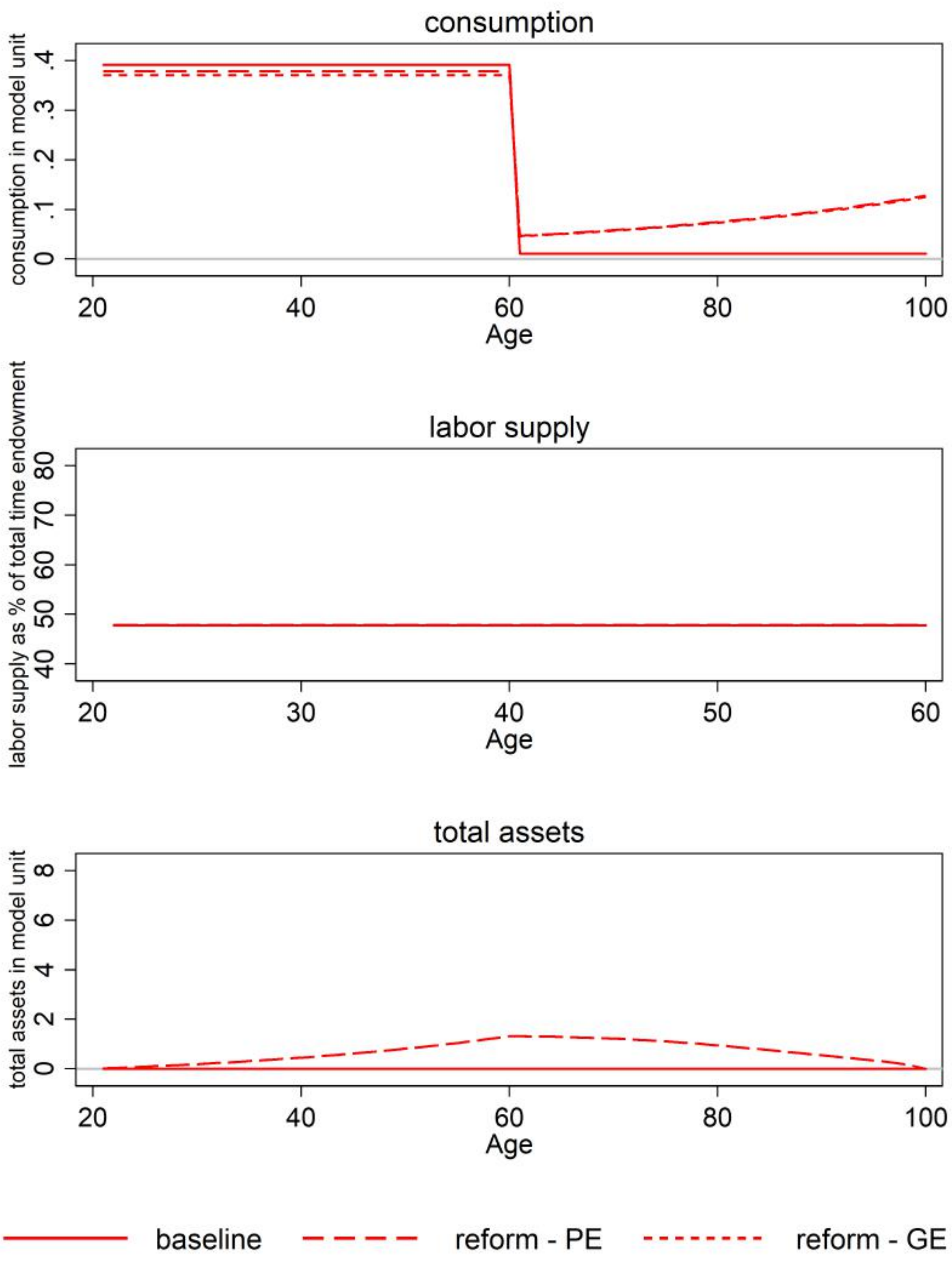
Note: We report the consumption equivalents (% of permanent consumption in baseline scenario). Vertical lines mark the birth cohort which may join the government-subsidized old-age saving instrument (the first vertical line) and the first birth cohort which may participate in the government-subsidized old-age saving instrument for the entire lifetime (the second vertical line). Positive values signify welfare gains. Conversely, negative values signify that a given birth cohort is willing to give up certain fraction of their lifetime consumption to prevent government-subsidized old-age saving instrument from implementation.

Figure 5.6: Hand to mouth agents: Consumption equivalent by year of birth



Note: We report the consumption equivalents (% of permanent consumption in baseline scenario). Vertical lines mark the birth cohort which may join the government-subsidized old-age saving instrument (the first vertical line) and the first birth cohort which may participate in the government-subsidized old-age saving instrument for the entire lifetime (the second vertical line). Positive values signify welfare gains. Conversely, negative values signify that a given birth cohort is willing to give up certain fraction of their lifetime consumption to prevent government-subsidized old-age saving instrument from implementation.

Figure 5.7: Hand to mouth agents: lifetime profiles for consumption, labor supply and assets



Notes: Figure portrays life-time pattern in the final steady state for consumption, labor supply and total assets (i.e. voluntary savings subjected to capital income taxation and government-subsidized old-age saving instrument).

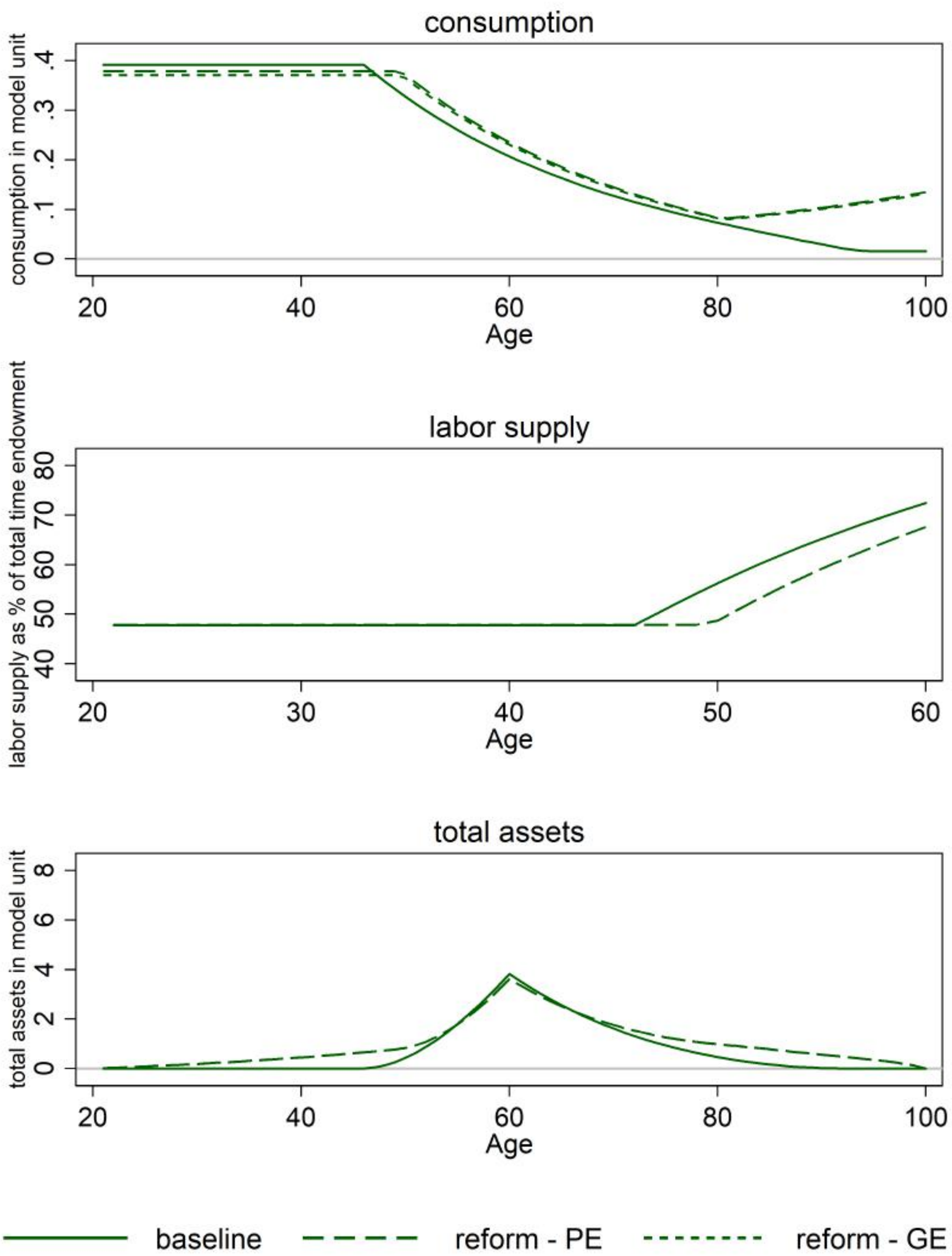
accumulate slightly less than a third of assets that a fully rational agent accumulates, obtaining additional source of income: interest on these assets (including the annuity premium, $\mu_{j,t} + \bar{r}_{j,t}$ in every age j and period t). This changes their lifetime income substantially. Second, we treat the increase in welfare as if HTM agents actually *wanted to* save (they have preference for smooth consumption), but could not (they are forced to consume all disposable income instantaneously). Meanwhile, compulsive consumption and other behavioral patterns underlying operationalization of households with no observable assets as HTM households – does not have to follow from the limitations on the budget constraint. For example, HTM behavior could be obtained through extremely high inter-temporal discounting. If HTM agents were modeled this way rather than through the budget constraint, their welfare accounting would be equivalent to fully rational agents with very high δ rather than what we portray in Figure 5.6.

5.1.6 Financially illiterate agents

As discussed in section 3.1.5, financially illiterate agents delay and reduce savings relative to fully rational agents solely due to no access to interest. They do accumulate assets towards the end of their working period, because consumption smoothing is still an important driver of household optimization, but with regular time preference they are not willing to give up consumption in the young ages in exchange for consumption in the old ages. Introducing a government-subsidized old-age saving instrument provides financially illiterate agents with a return to postponing consumption. For this reason, they choose to participate in the instrument and contribute $\tau^{ref} w_t l_{j,m,t}$ despite continuing to delay private voluntary savings. In presence of instrument, financially illiterate agents still accumulate bulk of their assets in the years prior to the retirement. In order to be able to accumulate these assets, they also continue to increase labor supply towards the end of the working ages. Admittedly, with the government-subsidized old-age saving instrument, labor supply increases later.

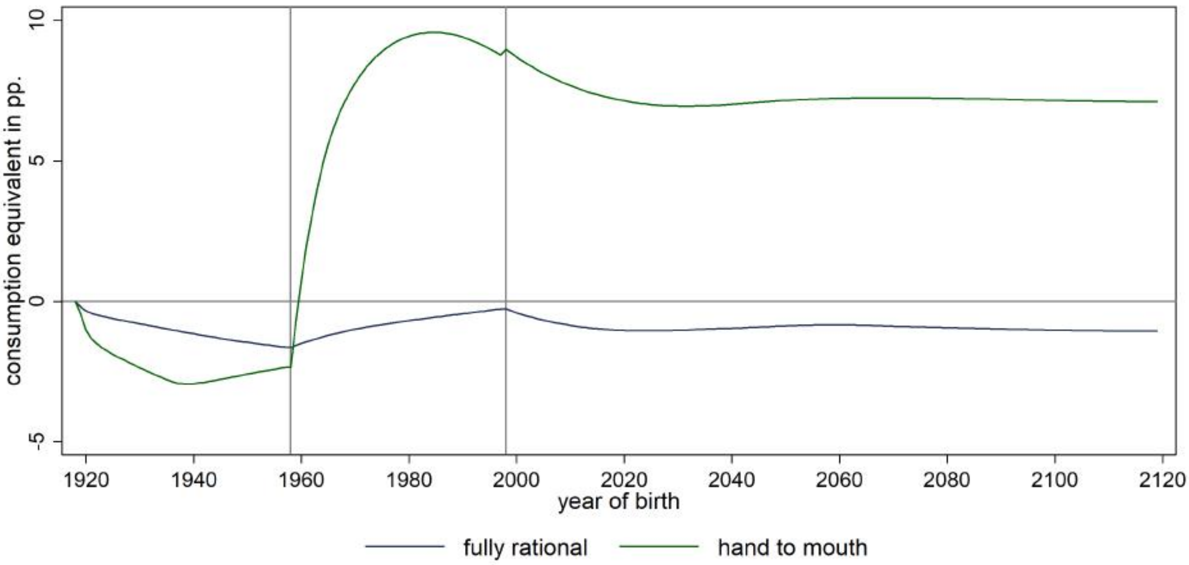
Welfare accounting for financially illiterate agents is effectively accounting for the expanded choice set due to the access to interest-bearing savings instrument. The ability to accrue interest raises the gains from the instrument from large negative for the birth cohorts without access to the instrument – turns large and positive for the subsequent birth cohorts, see Figure 5.9. These large positive effects amount to roughly 7% of lifetime consumption in the baseline, which is consistent with the observed magnitude of the increase in consumption in the old-age, as portrayed in Figure 5.8.

Figure 5.8: Financially illiterate agents: lifetime profiles for consumption, labor supply and assets



Notes: Figure portrays life-time pattern in the final steady state for consumption, labor supply and total assets (i.e. voluntary savings subjected to capital income taxation and government-subsidized old-age saving instrument).

Figure 5.9: Financially illiterate agents: Consumption equivalent by year of birth



Note: We report the consumption equivalents (% of permanent consumption in baseline scenario). Vertical lines mark the birth cohort which may join the government-subsidized old-age saving instrument (the first vertical line) and the first birth cohort which may participate in the government-subsidized old-age saving instrument for the entire lifetime (the second vertical line). Positive values signify welfare gains. Conversely, negative values signify that a given birth cohort is willing to give up certain fraction of their lifetime consumption to prevent government-subsidized old-age saving instrument from implementation.

5.1.7 Commonalities and differences in adjustment to policy reform

Analyzing the microeconomic effects of government-subsidized old-age saving instrument reveals several important observations. First, providing the instrument generates large inter-generational and intra-generational redistribution. Due to large fiscal costs, agents with fully rational preferences and agents with time inconsistent preferences observe welfare losses, whereas agents with limitations on access to savings vehicles in baseline observe large welfare gains in the reform scenario. The inter-generational redistribution of the welfare gains concerns mostly those agents, who observe gains: the initially old cohorts, who have no ability to achieve greater consumption smoothing thanks to the government-subsidized old-age saving instrument, nonetheless bear its fiscal cost and thus loose welfare. The longer the agents have access to the government-subsidized old-age saving instrument, the greater their welfare gains – in the case of HTM and financially illiterate agents, i.e. those with no or limited access to consumption smoothing in the baseline.

Second, note that the peak of accumulated assets with and without government-subsidized old-age saving instrument is virtually the same for all types of agents, with the trivial exception of HTM agents. The fact that the fully rational agents do not increase their stock of wealth is consistent with common sense intuition: their lifetime optimization in baseline is the same as in reform, with the trivial exception of the general equilibrium effects. The frequent premise for the government-subsidized old-age saving instrument is that with declining pension benefits, old-age poverty becomes a policy challenge and private (subsidized and voluntary) old-age savings are a potential policy instrument. If at the peak the accumulated stock of assets is the same with and without instrument, then indeed government-subsidized old-age saving vehicles cannot serve the purpose of reducing old-age poverty through *raising* incomes. What they help to achieve is to change the distribution of consumption over the life cycle, because accumulation of the peak wealth is less burdensome.

Note that this information is particularly relevant for the interpretation of the crowd-out effects studied in the empirical literature and often raised as a concern for the effectiveness of government-subsidized old-age saving instrument. If maximum wealth is not raised across agents of various types through the implementation of the instrument, then such instruments are not actually raising wealth within life-time. They change the distribution of this wealth across age groups, hence raising wealth through cross-sectional composition effects (agents in some age groups hold assets in the instrument in the reform scenario, but would have held no assets in the baseline scenario in these age groups). Consequently, in order to encourage actual increase in wealth stock across groups of agents, alternative mechanisms need to be designed.

There is also a methodological conclusion from our analysis. Namely, unlike other agents with incomplete rationality, the hand to mouth agents are particularly useful for observing the macroeconomic effects, because they do not adjust labor supply. By the same token, they are not very useful for observing the welfare effects, because any policy instrument effectively massively expands their choices set (lifetime income) thus allowing substantially higher utility. Meanwhile, agents with time inconsistent preferences, adaptive learners do not differ substantially from fully rational agents in their behavioral patterns, which implies that fully rational agents provide a reliable source of intuition on what can be expected in case agents departed from full rationality to time inconsistency or inability to forecast and process new information. Finally, financially illiterate agents display very peculiar life-cycle labor supply functions which are not inline with observational data.

While our model fully endogenizes voluntary participation, the actual choices of the agents show that full participation is always optimal. On the one hand, this is a consequence of endogenizing the participation decision in partial equilibrium: the agents compare if they prefer to participate or not, given the state of the world. In other words, the agents know they will live in a world with higher taxation (less favorable general equilibrium) and effectively choose between accruing or not the fiscal relief within the instrument. If the government-subsidized old-age saving instrument was large (high τ^{ref}), the agents may prefer to enter the instrument later in the life-time, but not accruing the fiscal relief if one hast to pay its cost would be clearly suboptimal. On the other hand, such decision to endogenize participation decision is internally consistent in the model and the fixed point approach assures that we find equilibrium.

5.2 Macro adjustments

5.2.1 Fiscal effects

Policy reform is fiscally costly for a number of reasons. First and foremost, tax exemption provided in the government-subsidized old-age saving instrument reduces tax revenues from capital income taxation. This effect is strong given the nature of the crowding out in this economy. Second, due to adjustments in labor supply and wages, revenues from labor income tax change as well. Finally, consumption changes as well: raises in the old ages, but declines in the young ages. Overall, these three mechanisms necessitate adjustments in consumption tax rate to accommodate for the government budget constraint. As per Figure 5.10, consumption taxes increase by 2.5 percentage point relative to baseline. Due to increased consumption taxation, second round effects are generated for the overall consumption: it decreases relative to baseline in the long-run. If our economy had a higher share of HTM agents and/or fully rational agents, the growth in their consumption in the old age could partially compensate for the decline in consumption by the fully rational agents.

In the short-run, at the beginning of the transition path, consumption increases relative to baseline. This is because the policy reform brings higher effective interest rate (due to capital gain tax exemption in the policy instrument). The same target assets stock upon retiring my be achieved with lower net savings rate. This encourages agents to reduce savings efforts relative to baseline.

Note that this adjustment in consumption taxation generates large negative welfare effects: roughly 1.9% of lifetime consumption in the final steady state as reported in the last column of Table 5.1. The decline in consumption reaches 3% relative to baseline, that is considerable drop in total consumption in the economy, especially given lower decline in output. Given that the effects on improving the lifetime paths for the agents are rather small, this large welfare cost suggests that the instrument studied in our paper is not optimal.

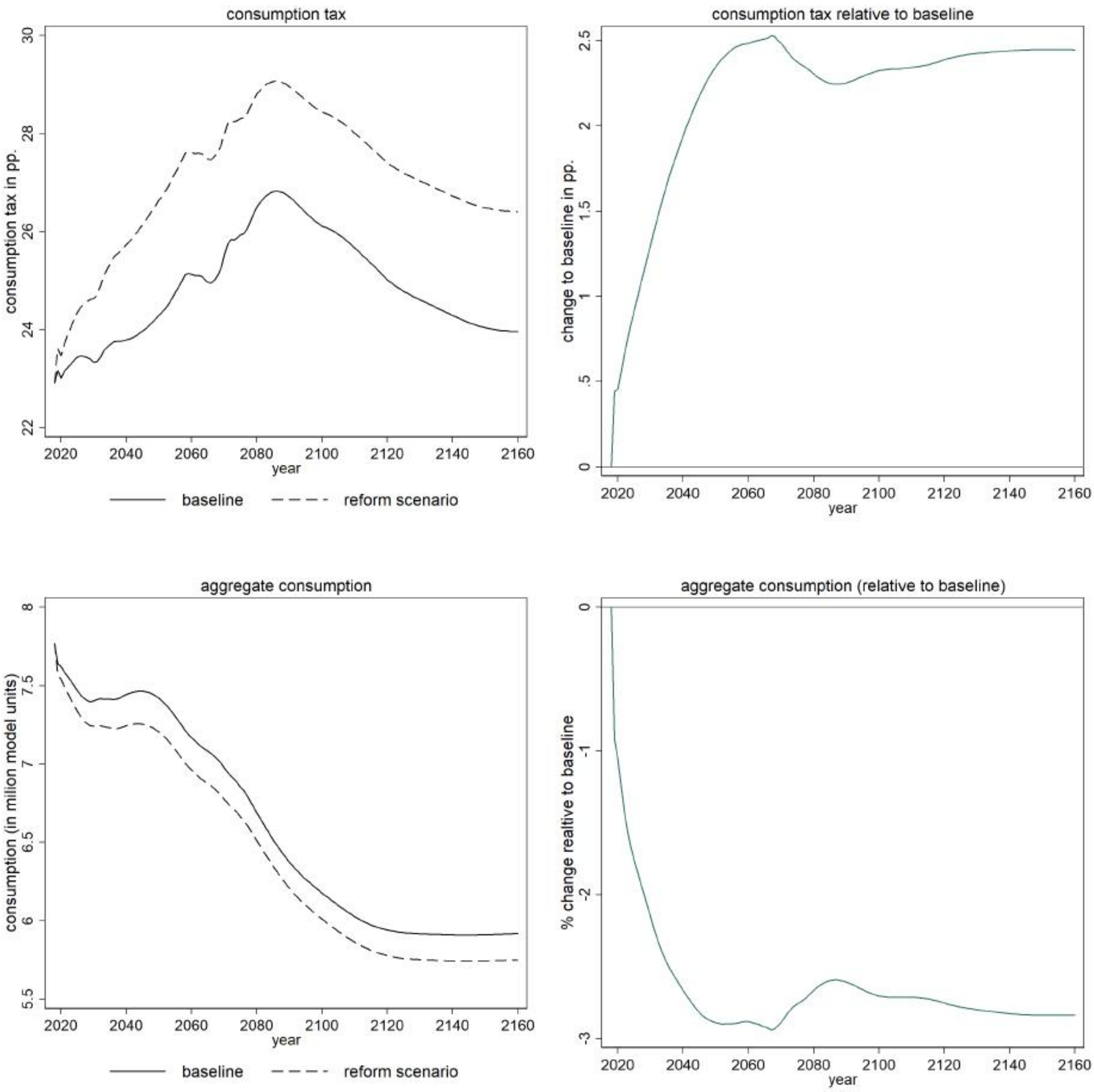
5.2.2 Crowd-out

Recall that for all types of agents in our economy, the peak stock of assets was the same with and without the government-subsidized old-age saving instrument. HTM agents are the only exception. Thus, one should expect no crowd-out for the HTM agents and almost complete crowd-out for the rest of the agents. However, as portrayed in sections 5.1.2 - 5.1.6, adjustment to *life-cycle* assets accumulation patterns occur once the instrument is introduced. This cross-sectional dimension has two components: changes in the asset holdings across age groups for each type of agents, and the changing composition of age groups due to demographic processes (longevity and declining number of youth entries).

To capture the combined effect of these two components, we obtain a macroeconomic measure of crowd-out, which is based on the notion of effective capital growth. Comparing aggregate capital in baseline and in reform scenarios and measuring how much of the capital is the government-subsidized old-age saving instrument we measure how much capital is generated from one unit of assets in the instrument. This parameter measures the total impact of the reform. For example, consider assets accumulated in the scheme amount to 10PLN in the long run and the private assets holdings outside the scheme decreased by 9PLN relative to baseline. Our measure captures that 1PLN out of those 10PLN accumulated in the government-subsidized old-age saving instrument is actually increased savings. In this case, the effective capital growth out of 1PLN amounts to 0.1PLN. We portray this measure in Figure 5.11

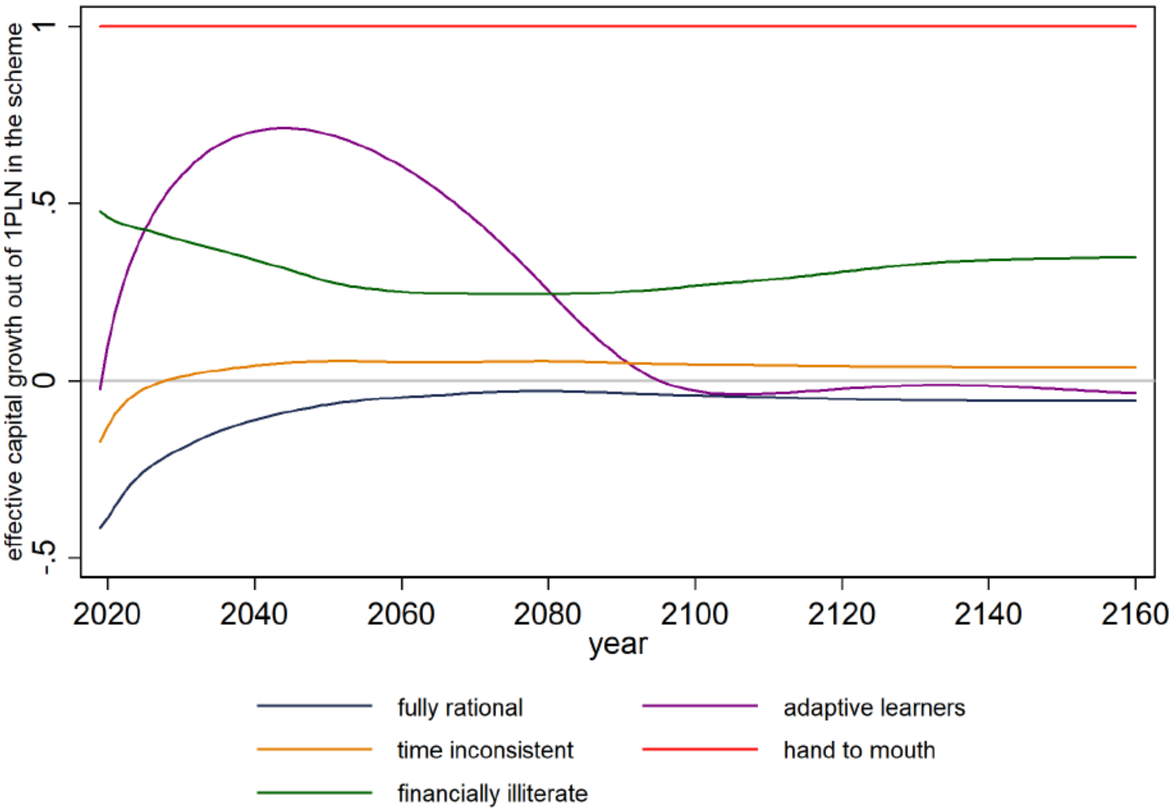
Effective capital creation due to the introduction of government-subsidized old-age saving instrument differs between agents types by orders of magnitude. It varies between 0% and 5% for fully

Figure 5.10: Fiscal adjustment (top) and consumption adjustment (bottom)



Note: baseline scenario comprises demographic change and decline in exogenous technological progress. Reform scenario comprises additionally the government-subsidized old-age saving instrument. The adjustment in τ_t^c satisfy equations (3.13),(3.14) and (3.15).

Figure 5.11: Effective capital creation due to government-subsidized old-age saving instrument



Note: Crowding out portrayed as a fraction of 1PLN accumulated in the government-subsidized old-age saving instrument that actually contributes to increase in capital stock between baseline and reform scenario. Due to low shares of incompletely rational agents in the economy, the total effect is equivalent to the one portrayed for fully rational agents.

rational agents and adaptive learners, between 23% and 45% for time inconsistent and financially illiterate agents. Mechanically the effective capital creation due to policy reform for HTM agents is equal to 100%. They held no assets in the baseline scenario, hence there were nothing to be crowded out by the scheme.

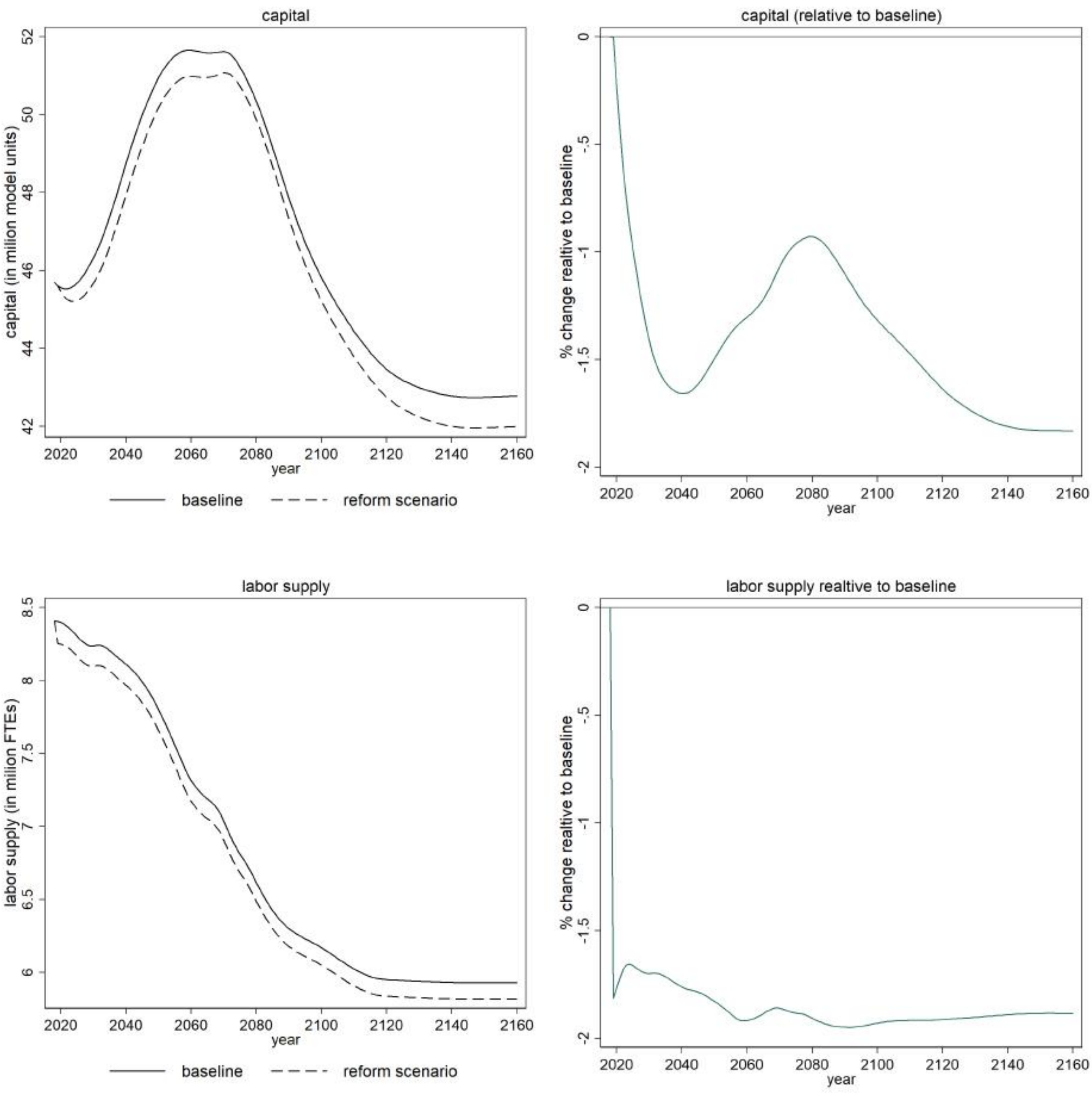
5.2.3 Capital, labor and their prices

Overall, the capital declines in our model economy, which is a consequence of high share of the fully rational agents and negative creation of wealth for this type of agents, subsequent the introduction of government-subsidized old-age saving instrument, see Figure 5.12. In an economy populated exclusively by agents with no crowd-out (e.g. HTM agents), the capital growth would amount to $\tau^{ref} \cdot w \cdot L$, which in our model economy is equivalent to roughly 1.78% in the long run.[1] The decline of capital stock relative to baseline of roughly 1.5% is large given that this economy increases capital overall by roughly 13% due to longevity. With an instrument aimed at fostering savings, the growth in capital is roughly 10% *lower* than without this instrument. Admittedly, majority of the changes in both baseline and reform scenario are governed by demographic change.

The implementation of the government-subsidized old-age saving instrument gives raise to some general equilibrium effects: relative prices change in the economy. The reform raises consumption taxes rendering leisure relatively cheaper then consumption, hence aggregate labor supply drops in the long-run. Note that this effect would be stronger, if the model economy had a higher share of time inconsistent and financially illiterate agents.

[1] Note that if an economy was populated by exclusively HTM agents, it would have no capital and thus zero output, so this comparison serves just the illustrative purposes.

Figure 5.12: Capital and labor supply changes due to policy reform



Note: Left panels show that majority of the changes in both baseline and reform scenario are driven by demography. Right panels show that even though humble, the effect of the reform policy is non-negligible.

6 Conclusions

This study compares and contrasts completely rational agents with four various types of agents with incomplete rationality. This comparison is performed in terms of reaction of said agents to the introduction of a government-subsidized old-age saving instrument with voluntary participation. In the context of life-cycle optimization, rationality means that the agents are characterized by three basic features. First, agents have perfect foresight. Second, agents' time preferences are given by exponential discounting. Third, agents have access to financial markets.

We then relax those assumptions. First, we consider agents without perfect foresight. Second, we study agents with quasi-hyperbolic discounting. Third we consider hand to mouth agents whose access to financial markets is totally restricted, hence they cannot store wealth at all. Fourth, we study agents whose access to financial markets is limited, but less severely. The latter type of agents can store wealth but do not earn any interest on their assets.

Studying various types of incomplete rationality shows that welfare may be effectively improved for those agents who have restricted access to the financial markets, while the rest of modeled agents suffer welfare deterioration due to implementation of a government-subsidized old-age saving instrument with voluntary participation.

There are two particularly promising avenues for further research. First, our model does not calibrate the prevalence of behavioral heterogeneity. This is founded on so far insufficient empirical evidence documenting the prevalence of incompletely rational households in the economy. To provide reliable policy implications for any government-subsidized old-age saving instrument, one would require more input from observational data to calibrate the shares of the agents within each birth cohort. Second, our model is developed in deterministic setting. Thus, the only motivation for saving is related to life-cycle optimization. In a setup with idiosyncratic income shocks, the agents would have precautionary savings as well. Uncertainty plausibly differentiates further rational agents from agents with incomplete rationality and thus could influence the evaluation of the welfare effects of introducing the government-subsidized old-age saving instruments.

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