Statystyki z proby

Lucas van der Velde

Task 1 Students believe that the local bartender is not honest while serving drinks. In order to check their beliefs, they conduct a short experiment. In one night, they ask 16 beers and measure how much beer was on there. If the bartender was honest, the distribution in each glass should be normal with a mean of half a liter and a standard deviation of 0.03, as some part of the beer might be lost during the measuring.

(a) What is the probability that the average amount of beer is below .490 l?

We find first the distribution

$$\bar{x} \sim N(0.5; 0.03/\sqrt{16})$$

And now we find the probability

$$\begin{aligned} P(\bar{x} \leq 0.490) = &\quad P(\tfrac{\bar{x}-\mu}{\sigma_{\bar{x}}} \leq (0.490 - 0.5)0.03 * \sqrt{16}) \\ = &\quad P(u < -.1/.03 * 4) = 1 - P(u \leq 12) \\ = &\quad 1 - \Phi(12) = 0.000 \end{aligned}$$

(b) What would be the consequence of using the sample standard deviation (.05 liters) instead of the assumed one in the computation of the probability?

We have a small sample, and we do not know the standard deviation. We need to use the t distribution

$$\frac{\bar{x} - \mu}{s} * \sqrt{n} \sim t_{n-1}$$

Filling the gaps we obtain our value t_{14}

$$t_{14} = \frac{.5 - 0.49}{0.05} * \sqrt{16} = \frac{.10}{.05} * 4 = 8$$

And then we get the probability $P(t < 8) \sim 0$

Task 2 A firm hires two types of workers: tenured and interns. The time it takes tenured workers to complete a task is a random variable distributed normally with a mean of 10 minutes and a standard deviation of 5 minutes. Among interns, the parameters of the distribution are 13 and 6 minutes. Assume that there are 16 tenured workers and 64 interns working at the firm

(a) Find the probability that the average time taken to complete the task for interns is smaller than the average task among tenured workers.

We find first the distribution

$$\bar{x} - \bar{y} \sim N(10 - 13; \sqrt{\frac{5^2}{16} + \frac{6^2}{64}})$$

And now we find the probability

$$
\begin{aligned}
P(\bar{x}-\bar{y} \geq 0) = & \quad P(\tfrac{\bar{x}-\bar{y}-(\mu_x-\mu_y)}{\sigma_{\bar{x}-\bar{y}}} \geq \tfrac{0-(10-13)}{\sqrt{(\frac{25}{16}+\frac{36}{64})}}) \\
= & \quad P(u \geq \tfrac{3}{\sqrt{\frac{14}{16}}}) = P(u \geq \tfrac{3}{0.94}) \\
= & \quad 1 - P(u \leq 3.21) = 1 - \Phi(3.21) = \quad 0.0003
\end{aligned}
$$

(b) Find the probability that the overall average (tenured and interns) is below 9 minutes.

We define the average as $\bar{Z} = \frac{\sum X + \sum Y}{n_x + n_y}$.

The expected value is

$$
E(\bar{Z}) = \frac{n_x * \mu_x + n_y * \mu_y}{n_x + n_y} = \frac{16 * 10 + 64 * 30}{16 + 64} = 12.4
$$

The variance is

$$
D^2(\bar{Z}) = (\frac{1}{n_x + n_y})^2 * (n_x * D^2(x) + n_y * D^2(y)) = \frac{16 * 5^2 + 64 * 6^2}{80^2} = 0.4225
$$

So, the standar deviation $D(\bar{Z}) = \sqrt(0.4225) = 0.65$. Since the time follows a normal distribution, the sample mean will also follow a normal distribution.

And now we find the probability

$$
\begin{aligned}
P(\bar{z} \leq 9) = & \quad P(\tfrac{\bar{Z}-E(\bar{Z})}{\sigma_{\bar{Z}}} \leq \tfrac{9-12.4}{0.65}) \\
= & \quad P(u \leq -\tfrac{3.4}{0.65}) = P(u \leq -5.23) \\
= & \quad 1 - P(u < 5.23) \sim 0
\end{aligned}
$$

Task 3 A producer of snacks conducts regular inspections to verify whether there is a problem with the machinery. Machines are programmed to pack 500 grams of product in each bag. The producer recognizes that due to irregular shapes it is possible to have some differences in the filling across packages. He believes these errors to follow a normal distribution with a zero mean and a standard deviation of 5 grams.

(a) What is the probability that a product is wrongly filled, i.e. that its filling deviates by more than 12 grams from the mean?

Filling (x) follows a normal distribution wiht a mean with parameter 500 and 5.

$$
\begin{aligned}
P(|x-\mu_x| \geq 12) = & \quad P(x-\mu_x > 12) + P(x-\mu_x < -12) \\
= & \quad 2 * P(x-\mu > 12)(\text{because of symmetry of the normal distribution}) \\
= & \quad 2 * P(\tfrac{x-\mu}{\sigma} > \tfrac{12}{5}) = 2 * P(u > 2.4) \\
= & \quad 2 * (1 - P(u \leq 2.4)) = 2 * (1 - \Phi(2.4)) \\
= & \quad 2 * (1 - 0.999) = 0.02
\end{aligned}
$$

(b) Find the probability that 72 or more products out of 1000 will be defective.

Y is the number of defective products.

$$
\begin{aligned}
Y \sim & \quad N(1000 * 0.02; \sqrt{1000 * 0.02 * 0.98}) \\
\sim & \quad N(20; 4.427)
\end{aligned}
$$

And to find the probability and answer the question...

$$\begin{aligned} P(Y \geq 72) = & P(\tfrac{Y-\mu_Y}{\sigma_Y} \geq \tfrac{72-20}{4.427}) \\ = & P(u \geq \tfrac{52}{4.427}) = P(u \geq 11.75) \\ = & 0 \end{aligned}$$

(c) What is the probability that in a sample of 1000 products less than 3% will be defective?

Z is the probability of finding a defectively filled product.

$$\begin{aligned} Z \sim & N(0.02; \sqrt{\tfrac{0.02*0.98}{1000}}) \\ \sim & N(0.02; .004427) \end{aligned}$$

In order to answer the question

$$\begin{aligned} P(Z \leq 0.03) = & P(\tfrac{Z-\mu_Z}{\sigma_Z} \geq \tfrac{0.03-0.02}{.004427}) \\ = & P(u \leq \tfrac{.01}{.004427}) = P(u \leq 2.26) \\ = & \Phi(2.26) = 0.988 \end{aligned}$$