STATISTICS
Lecture 2

Random variable and probability distributions

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Lecture is based on J.T. Mc Clave, P.G. Benson,T. Sincich: Statistics for Business and Economics, 11th Edition, 2010

Introduction

1. Definition of propability distribution

2. Probability distribution for a discrete random variable
   - Binomial distribution

3. Probability distribution for a continuous random variable
   - Normal distribution

4. Approximating a Binomial Distribution with a Normal Distribution

Definitions

Random variable - is a variable that assumes numerical values associated with the random outcomes of an experiment, where one (and only one) numerical value is assigned to each sample point.

If we can list the values of a random variable x, even though the list is never ending….

Discrete random variable – variable that assumes a countable number of values, e.g. marks at school, number of sold products, number of errors

Continuous random variable – assumes uncountable number of values, e.g. length of time interval, weight of food item, height of block of flats, depth of lake.

Definitions

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Example

Examples of discrete variables:

- The number of clicks on a given advertisement: 0,1,2…
- The number of consumers in a sample of 500 who favours a particular product over all competitors: x=0,1,2,…,500
- The number of bids received in a bond offering, x=0,1,2,…
- The number of errors on a page of an accountant’s ledger: x=0,1,2,…
- The number of customers waiting to be served in a restaurant at a particular time: x=0,1,2,…

Examples of continuous variables:

- The length of time between arrivals at a hospital clinic
- The depth at which a successful oil drilling venture first strikes oil
- The exact weight of a food item bought in a supermarket
- The exact amount of carbonated beverage loaded into a 12-ounce can-filling operation
- The exact temperature at which a room is heated

Definitions

Probability distribution:

is a function that describes probabilities of different outcomes in the experiment

Probability of each is between 0-1.

The sum of all probabilities equals one.

Probability distributions for many discrete and random variables that occur have been already derived.

Sample Space

An experiment is an act or process of observation that leads to a single outcome that cannot be predicted with certainty

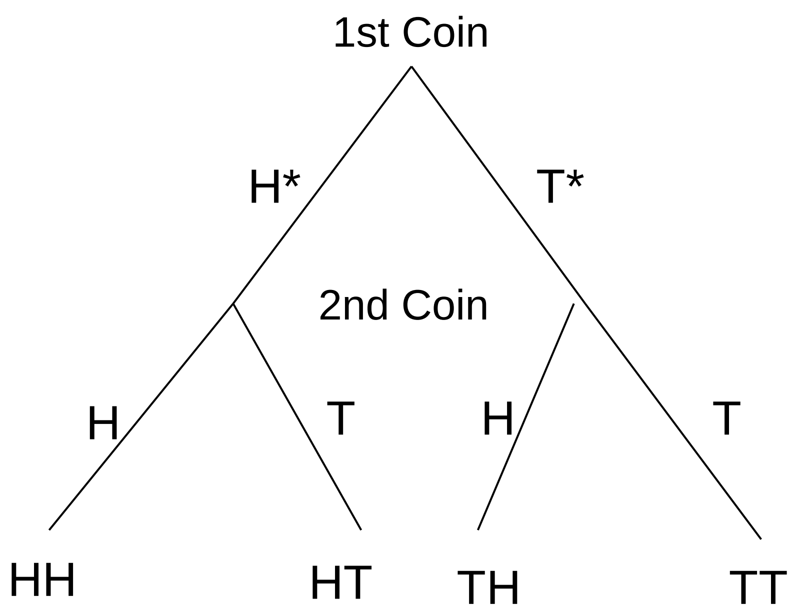
A sample point is the most basic outcome of an experiment
Requirements for the probability distribution of a discrete random variable.

Sample space is the set containing all sample points (possible outcomes)

Sample Space - example

Sample points from an experiment were we record the results of throwing two coins:

H – up face: „Head", T- up face: „Tail"



Sample points

Sample space - S:{HH, HT, TH, TT}

¼ ¼ ¼ ¼

Probability distribution

Expected Value

- Long-run average value of repetitions of an experiment.
- E(X) = mean, as n goes to infinity (law of large numbers)
- Properties
  - E(aX+bY+c) = aE(X) + bE(Y) + c
  - E(1(A)) = Pr(A)
  - E(X*Y) = E(X)*E(Y) only if X and Y are independent

Variance of a random variable

How much outcomes spread around the expected value.

$$Var(X) = E\big(x - E(X)\big)^2 = E(X^2) - E(X)^2$$

Properties:

$$Var(X) \geq 0$$

$$Var(aX + bY + c) = a^2 Var(X) + b^2 Var(Y) + 2ab\ Covar(XY)$$

If variables are independent: Covar(X,Y) = 0

Discrete random variable

Probability distribution

To describe a discrete random variable, we need to specify sample space and the probability associated with each element.

Requirements for the probability distribution of a discrete random variable:

$$p(x) \geq 0 \quad \text{for all values of x}$$

$$\sum p(x) = 1$$

Example

Experiment: tossing two balanced coins and observing the total number of heads

X – obtained number of heads, X = 0, 1, 2

Probability distribution:

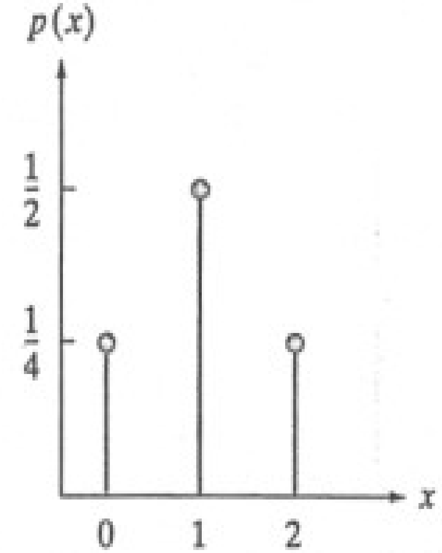
$$P(x = 0) = P(TT) = \frac{1}{4}$$

$$P(x = 1) = P(TH) + P(HT) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

$$P(x = 2) = P(HH) = \frac{1}{4}$$

Tabular Form

x	$p(x)$
0	$\frac{1}{4}$
1	$\frac{1}{2}$
2	$\frac{1}{4}$



a. Point representation of $p(x)$

Mean and variance

- The **mean** or **expected value**

$$\mu = EX = \sum x \cdot p(x)$$

- **The variance**

$$\sigma^2 = E\left[(x-\mu)^2\right] = \sum (x-\mu)^2 \cdot p(x)$$

- **Standard deviation**

$$\sigma = \sqrt{\sigma^2}$$

Example

Experiment: tossing two balanced coins and observing the total number of heads

X – obtained number of heads, X = 0, 1, 2

Prob. distribution:

Tabular Form

x	$p(x)$
0	$\frac{1}{4}$
1	$\frac{1}{2}$
2	$\frac{1}{4}$

$$\mu = 0\cdot\frac{1}{4} + 1\cdot\frac{1}{2} + 2\cdot\frac{1}{4} = 1$$

$$\sigma^2 = (0-1)^2\cdot\frac{1}{4} + (1-1)^2\cdot\frac{1}{2} + (2-1)^2\cdot\frac{1}{4} = \frac{1}{2}$$

$$\sigma = \sqrt{\frac{1}{2}}$$

Combonatorics Rule

A sample of n elements is to be drawn from a set of N elements. Then the number of different combinations (possible samples) is denoted by $\binom{N}{n}$

$\binom{N}{n} = \frac{N!}{n!(N-n)!}$, where factorial symbol (!) means:

$$N! = 1 * 2 * 3 * \cdots * N$$

Example:

What is the number of combinations of the 6-elemental sample from 49 elemental set?

Binomial distribution

- The experiment consists of n identical independent trials
- 2 possible outcomes on each trial: S- success and F – failure
- The probability of S (p) and F (q = 1-p) are constant.
- A binomial random variable measures the number of successes x in n trials
- The probability of x successes is computed as:

$$P(X = x) = \binom{n}{x} p^x (1 - p)^{n-x}$$

Binomial distribution

- **Mean** $$\mu = np$$
- **Variance** $$\sigma^2 = np(1-p)$$
- **Standard deviation** $$\sigma = \sqrt{np(1-p)}$$

Example

A machine that produces stampings for automobile engines is malfunctioning and producing 10% defectives. The defective and nondefective stampings proceed from the machine in a random manner. If the next five stampings are tested, find the probability that three of them are defective.

- X – the number of defectives in n=5 trials. It is a binomial random variable with p=0.1.

$$P(X=x)=\binom{n}{x}p^x(1-p)^{n-x}$$

$$P(X=3)=\frac{5!}{3!(5-3)!}(0.1)^3(0.9)^{5-3}=\frac{5!}{3!2!}(0.1)^3(0.9)^2=$$

$$=\frac{5\cdot 4\cdot 3\cdot 2\cdot 1}{(3\cdot 2\cdot 1)(2\cdot 1)}(0.1)^3(0.9)^2=0.0081$$

Example

Find the remaining probabilities, graph P(x), calculate the mean and standard deviation.

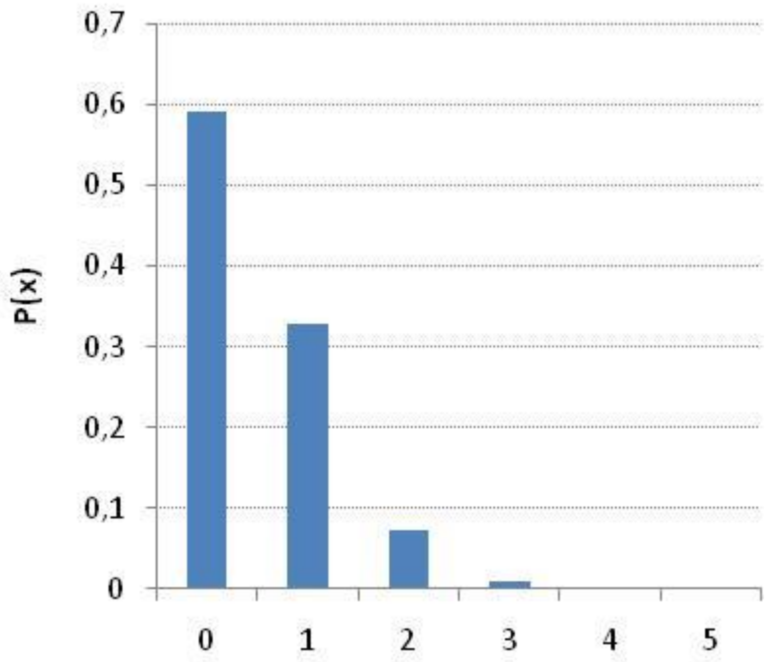
$$p(0) = \frac{5!}{0!(5-0)!}(.1)^0(.9)^{5-0} = \frac{5\cdot4\cdot3\cdot2\cdot1}{(1)(5\cdot4\cdot3\cdot2\cdot1)}(1)(.9)^5 = .59049$$

$$p(1) = \frac{5!}{1!(5-1)!}(.1)^1(.9)^{5-1} = 5(.1)(.9)^4 = .32805$$

$$p(2) = \frac{5!}{2!(5-2)!}(.1)^2(.9)^{5-2} = (10)(.1)^2(.9)^3 = .07290$$

$$p(4) = \frac{5!}{4!(5-4)!}(.1)^4(.9)^{5-4} = 5(.1)^4(.9) = .00045$$

$$p(5) = \frac{5!}{5!(5-5)!}(.1)^5(.9)^{5-5} = (.1)^5 = .00001$$



Example

Find the remaining probabilities, graph P(x), calculate the mean and standard deviation.

$$\mu = np = 5 \cdot 0.1 = 0.5$$

$$\sigma = \sqrt{np(1-p)} = \sqrt{5 \cdot 0.1 \cdot (1-0.1)} = 0.67$$

One may expect that out of 5 stampings 0.5 will be defective
with a standard deviation 0.67.

Example

For the same example find the probability that:

a) Less than two stampings will be defective
b) At most two stampings will be defective
c) More than 2 stampings will be defective
d) At least 2 stampings will be defective

Example

For the same example find the probability that:

a) **Less than two stampings will be defective**
b) At most two stampings will be defective
c) More than 2 stampings will be defective
d) At least 2 stampings will be defective

x	p(x)
0	0,59049
1	0,32805
2	0,0729
3	0,0081
4	0,00045
5	0,00001

a) $$P(X < 2) = P(X = 0) + P(X = 1) = 0.91854$$

Example

For the same example find the probability that:

a) Less than two stampings will be defective

b) At most two stampings will be defective

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x	p(x)
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a) $$P(X < 2) = P(X = 0) + P(X = 1) = 0.91854$$

b) $$P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2) = 0.99144$$

Example

For the same example find the probability that:

a) Less than two stampings will be defective
b) At most two stampings will be defective
c) **More than 2 stampings will be defective**
d) At least 2 stampings will be defective

x	p(x)
0	0,59049
1	0,32805
2	0,0729
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4	0,00045
5	0,00001

a) $$P(X < 2) = P(X = 0) + P(X = 1) = 0.91854$$

b) $$P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2) = 0.99144$$

c) $$P(X > 2) = P(X = 3) + P(X = 4) + P(X = 5) = 1 - P(X \leq 2) =$$
$$= 1 - 0.99144 = 0.00856$$

Example

For the same example find the probability that:

a) Less than two stampings will be defective
b) At most two stampings will be defective
c) More than 2 stampings will be defective
d) At least 2 stampings will be defective

x	p(x)
0	0,59049
1	0,32805
2	0,0729
3	0,0081
4	0,00045
5	0,00001

a) $$P(X < 2) = P(X = 0) + P(X = 1) = 0.91854$$

b) $$P(X \leq 2) = P(X = 0) + P(X = 1) + P(X = 2) = 0.99144$$

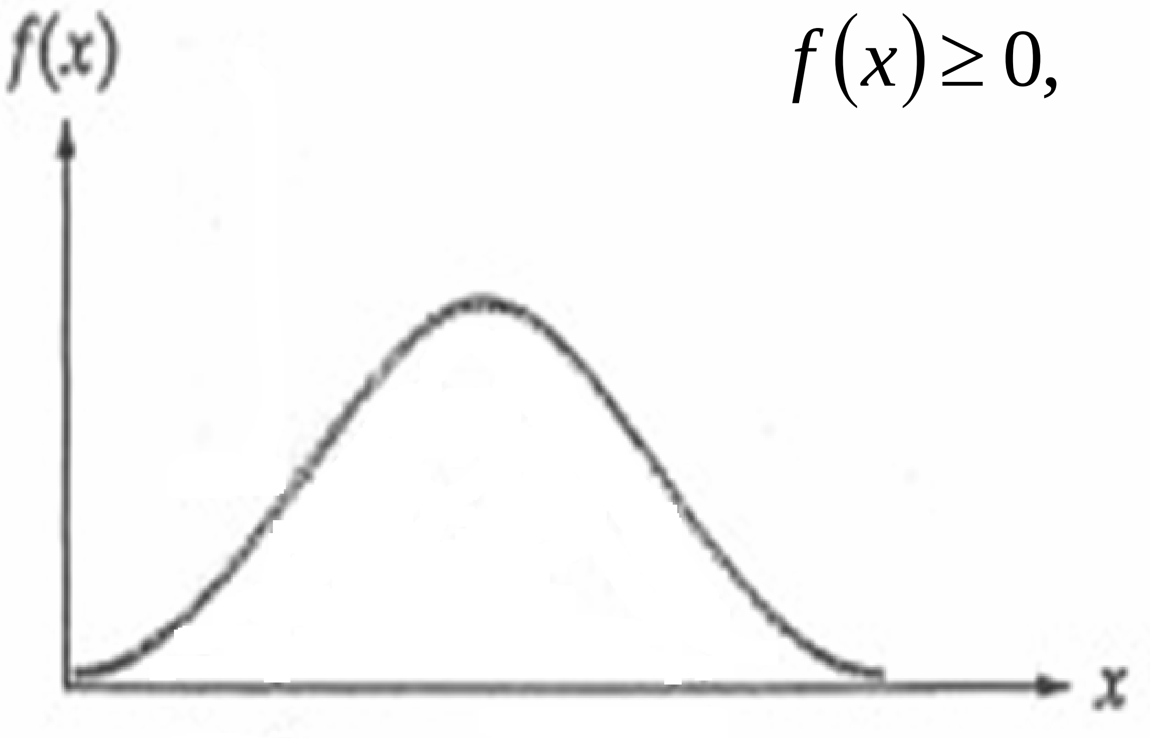
c) $$P(X > 2) = P(X = 3) + P(X = 4) + P(X = 5) = 1 - P(X \leq 2) =$$
$$= 1 - 0.99144 = 0.00856$$

d) $$P(X \geq 2) = P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5) = 1 - P(X < 2) =$$
$$= 1 - 0.91854 = 0.08146$$

Continuous random variable

Probability distribution

for a continuous random variable is represented by a **probability density function** (pdf) *f(x).*



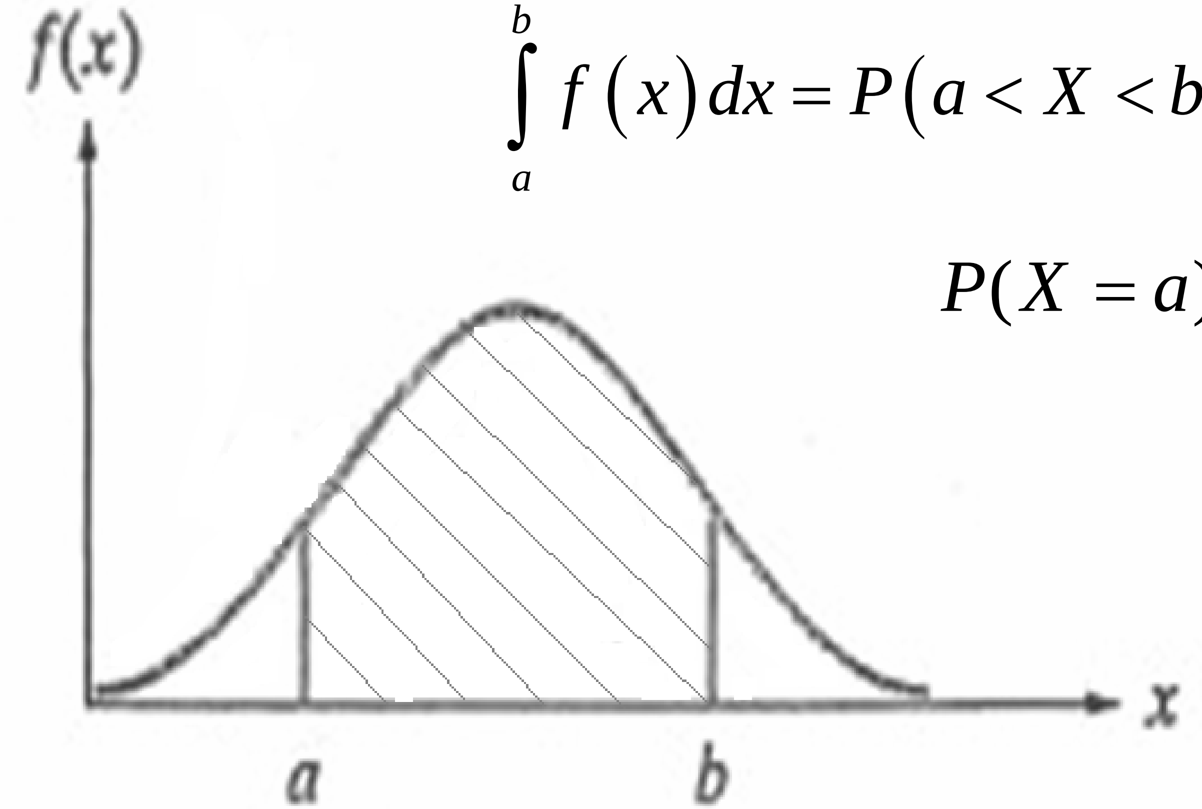
$$f(x) \geq 0,$$

Probability distribution

For a continuous random variable is represented by a **probability density function** (pdf) *f(x).*

$$\int_a^b f(x)\,dx = P(a < X < b) = P(a \le X \le b)$$

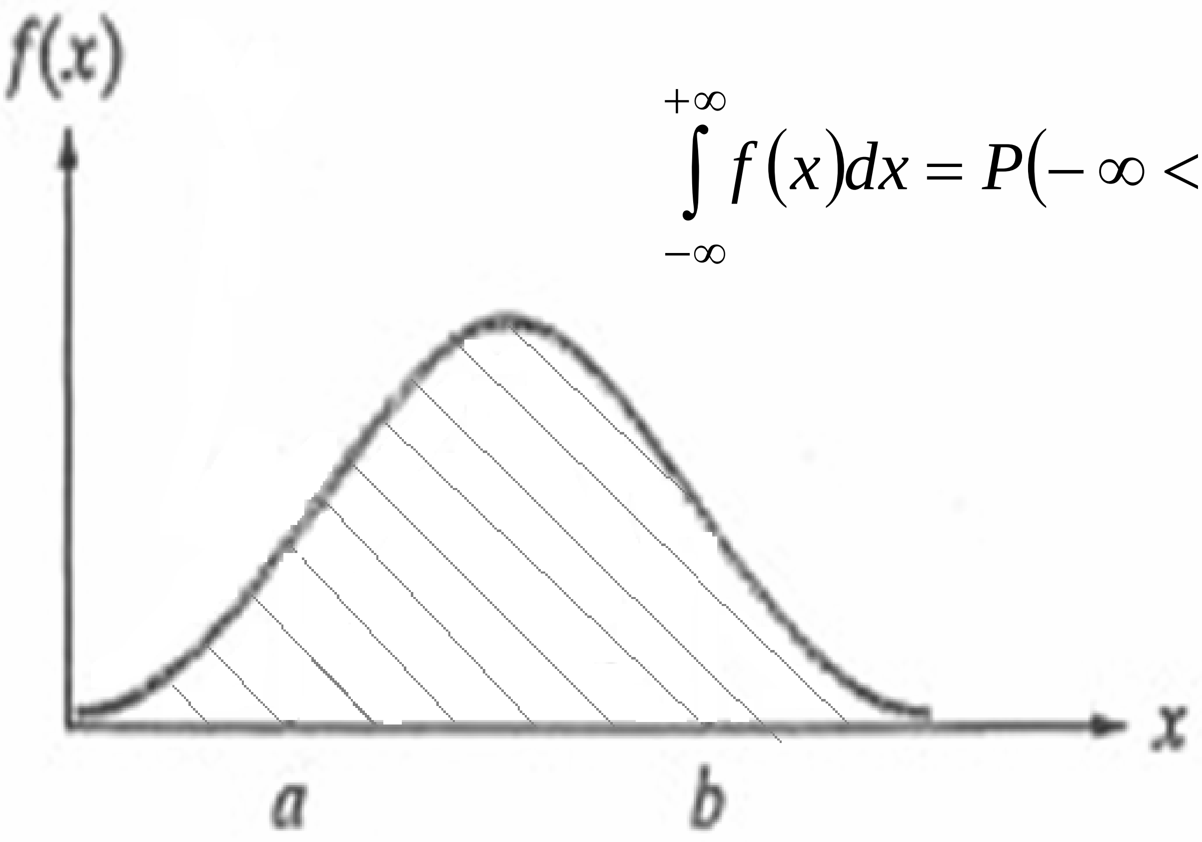
$$P(X = a) = \int_a^a f(x)\,dx = 0$$



Probability distribution

For a continuous random variable is represented by a **probability density function** (pdf) *f(x).*

$$\int_{-\infty}^{+\infty} f(x)dx = P(-\infty < X \leq +\infty) = 1$$



Mean and variance

- **The mean** or **expected value**

$$\mu = \int_{-\infty}^{\infty} x f(x)\, dx$$

- **The variance**

$$\sigma^2 = \int_{-\infty}^{\infty} [x-\mu]^2 f(x)\, dx$$

- **The standard deviation**

$$\sigma = \sqrt{\sigma^2}$$

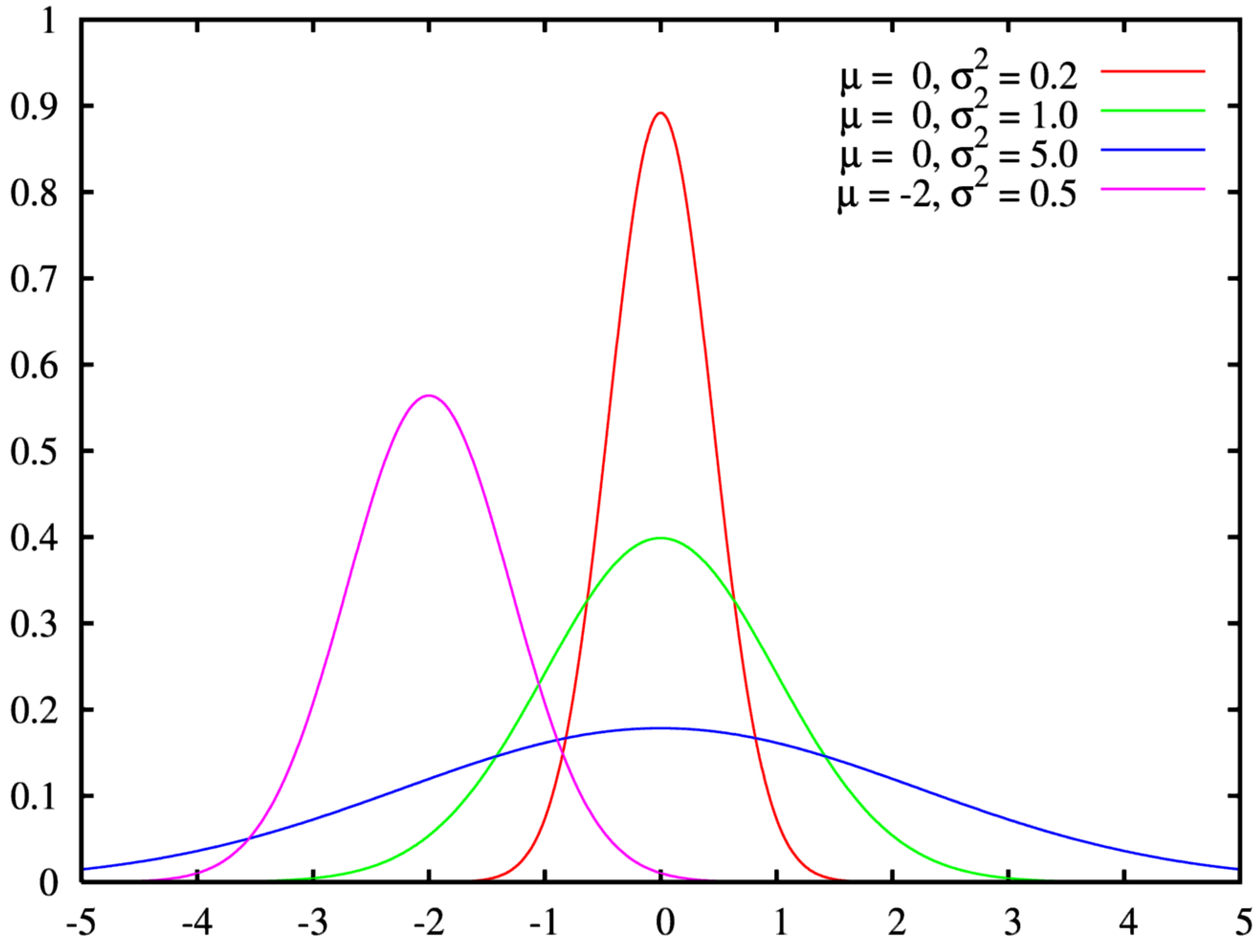
Normal distribution

- the most common distribution for continuous random variables
- characterised by two parameters, μ (mean) and σ (standard deviation), $X \sim N(\mu, \sigma)$
- symmetric around a mean
- its density function has a form:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-m)^2}{2\sigma^2}}, \qquad -\infty < x < +\infty$$

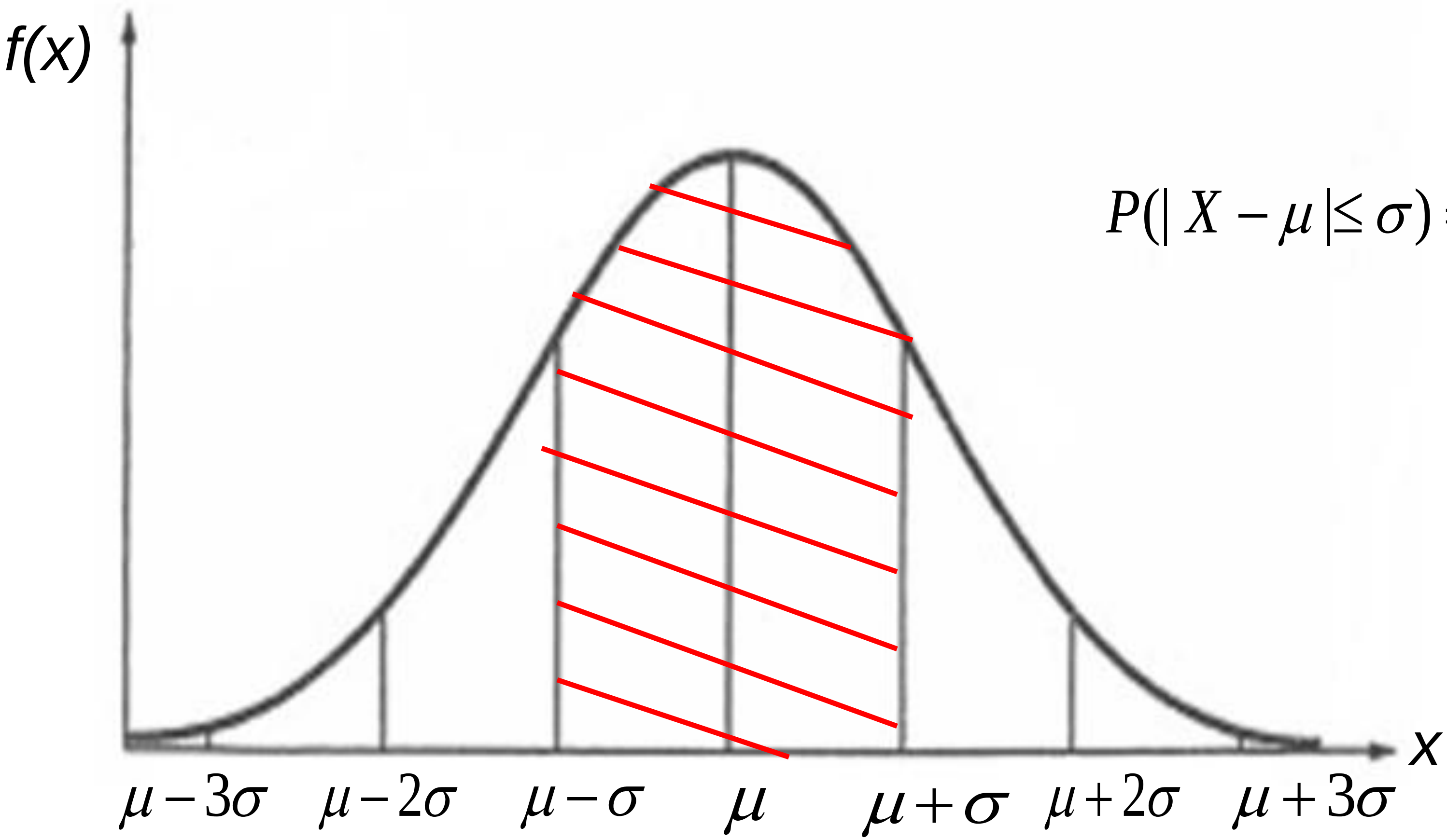
Normal distribution

Density function of the normal random variable,
different means and standard deviations



Normal distribution

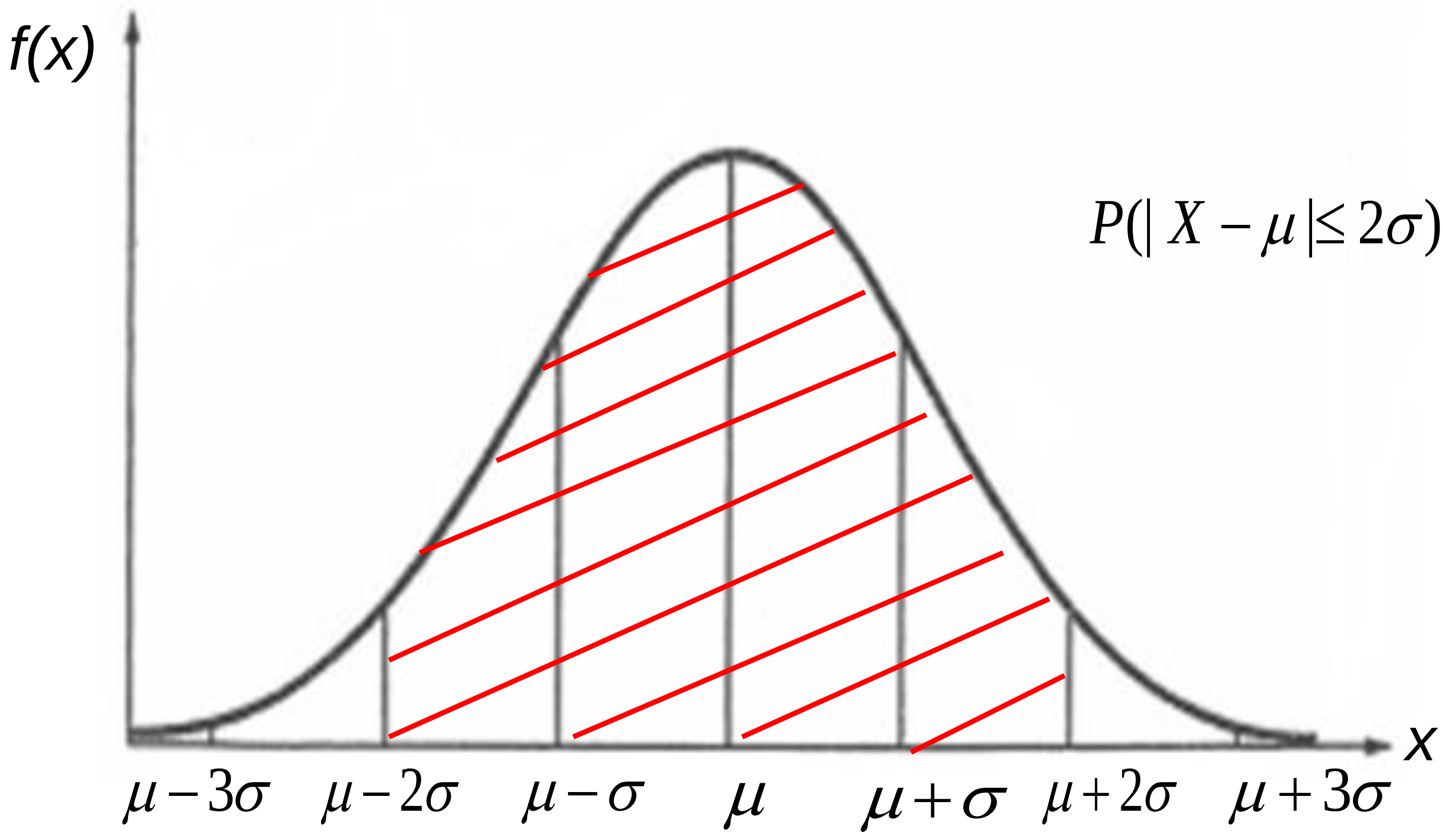
$\mu \pm 3\sigma$ rule



$$P(|X-\mu| \le \sigma) = 0.6827$$

Normal distribution

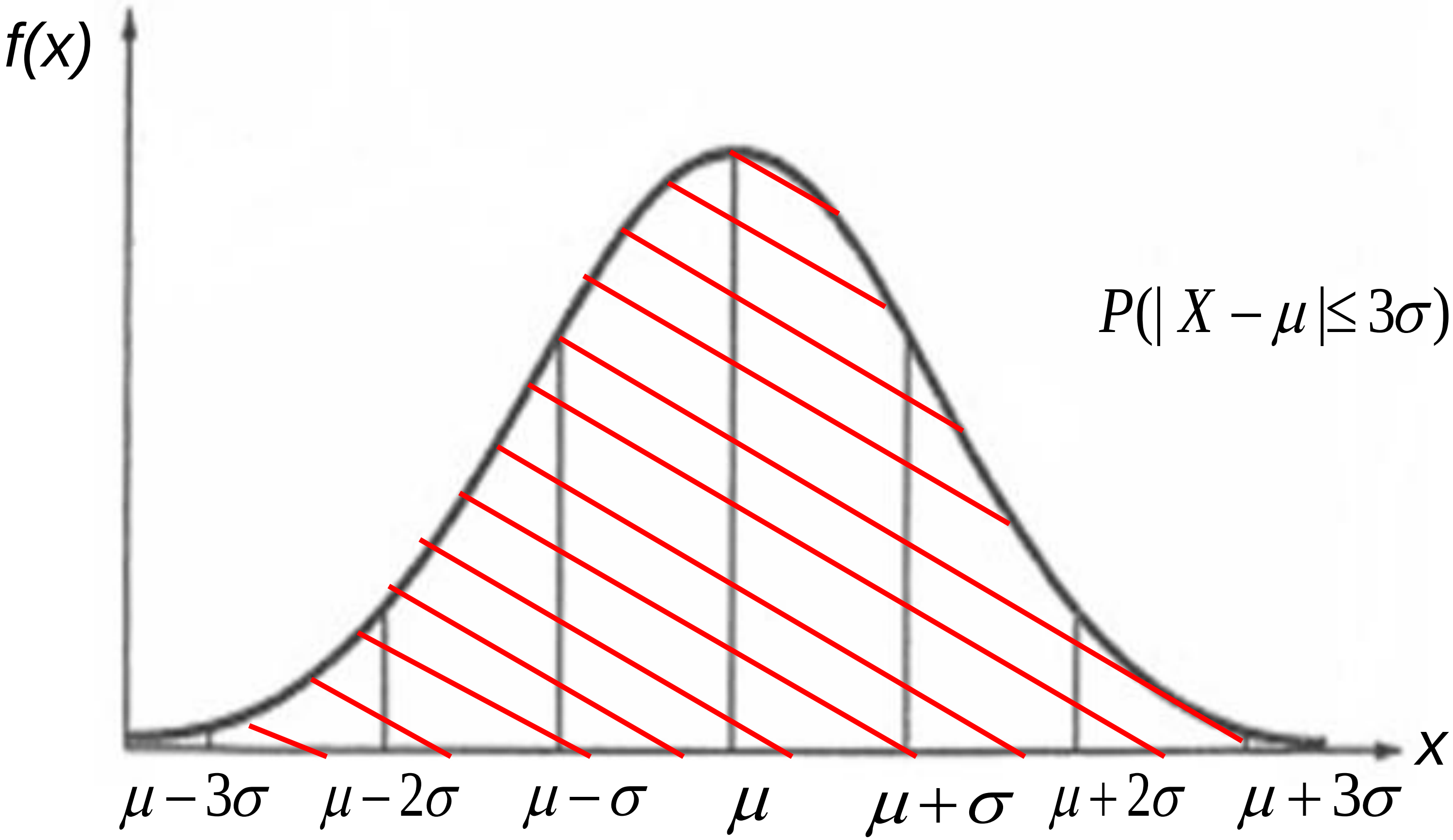
$\mu \pm 3\sigma$ rule



$$P(|X-\mu| \le 2\sigma) = 0.9545$$

Normal distribution

$\mu \pm 3\sigma$ rule



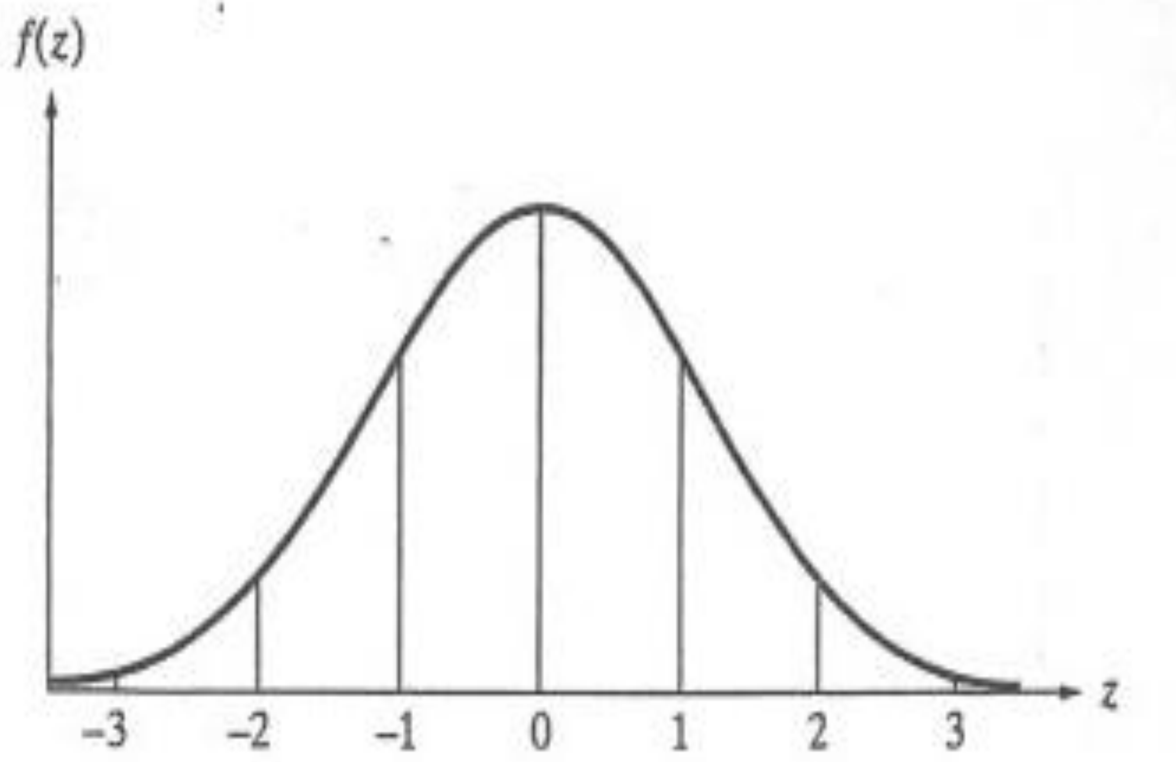
$P(|X-\mu| \le 3\sigma) = 0.9973$

Normal distribution

- Computing the area under the normal probability distribution is difficult.
- But… probabilities for a standard normal distribution were computed and are printed in statistical tables.
- Hence, in order to compute probabilities for a $X \sim N(\mu, \sigma)$ we first transform this distribution into a standard normal distribution.

Standard normal distribution

- A normal distribution with $\mu = 0$ and $\sigma = 1$



Standardisation

If $X \sim N(\mu, \sigma)$ then: $Z = \dfrac{X - \mu}{\sigma} \sim N(0,1)$

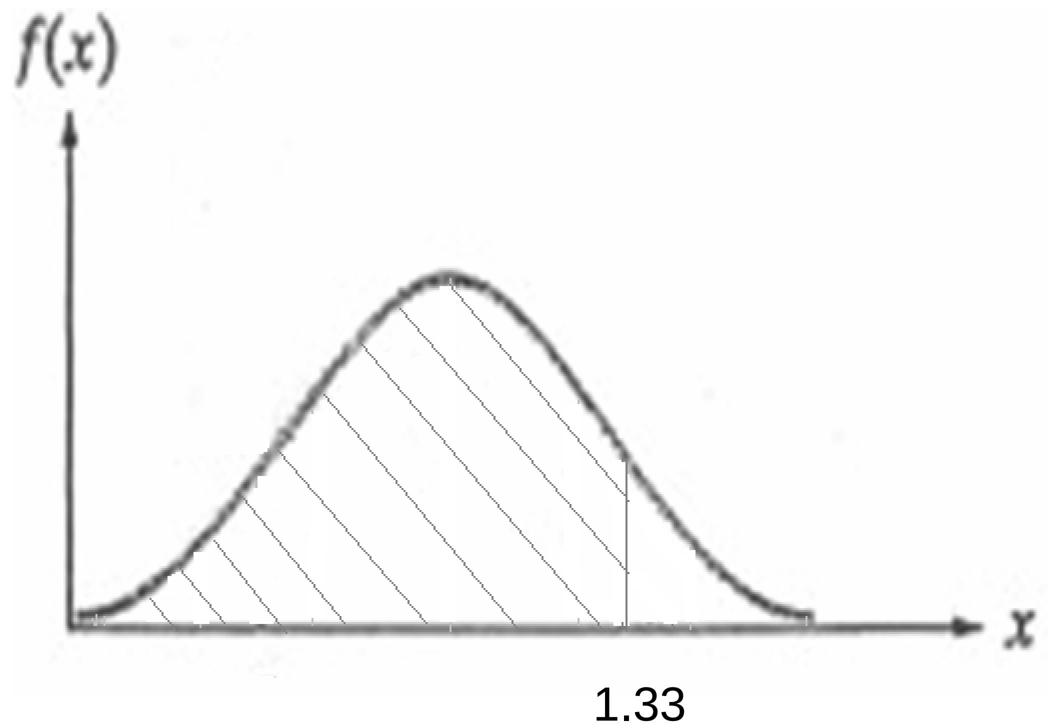
The probabilities P(Z<z) can be found in a statistical table "Cumulative normal distribution"

(We can always use specialized software too)

Example 1

Find the probability that a variable with a standard normal distribution takes values lower than 1.33.

P(Z<1.33)=?



Example 1

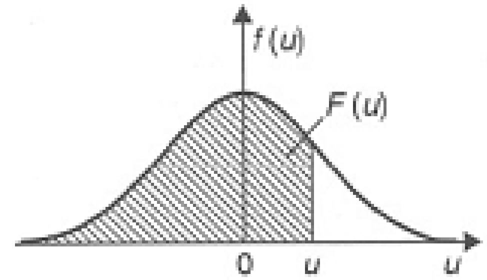


Table 1. Cumulative normal distribution

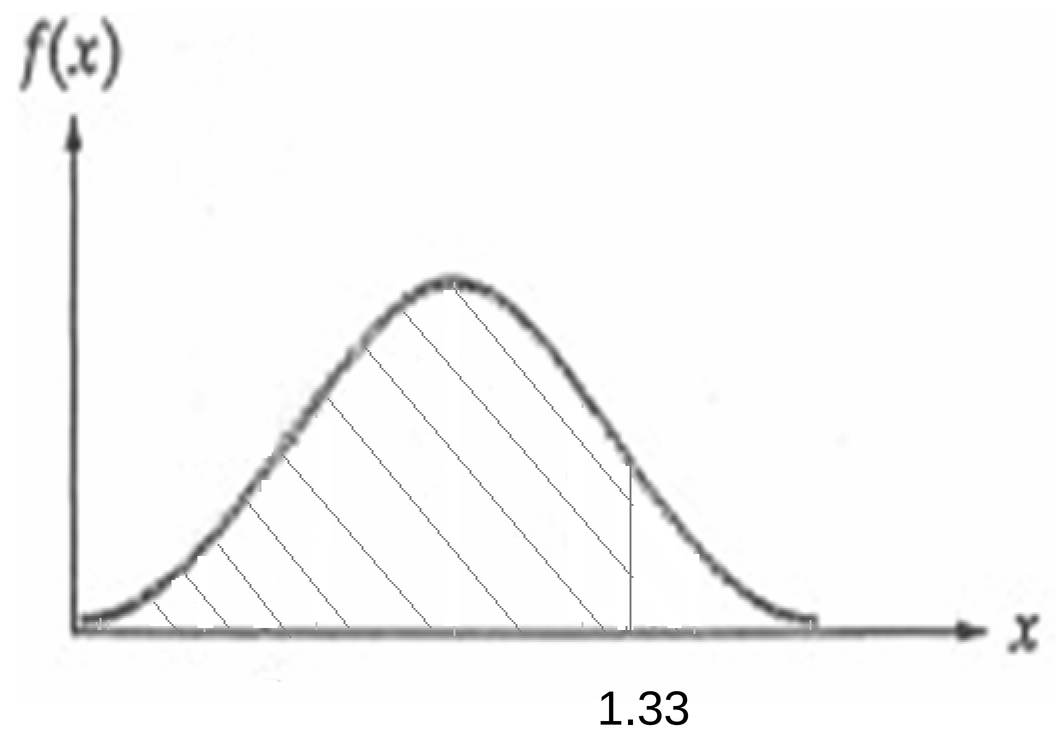
$F(u) = \mathrm{P}(U \leqslant u)$ dla $u > 0$

u	0,00	0,01	0,02	0,03	0,04	0,05	0,06	0,07	0,08	0,09	u
0,0	0,5000	0,5040	0,5080	0,5120	0,5160	0,5199	0,5239	0,5279	0,5319	0,5359	0,0
0,1	,5398	,5438	,5478	,5517	,5557	,5596	,5636	,5675	,5714	,5753	0,1
0,2	,5793	,5832	,5861	,5910	,5948	,5987	,6026	,6064	,6103	,6141	0,2
0,3	,6179	,6217	,6255	,6293	,6331	,6368	,6406	,6443	,6480	,6517	0,3
0,4	,6554	,6591	,6628	,6664	,6700	,6736	,6772	,6808	,6844	,6879	0,4
0,5	,6915	,6950	,6985	,7019	,7054	,7088	,7123	,7157	,7190	,7224	0,5
0,6	,7257	,7291	,7324	,7357	,7389	,7422	,7454	,7486	,7517	,7549	0,6
0,7	,7580	,7611	,7642	,7673	,7703	,7734	,7764	,7794	,7823	,7852	0,7
0,8	,7881	,7910	,7939	,7967	,7995	,8023	,8051	,8078	,8106	,8133	0,8
0,9	,8159	,8186	,8212	,8238	,8264	,8289	,8315	,8340	,8365	,8389	0,9
1,0	,8413	,8438	,8461	,8485	,8508	,8531	,8554	,8577	,8599	,8621	1,0
1,1	,8643	,8665	,8686	,8708	,8729	,8749	,8770	,8790	,8810	,8830	1,1
1,2	,8849	,8869	,8888	,8907	,8925	,8944	,8962	,8980	,8997	,90147	1,2
1,3	,90320	,90490	,90658	,90824	,90988	,91149	,91309	,91466	,91621	,91774	1,3
1,4	,91924	,92073	,92220	,92354	,92507	,92647	,92785	,92922	,93056	,93189	1,4

Example 1

Find the probability that the standard normal random variable is less than 1.33.

$P(Z<1.33)=0.9082$



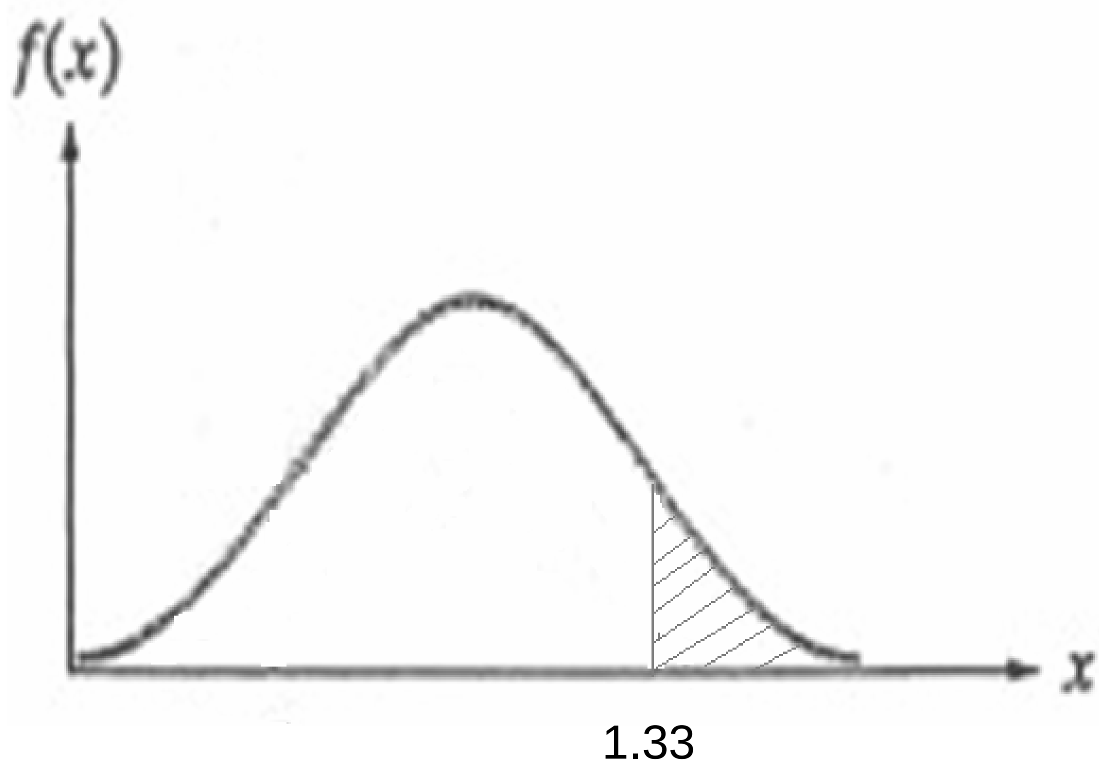
Example 1

Find the probability that the standard normal random variable is more than 1.33.

P(Z>1.33)=

= 1-P(Z<1.33)=

=1-0.9082=0.0918



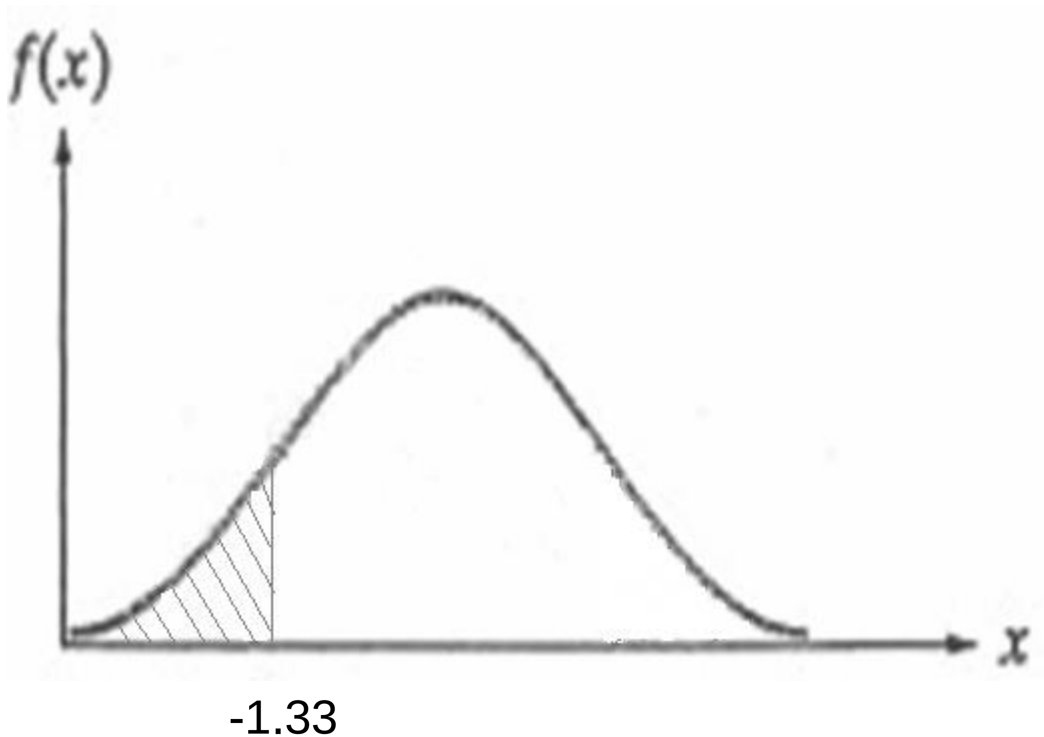
Example 1

Find the probability that the standard normal random variable is less than -1.33.

$P(Z<-1.33)=$

$=P(Z>1.33)=$

$=0.0918$



Example 1

Find the probability that the standard normal random variable is lower than -1.33 or higher than 1.33.

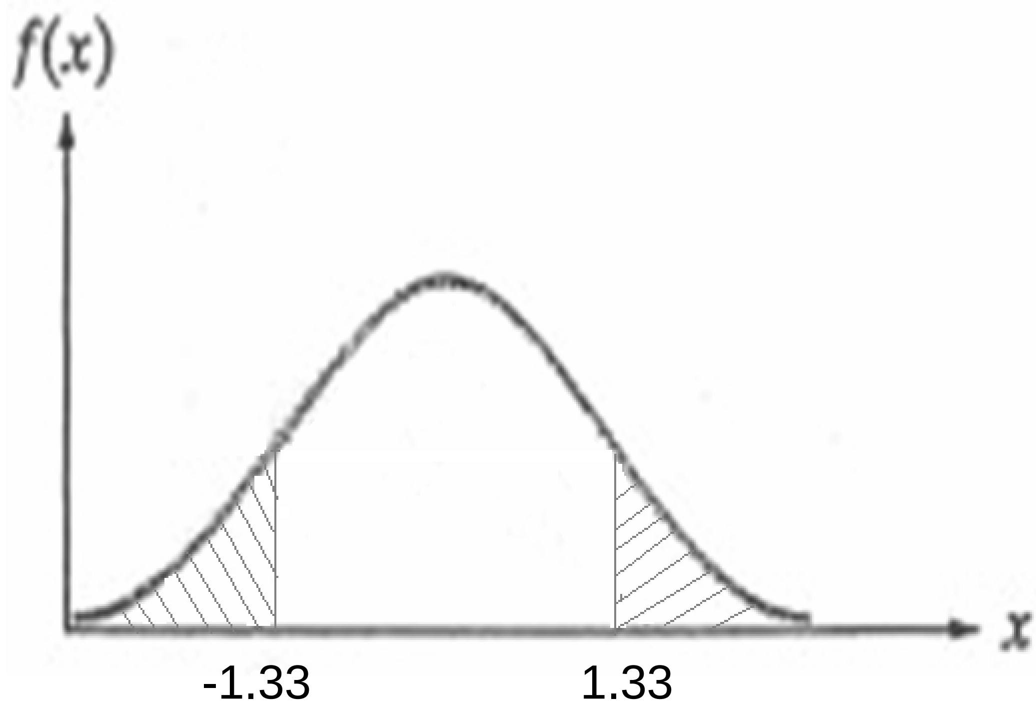
$P(|Z|>1.33)=$

$=P(Z<-1.33)+$

$+P(Z>1.33)=$

$=2*0.0918=$

$=0.1836$



Example 1

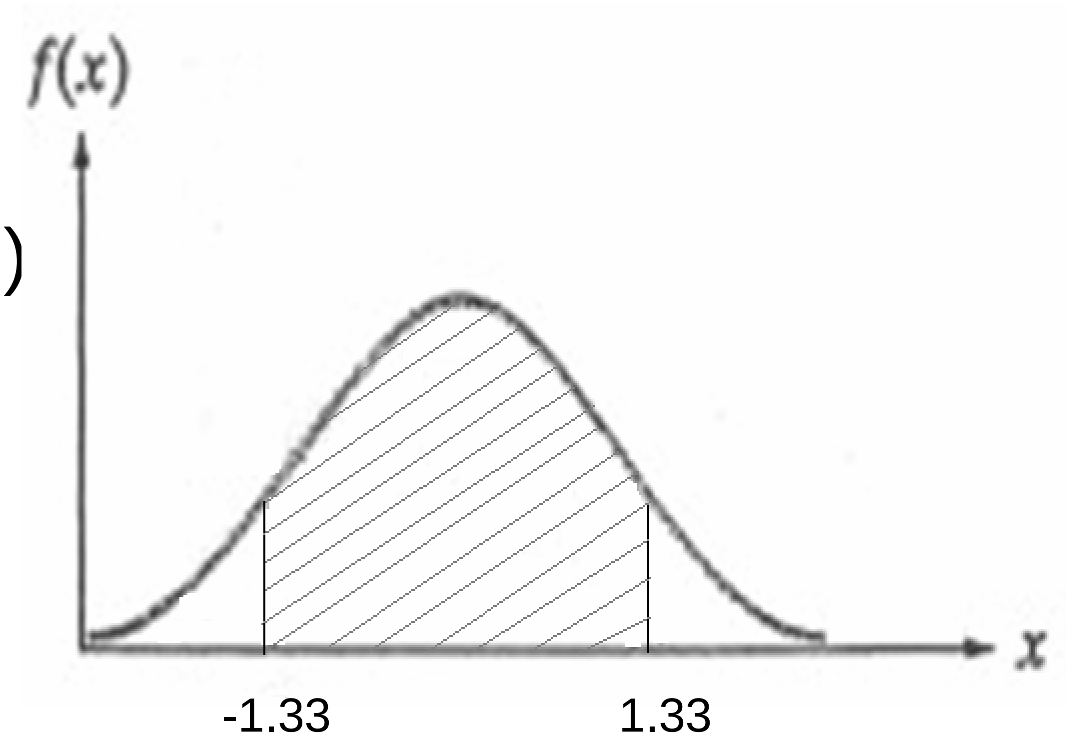
Find the probability that the standard normal random variable falls between -1.33 and 1.33.

$P(-1.33<Z<1.33)=$

$=P(Z<1.33)-P(Z<-1.33)$

$= 0.9082-0.0918=$

$=0.8164$



Example 2

Assume that the length of time, x, between charges of a mobile phone is normally distributed with a mean of 10 hours and a standard deviation of 1.5 hours. Find the probability that the battery will last between 8 and 12 hours.

X – length of time between charges of a mobile phone

$$X \sim N(10, 1.5)$$

$$P(8 < X < 12) = P\left(\frac{8-10}{1.5} < Z < \frac{12-10}{1.5}\right) =$$

$$= P(-1.33 < Z < 1.33) = 0.8164$$

Example 3

Suppose a return on 100 EUR investment made in a CEO company 3 years earlier. The data on return were shown to be normally distributed with a mean of 112 EUR and a standard deviation 70 EUR. A CEO company is randomly selected from the dataset.

- Find the probability that the return will be less than 40 EUR.
- Find the probability that the return will exceed 200 EUR.
- Find the probability that the return will fall between 40EUR and 120 EUR.

Example 3

Suppose a return on 100 EUR investment made in a CEO company 3 years earlier. The data on return were shown to be normally distributed with a mean of 112 EUR and a standard deviation 70 EUR. A CEO company is randomly selected from the dataset.
Find the probability that the return will be less than 40 EUR.

X – return on 100 EUR

$$X \sim N(112, 70)$$

$$P(X<40)=P\left(Z<\frac{40-112}{70}\right)=P\left(Z<-1.028\right)=1-P\left(Z<1.028\right)=$$

$$=1-0.8485=0.1516$$

Example 3

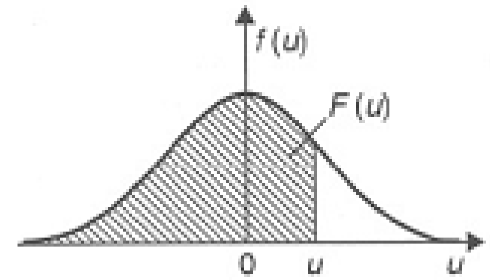


Table 1. Cumulative normal distribution

$F(u) = \mathrm{P}(U \leqslant u)$ dla $u > 0$

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0,2	,5793	,5832	,5861	,5910	,5948	,5987	,6026	,6064	,6103	,6141	0,2
0,3	,6179	,6217	,6255	,6293	,6331	,6368	,6406	,6443	,6480	,6517	0,3
0,4	,6554	,6591	,6628	,6664	,6700	,6736	,6772	,6808	,6844	,6879	0,4
0,5	,6915	,6950	,6985	,7019	,7054	,7088	,7123	,7157	,7190	,7224	0,5
0,6	,7257	,7291	,7324	,7357	,7389	,7422	,7454	,7486	,7517	,7549	0,6
0,7	,7580	,7611	,7642	,7673	,7703	,7734	,7764	,7794	,7823	,7852	0,7
0,8	,7881	,7910	,7939	,7967	,7995	,8023	,8051	,8078	,8106	,8133	0,8
0,9	,8159	,8186	,8212	,8238	,8264	,8289	,8315	,8340	,8365	,8389	0,9
1,0	,8413	,8438	,8461	,8485	,8508	,8531	,8554	,8577	,8599	,8621	1,0
1,1	,8643	,8665	,8686	,8708	,8729	,8749	,8770	,8790	,8810	,8830	1,1
1,2	,8849	,8869	,8888	,8907	,8925	,8944	,8962	,8980	,8997	,90147	1,2
1,3	,90320	,90490	,90658	,90824	,90988	,91149	,91309	,91466	,91621	,91774	1,3
1,4	,91924	,92073	,92220	,92354	,92507	,92647	,92785	,92922	,93056	,93189	1,4

Example 3

Suppose a return on 100 EUR investment made in a CEO company 3 years earlier. The data on return were shown to be normally distributed with a mean of 112 EUR and a standard deviation 70 EUR. A CEO company is randomly selected from the dataset.
Find the probability that the return will exceed 200 EUR.

X – return on 100 EUR

X~N(112, 70)

$$P(X > 200) = 1 - P\left(Z < \frac{200-112}{70}\right) = 1 - P(Z < 1.26) = 1 - 0.8962$$

Example 3

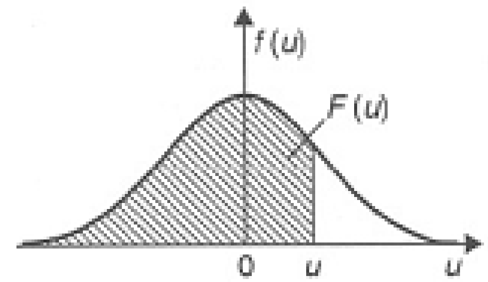

Table 1. Cumulative normal distribution

$F(u) = \mathrm{P}(U \leqslant u)$ dla $u > 0$

u	0,00	0,01	0,02	0,03	0,04	0,05	0,06	0,07	0,08	0,09	u
0,0	0,5000	0,5040	0,5080	0,5120	0,5160	0,5199	0,5239	0,5279	0,5319	0,5359	0,0
0,1	,5398	,5438	,5478	,5517	,5557	,5596	,5636	,5675	,5714	,5753	0,1
0,2	,5793	,5832	,5861	,5910	,5948	,5987	,6026	,6064	,6103	,6141	0,2
0,3	,6179	,6217	,6255	,6293	,6331	,6368	,6406	,6443	,6480	,6517	0,3
0,4	,6554	,6591	,6628	,6664	,6700	,6736	,6772	,6808	,6844	,6879	0,4
0,5	,6915	,6950	,6985	,7019	,7054	,7088	,7123	,7157	,7190	,7224	0,5
0,6	,7257	,7291	,7324	,7357	,7389	,7422	,7454	,7486	,7517	,7549	0,6
0,7	,7580	,7611	,7642	,7673	,7703	,7734	,7764	,7794	,7823	,7852	0,7
0,8	,7881	,7910	,7939	,7967	,7995	,8023	,8051	,8078	,8106	,8133	0,8
0,9	,8159	,8186	,8212	,8238	,8264	,8289	,8315	,8340	,8365	,8389	0,9
1,0	,8413	,8438	,8461	,8485	,8508	,8531	,8554	,8577	,8599	,8621	1,0
1,1	,8643	,8665	,8686	,8708	,8729	,8749	,8770	,8790	,8810	,8830	1,1
1,2	,8849	,8869	,8888	,8907	,8925	,8944	,8962	,8980	,8997	,90147	1,2
1,3	,90320	,90490	,90658	,90824	,90988	,91149	,91309	,91466	,91621	,91774	1,3
1,4	,91924	,92073	,92220	,92354	,92507	,92647	,92785	,92922	,93056	,93189	1,4

Example 3

Suppose a return on 100 EUR investment made in a CEO company 3 years earlier. The data on returns were shown to be normally distributed with a mean of 112 EUR and a standard deviation 70 EUR. A CEO company is randomly selected from the dataset. Find the probability that the return will fall between 40EUR and 120 EUR.

X – return on 100 EUR

X~N(112, 70)

$$P(40 < X < 120) = P\left(\frac{40-112}{70} < Z < \frac{120-112}{70}\right) = P\left(-1.028 < Z < 0.11\right) =$$

$$= P(Z < 0.11) - P(Z < -1.028) = 0.5438 - 0.1516 = 0.3922$$

Example 3

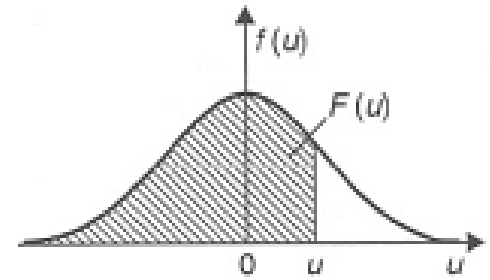

Table 1. Cumulative normal distribution

$F(u) = P(U \leqslant u)$ dla $u > 0$

u	0,00	0,01	0,02	0,03	0,04	0,05	0,06	0,07	0,08	0,09	u
0,0	0,5000	0,5040	0,5080	0,5120	0,5160	0,5199	0,5239	0,5279	0,5319	0,5359	0,0
0,1	,5398	$\boxed{,5438}$	,5478	,5517	,5557	,5596	,5636	,5675	,5714	,5753	0,1
0,2	,5793	,5832	,5861	,5910	,5948	,5987	,6026	,6064	,6103	,6141	0,2
0,3	,6179	,6217	,6255	,6293	,6331	,6368	,6406	,6443	,6480	,6517	0,3
0,4	,6554	,6591	,6628	,6664	,6700	,6736	,6772	,6808	,6844	,6879	0,4
0,5	,6915	,6950	,6985	,7019	,7054	,7088	,7123	,7157	,7190	,7224	0,5
0,6	,7257	,7291	,7324	,7357	,7389	,7422	,7454	,7486	,7517	,7549	0,6
0,7	,7580	,7611	,7642	,7673	,7703	,7734	,7764	,7794	,7823	,7852	0,7
0,8	,7881	,7910	,7939	,7967	,7995	,8023	,8051	,8078	,8106	,8133	0,8
0,9	,8159	,8186	,8212	,8238	,8264	,8289	,8315	,8340	,8365	,8389	0,9
1,0	,8413	,8438	,8461	,8485	,8508	,8531	,8554	,8577	,8599	,8621	1,0
1,1	,8643	,8665	,8686	,8708	,8729	,8749	,8770	,8790	,8810	,8830	1,1
1,2	,8849	,8869	,8888	,8907	,8925	,8944	,8962	,8980	,8997	,90147	1,2
1,3	,90320	,90490	,90658	,90824	,90988	,91149	,91309	,91466	,91621	,91774	1,3
1,4	,91924	,92073	,92220	,92354	,92507	,92647	,92785	,92922	,93056	,93189	1,4

Thank you for your attention