

STATISTICS

Lab4

Random variable and distributions

Warsaw School of Economics
Winter Semester 2017

based on J.T. Mc Clave, P.G. Benson, T. Sincich: Statistics for Business and Economics, 11th Edition, 2010

Exercise 1

Security analysts are professionals who devote full-time efforts to evaluating the investment worth of a narrow list of stocks. For example, one security analyst might specialize in bank stocks while another specializes in evaluating firms in the computer industry. The following variables are of interest to security analysts (Radcliffe, *Investments: Concepts, Analysis and Strategy*, 1994). Which are discrete and which are continuous random variables?

- a. The closing price of a particular stock on the New York Stock Exchange
- b. The number of shares of a particular stock that are traded each business day
- c. The quarterly earnings of a particular firm
- d. The percentage change in yearly earnings between 2002 and 2003 for a particular firm
- e. The number of new products introduced per year by a firm
- f. The time until a pharmaceutical company gains approval from the U.S. Food and Drug Administration to market a new drug

Source: J.T. Mc Clave,
P.G. Benson, T. Sincich:
Statistics for Business
and Economics, 9th
Edition, 2005

Exercise 1a

Which of the following describe continuous random variables, and which describe discrete random variables?

- a. The number of newspapers sold by the *New York Times* each month
- b. The amount of ink used in printing a Sunday edition of the *New York Times*
- c. The actual number of ounces in a one-gallon bottle of laundry detergent
- d. The number of defective parts in a shipment of nuts and bolts
- e. The number of people collecting unemployment insurance each month

Exercise 2 (a,b)



a) A player throws a dice.

- Please describe the sample space for a single toss of a single die.
- What is the probability of each of the sample points?
- What is the probability of scoring a number higher than 4?
- What is the sample space and probability distribution of single toss of two dice?



b) Please describe the probability that one card drawn from a typical pack of 52 cards is an ace, a spade, an ace and a spade at the same time?

Exercise 3



In Polish version of Lotto lottery the main prize amounts to at least 1 000 000 PLN. In order to win this prize you need to correctly select six numbers from a set of numbers ranging from 1 to 49. Correct selection means choosing the same ball numbers as appear in the draw few days later.

Use the Combination Rule to determine the total number of combinations of 6 numbers selected from 49. What is the probability of winning main prize if you buy a single ticket? How you assess the probability that one of the participants (usually several millions) will win this game?

* What is the probability of winning the lowest prize (about 10 PLN) for choosing the combination of 6 numbers that contain 3 correct numbers out of the total of 49?

Exercise 4

The probability of winning or losing a small amounts of money in a game can be described by the following random variable:

Result of the game (x_i)	-2	-1	0	1	2
Cumulative probability distribution $F(x_i)$	0,1	0,35	0,65	0,85	1

- Please calculate the probability of losing and winning in this game.
- Please calculate the expected value of this random variable (mean). What is the interpretation?
- Please calculate the standard deviation in this game. What is the interpretation?

Exercise 5

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Test of the “Chinese language course” consists of 10 questions. Each question has 4 answers and only one answer is correct each time. The student receives 1 point per each question with positive answer and 0 points if the answer is not correct. Suppose that you don’t know Chinese language?

- What is the name of the random variable that can be used to describe this situation?
- What is the probability that only one answer will be correct?
- What is the probability that the score will be “very good” (one or no mistakes) ?
- What is the probability of passing this exam (at least 6 correct answers)?
- What is the expected value of correct answers in that test?
- What is the standard deviation of correct answers?

Exercise 6

According to the *Journal of Business Venturing* (Vol. 17, 2002), 27% of all small businesses owned by non-Hispanic whites nationwide are women-owned firms.

- a. In a sample of 200 small businesses owned by non-Hispanic whites, how many would you expect to be female owned?
- b. If eight small businesses owned by non-Hispanic whites were randomly selected, what is the probability that none are female owned? That half are female owned?

Exercise 7

A Federal Trade Commission (FTC) study of the pricing accuracy of electronic checkout scanners at stores found that one of every 30 items is priced incorrectly (*Price Check II: A Follow-Up Report on the Accuracy of Checkout Scanner Prices*, Dec. 16, 1998). Suppose the FTC randomly selects five items at a retail store and checks the accuracy of the scanner price of each. Let x represent the number of the five items that is priced incorrectly.

- a. Show that x is (approximately) a binomial random variable.
- b. Use the information in the FTC study to estimate p for the binomial experiment.
- c. What is the probability that exactly one of the five items is priced incorrectly by the scanner?
- d. What is the probability that at least one of the five items is priced incorrectly by the scanner?

Exercise 8

- A poll by the Gallup Organization sponsored by Philadelphia-based CIGNA Integrated Care found that about 40% of employees have missed work due to a musculoskeletal (back) injury of some kind (*National Underwriter*, Apr. 5, 1999). Let x be the number of sampled workers who have missed work due to a back injury.
- Explain why x is approximately a binomial random variable.
 - Use the Gallup poll data to estimate p for the binomial random variable of part a.
 - A random sample of 10 workers is to be drawn from a particular manufacturing plant. Use the p from part b to find the mean and standard deviation of x , the number of workers that missed work due to back injuries.
 - For the sample in part c, find the probability that exactly one worker missed work due to a back injury. That more than one worker missed work due to a back injury.

Exercise 9

Find the following probabilities for the standard normal random variable z :

a. $P(-1 \leq z \leq 1)$

b. $P(-2 \leq z \leq 2)$

c. $P(-2.16 \leq z \leq .55)$

d. $P(-.42 < z < 1.96)$

e. $P(z \geq -2.33)$

f. $P(z < 2.33)$

Exercise 10

Suppose x is a normally distributed random variable with $\mu = 11$ and $\sigma = 2$. Find each of the following:

a. $P(10 \leq x \leq 12)$

b. $P(6 \leq x \leq 10)$

c. $P(13 \leq x \leq 16)$

d. $P(7.8 \leq x \leq 12.6)$

e. $P(x \geq 13.24)$

f. $P(x \geq 7.62)$

Exercise 11

The random variable x has a normal distribution with $\mu = 1,000$ and $\sigma = 10$.

- a. Find the probability that x assumes a value more than 2 standard deviations from its mean. More than 3 standard deviations from μ .
- b. Find the probability that x assumes a value within 1 standard deviation of its mean. Within 2 standard deviations of μ .
- c. Find the value of x that represents the 80th percentile of this distribution. The 10th percentile.

Exercise 12

The weight of the models in “Bony bodies” global agency can be describe by a Normal distribution $N(55, 10)$. Please calculate without computer and then using excel following probabilities:

- a) What is the probability of selecting a model with weight 50kg?
- b) What is the probability of selecting a model that weights less than 60kg?
- c) What is the probability of selecting a model that weights more than 60kg?
- d) What is the probability of selecting a model with a weight higher than 45kg?
- e) What is the probability of selecting a model that weights between 40 and 50kg?

Exercise 13

The problem of matching aircraft to passenger demand on each flight leg is called the *flight assignment problem* in the airline industry. *Spill* is defined as the number of passengers not carried because the aircraft's capacity is insufficient. A solution to the flight assignment problem at Delta Airlines was published in *Interfaces* (Jan.–Feb. 1994). The authors—four Delta Airlines researchers and a Georgia Tech professor (Roy Marsten)—demonstrated their approach with an example in which passenger demand for a particular flight leg is normally distributed with a mean of 125 passengers and a standard deviation of 45. Consider a Boeing 727 with a capacity of 148 passengers and a Boeing 757 with a capacity of 182.

- a. What is the probability that passenger demand will exceed the capacity of the Boeing 727? The Boeing 757?
- b. If the 727 is assigned to the flight leg, what is the probability that the flight will depart with one or more empty seats? Answer the same question for the Boeing 757.
- c. If the 727 is assigned to the flight, what is the probability that the spill will be more than 100 passengers?

Source: J.T. Mc Clave,
P.G. Benson, T. Sincich:
Statistics for Business
and Economics, 9th
Edition, 2005

Exercise 14

Before negotiating a long-term construction contract, building contractors must carefully estimate the total cost of completing the project. Benzion Barlev of New York University proposed a model for total cost of a long-term contract based on the normal distribution (*Journal of Business Finance and Accounting*, July 1995). For one particular construction contract, Barlev assumed total cost, x , to be normally distributed with mean \$850,000 and standard deviation \$170,000. The revenue, R , promised to the contractor is \$1,000,000.

- a. The contract will be profitable if revenue exceeds total cost. What is the probability that the contract will be profitable for the contractor?
- b. What is the probability that the project will result in a loss for the contractor?
- c. Suppose the contractor has the opportunity to renegotiate the contract. What value of R should the contractor strive for in order to have a .99 probability of making a profit?

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Exercise 15

The *monthly rate of return* of a stock is a measure investors frequently use for evaluating the behavior of a stock over time. A stock's monthly rate of return generally reflects the amount of money an investor makes (or loses if the return is negative) for every dollar invested in the stock in a given month. In his classic text, *Foundations of Finance* (1976), Eugene Fama demonstrated that the probability distribution for the monthly rate of return of a stock can be approximated by a normal probability distribution. Suppose the monthly rates of return to stock ABC are normally distributed with mean .05 and standard deviation .03, and the monthly rates of return to stock XYZ are normally distributed with mean .07 and standard deviation .05. Assume that you have \$100 invested in each stock.

- a. Over the long run, which stock will yield the higher average monthly rate of return? Why?
- b. Suppose you plan to hold each stock for only one month. What is the expected value of each investment at the end of one month?
- c. Which stock offers greater protection against incurring a loss on your investment next month? Why?

Source: J.T. Mc Clave, P.G. Benson, T. Sincich: *Statistics for Business and Economics*, 9th Edition, 2005

Exercise 16

Personnel tests are designed to test a job applicant's cognitive and/or physical abilities. An IQ test is an example of the former; a speed test involving the arrangement of

pegs on a peg board is an example of the latter (Cowling and James, *The Essence of Personnel Management and Industrial Relations*, 1994). A particular dexterity test is administered nationwide by a private testing service. It is known that for all tests administered last year the distribution of scores was approximately normal with mean 75 and standard deviation 7.5.

- a. A particular employer requires job candidates to score at least 80 on the dexterity test. Approximately what percentage of the test scores during the past year exceeded 80?
- b. The testing service reported to a particular employer that one of its job candidate's scores fell at the 98th percentile of the distribution (i.e., approximately 98% of the scores were lower than the candidate's, and only 2% were higher). What was the candidate's score?

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Exercise 17

Government data indicate that the mean hourly wage for manufacturing workers in the United States is \$14 (*Statistical Abstract of the United States: 2002*). Suppose the distribution of manufacturing wage rates nationwide can be approximated by a normal distribution with standard deviation \$1.25 per hour. The first manufacturing firm contacted by a particular worker seeking a new job pays \$15.30 per hour.

If the worker were to undertake a nationwide job search, approximately what proportion of the wage rates would be greater than \$15.30 per hour?