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Normative Inference in Efficient Markets

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Abstract
This paper develops a non-parametric method to infer social preferences over policies from prices of securities when agents have non-stationary heterogeneous preferences. We allow for arbitrary efficient risk-sharing mechanisms, formal and informal, and consider a large class of policies. We present a condition on the distribution of aggregate wealth that is necessary and sufficient for the revelation of social preferences over a universal set of policies. We also provide a weaker condition that is sufficient for revelation of social preferences for an arbitrary finite collection of policies.

Keywords:
Social preferences, normative predictions, asset prices

JEL Classification
D43, D53, G11, G12, L13

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"Everything changes and nothing stands still." – Heraclitus of Ephesus[1]

1 Introduction

Consider the following problem: a policy maker contemplates the construction of a large river dam, financing the project with a new consumption tax. Such a policy reallocates available resources towards future consumption during flood or drought. The policy maker can often accurately predict the impact of the policy on current and future (aggregate) consumption, but are the benefits worth the costs for a society? One potential source of information regarding the welfare effects of the policy is market prices. Unfortunately, in settings with structurally unstable preferences of consumers, inferring social preferences regarding economic policies from market-level data can be very challenging. In such settings, a consumption price during one period is not informative of the agents' demand during other periods, and information about social preferences (i.e., preferences of a society as a whole) does not accumulate over a long-term series of market-level data. Moreover, a demand function for past periods does not facilitate normative predictions for policies implemented after demand estimation. Thus, the existing normative econometric methods in the General Equilibrium and Industrial Organization literatures are not applicable to non-stationary settings. These backward-looking methods uncover social preferences by estimating an aggregate demand system using historical time series on consumption and prices. To identify the demand, they require structurally stable preferences for the time span in which available time-series are generated. To make predictions regarding future policies, they also require preference stability after the estimation.

In this paper we develop an alternative non-parametric[2] method to rank economic policies implemented in future periods within settings with preferences that change arbitrarily over time. Instead of estimating historical demand functions and extrapolating them to future periods, our *forward-looking* method reconstructs multi-period demand by utilizing information contained in the prices of aggregate securities (i.e., derivatives of aggregate consumption). The proposed method is applicable to all allocative mechanisms that result in (Pareto) efficient distributions of consumption among agents, including (dynamically) complete financial markets, efficient over-the-counter markets, and (in)formal risk-sharing networks. The method gives predictions for endowment, real asset and production economies, and for policies that affect economic systems arbitrarily as long as they preserve agents' preferences, and a policy maker can asses their impact on aggregate consumption.

[1]As quoted by Plato in Cratylus, 402a.

[2]In the analysis we only require that preferences satisfy concavity and Inada conditions.

We consider the set of all conceivable policies (the *universal set*) and assume that one of the policies from this set is implemented. We call the latter the factual policy. We present the following results regarding the inference of social preferences over the universal set of future policies from factual prices (i.e., prices observed given the factual policy). We give a necessary and sufficient condition for the complete revelation of social preferences over economic policies; the revelation holds provided that the factual aggregate consumption has full support. Next, we discuss the limits to normative inference in more realistic settings with less than full consumption support. We argue that the degree of completeness of the revealed social order depends on the denseness of the support and the strength of *a priori* assumptions on agents' preferences. We also provide an inference algorithm for general consumption distributions.

Our normative method draws heavily on the theory of asset pricing. In the absence of arbitrage, from prices of securities one can derive a pricing kernel, a strictly positive stochastic process that predicts the prices of all, possibly non-existing, cash flows. Pricing predictions require constant allocation of consumption among agents. Counterfactual policies with non-negligible welfare effects necessarily alter the factual allocation, making the original pricing kernel obsolete. Thus, the use of a pricing kernel in the existing literature on normative inference is very limited.

The paper is most closely related to Alvarez and Jermann (2004), who use pricing kernel to approximate the costs of business cycles. Our paper differs from the latter in the following respects: Alvarez and Jermann focus on two consumption flows (in our paper interpreted as policies), namely aggregate consumption c and its trend c'. To measure welfare gains from replacing the aggregate consumption with the smoother counterpart, they use a statistic called a marginal cost of consumption fluctuations, defined as $\omega_0 \equiv V_{0,c}(c')/V_{0,c}(c) - 1$ where function $V_{0,c}(\cdot)$ gives market value for any cash flow assuming the base pricing kernel realized under consumption c. In this paper, we consider a collection of *all* conceivable economic policies. On such a set statistic ω_0 is not additive, and it does not consistently rank policy pairs: i.e., its sign may depend on a choice of a base pricing kernel. More importantly, even though under certain assumptions, statistic ω_0 can be recast as a proxy of a utility of a positive representative agent, in general, the latter does not have any normative interpretation in terms of consumers' preferences in allocative mechanisms (e.g., competitive markets) with heterogeneous agents.[3] In this paper, we explicitly specify the class of admissible allocative

[3]In a consumption-based model with heterogenous agents, asset pricing theory demonstrates the existence of a *positive* representative consumer. In particular, given (fixed) efficient equilibrium allocation with strictly concave utilities, there exists a profile of weights such that the equilibrium allocation maximizes the sum of weighted utilities, subject to the constraint that total consumption does not exceed aggregate consumption. The value function of this program rationalizes the observed asset prices in a single-agent economy, and

mechanisms with heterogenous consumers. We aggregate consumers' preferences into social preferences over policies according to a money metric rule. Under our assumptions, social preferences are rational on the universal set of economic policies. To the best of our knowledge, this paper is the first to offer a comprehensive analysis of money metric social orders in the financial context.

The paper is also very closely related to the literature on the identification of consumer preferences based on consumer choices made in financial markets (Dybvig and Polemarchakis (1981); Geanakoplos and Polemarchakis (1990); Kübler et al. (2002); Kübler (2004)). Indeed, with quasilinear utilities assumed in this paper, normative analyses reduce to recovering von Neumann-Morgernstern utility function of a representative agent from prices in markets that are complete with respect to aggregate states. Our paper differs from this strand of literature both in terms of the assumptions on observables and the predictions. In particular, the literature assumes that an analyst, at least locally, can observe entire demand (or equilibrium) correspondence—an infinite number of choice-price (equilibrium prices-endowment) pairs—and shows that under certain conditions additively separable preferences can be fully recovered, even when markets are incomplete. We consider complete market settings in which consumer preferences can evolve arbitrarily, and thus the data on choices and prices do not accumulate over time. For such markets, the assumption that an analyst can observe demand/equilibrium correspondence is too strong—the analyst's knowledge is restricted to a single point on the respective correspondence, i.e. portfolio choice and prices that are realized in the factual equilibrium. Clearly, in general the preferences of a social planner cannot be fully recovered from this very limited data set. Our paper shows, however, that with some additional assumptions on wealth distribution, preferences can still be approximated very accurately. For this, we use novel arguments that rely on the global convexity of preferences or prudence.

The paper to some extent is related to the older literature on econometric modeling of aggregate consumers in general equilibrium (Deaton and Muellbauer (1980), Jorgenson (1990)). This literature develops (backward-looking) statistical tools to infer welfare effects from observed prices in competitive markets, and it assumes structurally stable preferences over time. Also, the adopted social welfare function is cardinal, and predictions hold only for particular non-generic cardinal representations of Gorman preferences.[4] We allow for

thus is often interpreted as a utility of a representative agent. The equilibrium pricing kernel is given by marginal rates of substitution of a representative agent. However, since the weights that define the value function depend on the original equilibrium, the fiction of a positive representative agent does not facilitate comparisons across equilibria as required by normative analyses (see, e.g., Duffie (2010), pp. 10 and 27).

[4]In this literature, social preferences are defined according to a cardinal criterion that assumes comparability of agents' utilities and hence requires particular representations of utility functions that allow Gorman aggregation (see, e.g., Jorgenson (1990)). Such normalizations are non-generic and inconsistent

arbitrary (efficient) allocative mechanisms and money metric social preferences from this paper are invariant to cardinal representations of consumers' preferences.

In terms of research question the paper is related to the Industrial Organization literature on normative structural analysis in differentiated product markets introduced by Lancaster (1971) and further developed by McFadden (1973) and Berry et al. (1995). Similar to our paper, this literature assumes heterogeneous quasilinear preferences and aggregates them into social preferences according to the money metric rule. The IO method, however, applies to static settings with discrete choice, where preferences are structurally stable over time. Neither discrete choice nor stability of utilities is realistic in the context of dynamically optimizing consumers, who trade arbitrary quantities of different types of securities.

The remainder of the paper is structured as follows: In Section 2 we define an efficient allocative mechanism with heterogenous agents and social preferences, and we state the problem of an analyst. In Section 3 we give our main inference results for the universal set of policies (Theorem 1) and discuss inference for general supports (Propositions 1-3). Section 5 concludes. All proofs are relegated to the appendix.

2 Preliminaries

2.1 Efficient allocative mechanism

Consumers hedge income shocks via many different channels: by trading assets in stock exchanges and over-the-counter markets, or by purchasing insurance policies (e.g., home or car insurance) from insurance agencies. In less developed economies, agents *ex post* redistribute consumption within communities of friends and family members or among strangers through a charity institution. Other agents rely on government transfers such as unemployment benefits or relief programs (e.g., FEMA). To study the problem of normative inference in the presence of different insurance channels, we work with an abstract *efficient allocative mechanism* defined as follows.

with the separable form of von Neumann-Morgenstern utility. Consider a canonical CRRA representation $U^i = E\sum_{t=1}^{\infty}\beta^t(c_t^i)^{1-\sigma}/(1-\sigma)$. Since utility functions are strictly concave, the utility possibility frontier is strictly convex. It follows that for different consumption distributions the corresponding points on the frontier are located on different isoquants (hyperplanes) of utilitarian welfare, and the latter varies in consumption distribution. For the representation $V^i = (U^i)^{\frac{1}{1-\sigma}}$, the utility possibility frontier becomes a hyperplane perfectly aligned with some utilitarian isoquant, and utilitarian welfare is invariant to all consumption distributions. Thus, for CRRA preferences, utilitarian analysis requires a V^i representation or its affine transformation identical for all agents. Such a family of cardinal utilities is non-generic within the class of all utility functions representing CARA preferences, and it does not have a separable form postulated by von Neumann and Morgenstern. Analogous considerations hold for other types of Gorman preferences with strictly concave instantaneous utilities.

We consider a dynamic setting with periods $t = 0, 1, ..., T$. All random variables are defined over abstract probability space $(\Omega, \mathcal{F}, \pi)$, with filtration $\{\mathcal{F}_t\}_{t=0}^T$ satisfying $\mathcal{F}_{t'} \subset \mathcal{F}_t$ for all $t > t'$, where there is no uncertainty in period zero, $\mathcal{F}_0 = \{\emptyset, \Omega\}$. Set $\mathcal{C}$ is the collection of all adapted processes, and $\mathcal{C}_{++} \subset \mathcal{C}$ is the collection of all processes that are strictly positive in periods $t \geq 1$ (processes in period $t = 0$ can be potentially negative). There are $I \geq 1$ agents, indexed by $i = 1, ..., I$. Agent $i's$ preferences $\succeq^i$ over the set of consumption processes, $c^i \in \mathcal{C}_{++}$, are represented by a quasilinear expected utility function $U^i(c^i) = \lambda^i c_0^i + E \sum_{t=1}^T u_t^i(c_t^i)$. Parameter $\lambda^i > 0$ and functions u_t^i can differ among agents. Observe that we allow for non-stationary preferences; i.e., functions u_t^i can vary across time. For each agent i and period t, function u_t^i is twice continuously differentiable, strictly increasing ($u_t^{i\prime} > 0$), strictly concave ($u_t^{i\prime\prime} < 0$), and satisfies Inada conditions.

Economic policies can affect different aspects of human interactions, including laws, transfers, tax systems and public spending. With preferences defined over material consumption, in normative analyses such policies matter to the extent they affect consumption flows for each agent. In the rest of the paper, each policy is therefore represented by the corresponding element $c \in \times_i \mathcal{C}_{++}$.

We focus on economic environments in which in period zero agents have access to efficient risk-sharing mechanisms that allow them to exhaust all potential insurance gains, such as (de)centralized financial markets or (in)formal insurance networks. For the sake of parsimony, instead of explicitly modeling how an efficient allocation is achieved, here a risk-sharing mechanism is modeled as a "black box" which for each policy produces some efficient profile of consumption processes c. Thus, any feasible policy satisfies the following assumption:

Assumption 1. *Policy c is a (Pareto) efficient allocation of aggregate consumption $C \equiv \sum_i c^i$.*

Observe that for each c Assumption 1 is conditional on the corresponding aggregate consumption C, and thus the assumption does not imply any efficiency rankings of policies; each policy defines its own utility possibility frontier and there exist pairs of policies that satisfy it, where one strictly dominates the other in Pareto sense.

Finally, we make a number of technical assumptions. For each period t and agent i consumption c_t^i is integrable (i.e., it has finite mean), and a cdf of a random vector $\{c_t^i\}_i$ is differentiable–except, potentially, on a countable subset of values. This assumption accommodates discrete, continuous and mixed distributions. The set of feasible policies, denoted by $\mathcal{P}$, is then given by a collection of all consumption profiles $c \in \times_i \mathcal{C}_{++}$ that satisfy Assumption 1 and the cdf differentiability assumption. We call set $\mathcal{P}$ the *universal set,* and each element $c \in \mathcal{P}$ we refer to as "policy." Throughout the paper, we illustrate our ideas in the following example of an efficient allocative mechanism:

Example. There are two periods $t = 0, 1$ and $I \geq 1$ agents. The set of policies $\mathcal{P}$ consists of efficient processes $c = \{c^i\}_i$ where for each $c^i = (c_0^i, c_1^i)$, component c_0^i is deterministic and c_1^i is a strictly positive random variable.

2.2 Social preferences

We next endow the efficient allocative framework with social preferences over policies, $\mathcal{P}$, by adapting the notion of equivalent variation (Hicks (1939)). One obstacle in applying money-metric social preferences to our setting is that the framework allows for all, possibly non-market, efficient allocative mechanisms, that need not rely on prices of consumption. The latter are needed to define equivalent variation. To overcome this problem we use an implication of efficiency, that the marginal rates of substitution of all consumers almost surely coincide. For any given preferences and policy $c \in \mathcal{P}$ there exists a process $\zeta^c \in \mathcal{C}_{++}$ such that, for each trader i and period t, the following equality holds in almost all states $\zeta_t^c \overset{a.s.}{=} u_t'^i\left(c_t^i\right)/\lambda^i$ (see Lemma 3 in the appendix). Under our assumptions, stochastic process ζ^c is uniquely defined up to zero-probability events. Common marginal rates of substitution can be interpreted as implicit prices of consumption in period t and state ω, in terms of period-zero consumption. When specialized to the context of complete financial markets, process ζ^c coincides with the equilibrium pricing kernel observed given policy c.

Social preferences over policies are derived according to the money metric rule with respect to prices ζ^c. Consider two policies, $c = \{c^i\}_i$ and $c' = \{c^{i\prime}\}_i$. For individual consumers, equivalent variation is defined as

$$EV_{c,c'}^i \equiv e^i\left(\zeta^c, U^i\left(c^{i\prime}\right)\right) - e^i\left(\zeta^c, U^i\left(c^i\right)\right), \tag{1}$$

where $e^i(\cdot,\cdot)$ is the consumer $i's$ expenditure function.[5] Heuristically, equivalent variation is an equivalent monetary payment to agent i given c, in lieu of implementing policy c', assuming prices ζ^c, and its negative value indicates that policy c is preferred to c'. Social preferences over the set of policies are determined according to the *aggregate* equivalent variation criterion:

Definition 1. *Preference relation* $c \succeq_* c'$ *holds if* $EV_{c,c'} \equiv \sum_i EV_{c,c'}^i \leq 0$.

[5] For any profile of implicit prices $\zeta^c \in \mathcal{W}_+$ and level of utility $U^{i\prime} \in \mathbb{R}_{++}$ expenditure function is

$$e^i\left(\zeta^c, U^{i\prime}\right) = \min_{c^i \in \mathcal{C}_{++}} c_0^i + \sum_{t \geq 1} E\zeta_t^c c_t^i,$$

subject to the inequality constraint $U^i\left(c^i\right) \geq U^{i\prime}$.

Policy c socially dominates c' if the required aggregate compensation to all agents for not implementing policy c' is negative. Alternatively, there exist zero-sum transfers redistributing consumption $C = \sum_i c^i$ among agents, for which all agents can be made better off under policy c.

It is well-known that, in general, money metric preferences need not be complete or transitive. The next lemma shows that, in the efficient allocative framework, the money metric rule aggregates individual preferences into a *rational* social order $\succeq_*$.

Lemma 1. *Social preferences $\succeq_*$ are complete and transitive on universal set $\mathcal{P}$. They are also Paretian.*

Social preferences are consistent with the Pareto criterion whenever the latter gives predictions. The money metric rule, commonly adopted in the IO literature, is measurable with respect to profiles of agents' preferences over consumption. It follows that social order is invariant to price normalizations (the same rankings obtain, if for any pair of policies $c, c' \in \mathcal{P}$ price vector ζ^c is scaled by a different positive constant) and cardinal representations of agents' preferences and inference of social preferences from prices does not require untestable assumptions about cardinal utility representations or price normalizations for counterfactual policies.

The parsimonious allocative framework is a reduced-form model for a wide variety of economic settings studied in the literature. Consider an exchange economy where economic policies affect initial endowment $x = \{x^i\}_i \in \times_i \mathcal{C}_{++}$. The canonical example of an efficient allocative mechanism is complete financial markets. Given quasilinear preferences, each profile x results in a unique competitive allocation, c, which, by *the First Welfare Theorem,* is efficient. Thus, each policy ultimately produces one profile of efficient allocation.

In contemporaneous economies a significant part of risk-sharing results from bilateral agreements among parties. For example, insurance companies offer individuals specific policies that can differ across buyers. For large coverages, companies may partly resell their insurance policies to other insurers, by which they share their risk exposure and premium. Our framework can be applied to such decentralized insurance networks, even if interactions are non-competitive, as long as they are efficient. Suppose that, in the exchange economy, instead of complete financial markets, allocation is determined in a game in which agent $i = 1$ (broker) is selling insurance to all other agents $i \geq 2$. The broker observes agents' initial endowments x and makes sequential take-it-or-leave-it offers to other agents. For each $i = 2, ..., I$, the idiosyncratic insurance contract specifies an obligation (payment, if negative) of the broker, contingent on period and state $\Delta x^i \in \mathcal{C}$. If all agents accept, broker's equilibrium consumption is given by $c^1 = x^1 - \sum_{i \neq 1} \Delta x^i$, and for agents $i \geq 2$, it is equal to $c^i = x^i + \Delta x^i$. If at least one agent rejects the offer, consumption distribution is as in autarky. In this complete information game, for any specification of x there exists a (Subgame Perfect Nash) equilibrium in which offered contracts $\{\Delta x^i\}_i$ jointly maximize total surplus, and surplus gains are extracted by the broker through period zero payments (for each i given by $-\Delta x^i_0$). In this equilibrium, the broker fully discriminates prices and the resulting consumption distribution $c = \{c^i\}_i$ is efficient. It follows that the game defines an efficient allocative mechanism.

For less developed economies, empirical literature highlights the role of informal insurance provided by family, friends and neighbors.[6] Even though implicit agreements are typically made locally among small social groups, the literature demonstrates that, with sufficient interconnectedness of the communities, social risk-sharing can be an effective instrument that results in an outcome that is nearly globally efficient (Ambrus et al. (2014)). The framework from this paper accommodates informal allocative mechanisms as well.

More generally, the allocative framework represents a class of economic settings that can differ in terms of the source of agents' endowments (exogenous endowment, real asset or production economies), risk-sharing mechanisms (centralized or decentralized financial markets, insurance markets, informal social risk-sharing mechanisms, or even benevolent planners) as well as economic policies. The defining feature of the class is some version of the First Welfare Theorem. Working with an abstract framework allows us to abstract away from the details of policymaking and risk-sharing and focus exclusively on the question of normative inference.

2.3 Problem of an analyst

We assume that only one among all the feasible policies is implemented, $\bar{c} = \{\bar{c}^i\}_i \in \mathcal{P}$, and we refer to that as the *factual policy*; all other policies $c \neq \bar{c}$ are called *counterfactual*. We consider the problem of an analyst whose objective is to infer social preferences from limited information about an allocative mechanism. In particular, for each policy $c \in \mathcal{P}$, an analyst *a priori* knows the resulting *aggregate* consumption process $C \equiv \sum_i c^i \in \mathcal{C}_{++}$ (i.e., the distribution of aggregate consumption in each period). He does not have information about how many agents participate in the mechanism, how individual preferences and consumption evolve in time, or what risk-sharing mechanism(s) is (are) used to achieve allocative efficiency. Also, the analyst does not have information about the probabilities distribution of future states. For the analyst, the primary source of information about agents' preferences is the

[6]De Weerdt and Dercon (2006), Fafchamps and Lund (2003) and Barr and Attanasio (2009) study risk sharing in villages while Mazzocco (2007) looks at transfers within castes.

prices of securities in financial markets,[7] which open in period zero once factual policy $\bar{c}$ is known by all agents.[8] The central question of this paper is what an analyst can infer from *factual* prices—effectively, a single observation of the future multi-period aggregate demand—about social preferences.

It is assumed that, for factual $\bar{c}$, all gains-to-trade are exhausted, and securities are priced but not traded in financial markets. At this stage, markets play a purely informative role.[9] Clearly, prices are the most informative when the collection of traded securities is complete. In settings with many heterogenous agents, however, the assumption of market completeness is not very plausible. Agents rarely hedge their income shocks by trading idiosyncratic securities in centralized exchanges. Moreover, even if consumer-specific assets are listed in such markets, monitoring prices for all the agents would be very demanding. The assumption of market completeness is also unnecessarily strong. From the asset pricing literature it is well-known that in absence of arbitrage the information contained in prices of all securities is summarized in prices of a small sub-collection of *aggregate* securities, e.g., options on aggregate consumption, Breeden and Litzenberger (1978)). In the context of efficient allocative mechanism this implies that prices of aggregate securities are also sufficient for money metric preferences. For any $x \in \mathcal{C}$, let $\mathcal{C}^x \subset \mathcal{C}$ be the set of stochastic processes adapted to a filtration generated by process x. A collection of securities is said to be *x-spanning* if, for any dividend process $x' \in \mathcal{C}^x$, one can find a portfolio of assets that gives rise to a dividend cash flow x'. Intuitively, trading x-spanning securities can result in arbitrary derivative cash flow of x.[10] Let the factual aggregate consumption process be denoted by $\bar{C} \equiv \sum_i \bar{c}^i \in \mathcal{C}_{++}$. In what follows, we restrict our attention to financial markets in which asset structure is complete with respect to *aggregate events*.

Assumption 2. *The asset structure is $\bar{C}$-spanning.*

An example of a collection of $\bar{C}$-spanning derivatives is that of European options with underlying cash flow $\bar{C}$, with arbitrary strike prices and expiry dates. Securities beyond any

[7] Note that by this assumption an analysts can infer marginal utility of a representative agent for all realizations of aggregate wealth. Importantly, it for our results it would not suffice to know the probability distributions of such marginal utilities.

[8] We also make a technical assumption that, whenever an analyst observes a convergent sequence, he can also infer its limit.

[9] Centralized financial markets can be a part of an allocative mechanism as well. In such cases, apart from being informative about preferences, they also provide risk-sharing opportunities for the agents. Our formulation, which separates allocation and price determination, is more general, because it allows for all types of risk-sharing mechanisms, and it allows us to make weaker and more realistic assumptions about asset structure that are sufficient for the revelation of preferences but not for efficiency (Assumption 2).

[10] For any process $x \in \mathcal{C}$, a derivative cash flow is any process $y(x) \in \mathcal{C}$, where $y : \mathbb{R} \to \mathbb{R}$ can be an arbitrary function.

$\bar{C}$-spanning collection are redundant and can be ignored in normative analysis, an implication of the existence of a *positive* representative consumer (Duffie (2010), p. 27, see also Lemma 5 in the appendix).

Example. (cont.) For factual policy the distribution of aggregate consumption $\bar{C}_1$ is absolutely continuous with density $\bar{f}$. The analyst observes prices of call options on consumption $\bar{C}_1$ for all strike prices $y \in \mathbb{R}_{++}$. Analyst infers social preferences over two counterfactual policies, c and c'. Policy c stabilizes aggregate consumption in both periods; i.e., $C_0 = C_1 = 1$. Alternative c' in addition involves the construction of a river dam that fosters economic growth and future consumption. The public expenditure is financed by a period-zero consumption tax. The overall impact of the policy on aggregate consumption, known to the analyst, is $C_0' = 0.2, C_1' = 2$.

Suppose agents have identical instantaneous utility functions $u_1^i(c_1^i) = \ln c_1^i$ and heterogenous utility of consumption, λ^i, satisfying $\sum_i (\lambda^i)^{-1} = 1$. In the assumed allocative mechanism equivalent variation is $EV_{c,c'} \simeq -0.106$, and therefore construction of a river dam is not socially desirable, $c \succ_* c'$.

In the example, the collection of options is $\bar{C}$-spanning but not complete, and in exchange economies with arbitrary initial endowments without prior engagement in some efficient risk-sharing mechanism, trading options alone would generically result in an inefficient outcome.

3 Complete revelation

In this section we give conditions under which social preferences are completely revealed in prices for the universal set $\mathcal{P}$. For any $t \geq 1$ and profiles $\{u_t^i, \lambda^i\}_i$, we define mapping $u_t : \mathbb{R}_{++} \to \mathbb{R}_{++}$ as a value function of program $u_t(y) \equiv \max_{\{y^i\}_i} \sum_i u_t^i (y^i) / \lambda^i$ subject to a non-negativity constraint $y^i \geq 0$ for all i and an aggregate feasibility $\sum_i y^i \leq y$. By the standard arguments, the derivative of the value function given by $v_t(y) \equiv u_t'(y)$ is well-defined, smooth, strictly positive, strictly decreasing and satisfies limit conditions $\lim_{y \to 0} v_t(y) = \infty$ and $\lim_{y \to \infty} v_t(y) = 0$ (Lemma 2 in the appendix, property 2). In the example, this index is given by $v_1(y) = 1/y$.

It turns out that the collection of functions $\{v_t\}_{t=1}^T$ summarizes all the information in agents' utilities required for social preferences. For any pair of policies $c, c' \in \mathcal{P}$, decompose the change in aggregate consumption in period t resulting from implementation of c' into a positive and a negative part, $C_t^+ \equiv \max(C_t, C_t')$ and $C_t^- \equiv \min(C_t, C_t')$, respectively. In terms of indices v_t, equivalent variation can be written as

$$EV_{c,c'} \equiv \sum_{t=1}^{T} E \left[\underbrace{\int_{C_t}^{C_t^+} v_t(y)dy - \int_{C_t^-}^{C_t} v_t\left(y\right)dy}_{\text{welfare change in } t \text{ and state } \omega} \right] + C_0' - C_0. \tag{2}$$

Conditional on period $t \geq 1$ and state $\omega \in \Omega$, the welfare effect of policy c' relative to c is geometrically represented by the area below derivative v_t where the horizontal limits of the area are determined by the realizations of aggregate consumption for policy c and c', respectively (see Figure 1). Summing up these areas for all states and periods gives the value of equivalent variation.

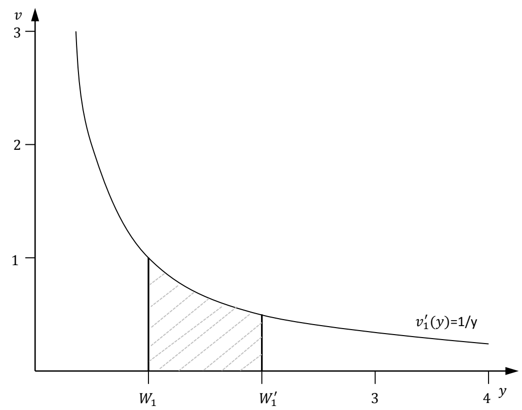

Figure 1: Surplus change in state ω

We assume that individual preferences and hence indices $\{v_t\}_{t=1}^{T}$ are not *a priori* known to the analyst. However, the analyst can use the fact that stochastic process $\zeta_t^{\bar{c}} \equiv \left\{v_t(\bar{C}_t)\right\}_{t=1}^{T}$ gives the factual pricing kernel; for any cash flow x traded in financial markets, its price is given by

$$p_x = x_0 + \sum_{t=1}^{T} Ex_t \times v_t(\bar{C}_t). \tag{3}$$

From the equation it is clear that, at least to some extent, prices can reveal values of index v_t. Indeed, in the appendix we show that value $v_t\left(y\right)$ can be uniquely pinned down from prices of $\bar{C}$-spanning asset structures whenever $y \in \bar{S}_t$, where $\bar{S}_t$ is the support of factual consumption $\bar{C}_t$. It follows that the restrictions $\{v_t|_{\bar{S}_t}\}_{t\geq 1}$ are observable by the analyst. By equation (3) the restrictions are also sufficient for factual prices of arbitrary cash flows, and cash flows outside of $\bar{C}$-spanning collection do not contain any additional information about social preferences.

Prices *reveal* social preferences if, for any pair of policies $c, c' \in \mathcal{P}$, only one of the three alternatives $c \succ_* c'$, $c' \succ_* c$ or $c' \sim_* c$ is consistent with factual prices of $\bar{C}$-spanning securities. Our next result gives a necessary and sufficient condition for the revelation of social preferences.

Theorem 1. *Prices* reveal *social preferences if and only if factual aggregate consumption has full support, i.e.,* $\bar{S}_t = \mathbb{R}_+$ *for all* $t \geq 1$.

In an allocative mechanism with von Neumann-Morgenstern quasilinear preferences, in period t, the *factual* price of consumption in states $\omega' \neq \omega$ reveals the shape of index $v_t(\cdot)$ and hence contains some information about a *counterfactual* welfare effect that occurs within state ω. Such cross-state inference facilitates welfare predictions based on a single observation of factual prices of securities, provided that aggregate consumption is sufficiently variable.

Theorem 1 can be interpreted as a positive or a negative result. On one hand side it shows that under certain assumptions on consumption distribution, economic theory imposes sufficient structure on prices to give predictions about social preferences over future policies in settings with with structurally unstable consumers' preferences. On the other hand, the preference revelation holds only with an extreme variability of the aggregate consumption, as the latter plays a role of an endogenous demand shifter that identifies preferences. The assumption of full support is not realistic as it requires arbitrary large changes of consumption within short time horizon (with strictly positive probability).

The revelation theorem highlights the qualitative difference between normative analysis and asset pricing. With $\bar{C}$-spanning asset structure, factual prices of securities allow to predict value of an *arbitrary* counterfactual cashflow, *regardless* of the support of aggregate consumption. Pricing of securities requires weaker assumptions as contrary to the normative inference implicitly assumes constant allocation in counterfactual experiments, i.e., consumption processes is not affected by the introduction of a new security.

We next illustrate the theorem in the example

Example. (cont.) For any $y > y' > 0$, a portfolio with $1/(y - y')$ of a call option with strike price y', combined with the negative of this amount of the option with strike price y, in the limit $y' \to y$, yields a payoff $x_1 = 1$ in all states for which $\bar{C}_1 \geq y$, and zero otherwise. By the pricing equation (3), the observable price is

$$p_x(y) = Ex_1 v_1(\bar{C}_1) = \int_y^{\infty} v_1(y) \bar{f}\left(\bar{C}_1\right) d\bar{C}_1, \tag{4}$$

and its derivative is $p_x'(y) = -v_1(y)\bar{f}(y)$. It follows that index $v_1(y)$ is identified whenever the density is strictly positive. With full support, ratio $v_1(y) = -p_x'(y)/\bar{f}(y) = 1/y$ is well defined for the entire domain, and for policies c, c', the corresponding equivalent variation can be expressed in terms of observables,

$$EV_{c,c'} \equiv E \int_{C_t}^{C_t^+} -\frac{p_x'(y)}{\bar{f}(y)} dy - \int_{C_t^-}^{C_t} -\frac{p_x'(y)}{\bar{f}(y)} dy + C_0' - C_0 = E \int_1^2 \frac{1}{x} dy - 0.8 = -0.106. \qquad (5)$$

Therefore, social preferences are revealed for the policy pair (c, c') as well as for all other pairs in $\mathcal{P}$. On the other hand, if for some y, the density is $f(y) = 0$, then the derivative condition becomes a tautology, $0 = 0$. Outside of the support, index values are not identified, and, for policies with aggregate consumption in this range, social rankings might not be revealed.

4 Normative analysis in realistic settings

As we argued in the previous section revelation of social preferences on the universal set $\mathcal{P}$ requires unrealistic assumptions on the statistical properties of aggregate consumption. We now turn to the question of inference of social preferences under more plausible (in fact arbitrary) consumption processes. With general distributions, the inference is more complicated as, for some pairs of policies, equivalent variation is integrated over a domain outside of the set $\bar{S}_t$ for which v_t are known. For these policies, the analyst needs to non-parametrically extend observed restrictions $v_t|_{\bar{S}_t}$ to an arbitrary interval, using some general properties of an efficient mechanism. In this paper, the extension takes the form of a lower and upper bound on a data-generating index v_t. For period t, define function $v_t^+ : \mathbb{R}_{++} \to \bar{\mathbb{R}}_+$ as

$$v_t^+(y) \equiv \inf\{v' \in \mathbb{R}_+ | v' = v_t(y') \text{ for some } y' \in \bar{S}_t \cap [0, y]\}. \qquad (6)$$

For any value outside support, $y \notin \bar{S}_t$, the function assigns value $v_t(y')$, where $y' \in \bar{S}_t$ is the element of the support that is the closest to but no larger than y. To close the definition, for y smaller than any element in the support, the extension is $v_t^+(y) = \inf(\emptyset) = \infty$. Function v_t^+ is right-continuous, non-increasing, and has flat segments on the intervals outside the support. Similarly, let $v_t^- : \mathbb{R}_{++} \to \bar{\mathbb{R}}_+$ be given by

$$v_t^-(y) \equiv \sup\{v \in \mathbb{R}_+ | v = v_t(y') \text{ for some } y' \in \bar{S}_t \cap [y, \infty]\}, \qquad (7)$$

where, now, for the values y greater than those in support $\bar{S}_t$, we adopt a non-standard convention that $v_t^-(y) = \sup\{\emptyset\} = 0$. For discrete distributions, extensions v_t^+ and v_t^- are given by the maximal and the minimal non-increasing step function, respectively, whose values on support $\bar{S}_t$ coincide with the actual revealed index v_t.

The extensions can be derived from factual prices, (Lemma 5 in the appendix). They separate agents' preferences that rationalize observed prices from the ones that are inconsistent with market level data. In particular for any data generating indices $\{v_t\}_{t\geq 1}$ and the corresponding observable bounds v_t^- and v_t^+, any allocative mechanism generates observed prices whenever corresponding indices $\{\tilde{v}_t\}_{t\geq 1}$ satisfy $v_t^- \leq \tilde{v}_t \leq v_t^+$ for all $t \geq 1$. The bounds are tight in the sense that each of the functions v_t^-, v_t^+ can be arbitrary closely approximated by an index $\tilde{v}_t$ corresponding to some allocative mechanism. Observe that for any two policies c, c', equivalent variation cannot be greater than the following statistic,

$$EV_{c,c'}^+ \equiv \sum_{t=1}^{T} E \int_{C_t}^{C_t^+} v_t^+(y)dy - \sum_{t=1}^{T} E \int_{C_t^-}^{C_t} v_t^-(y)dy + C_0' - C_0, \tag{8}$$

and for some policies social preferences can be recovered from prices by using the following criterion:

Proposition 1. *For any two policies c and c', inequality $EV_{c,c'}^+ \leq 0$ implies $c \succeq_* c'$.*

For other policies, however, statistics $EV_{c,c'}^+$ and $EV_{c',c}^+$ can both be strictly positive and therefore not revealing with respect to social rankings—the revealed social order is not complete in the universal set.[11]

4.1 Completeness of the revealed order

We next discuss factors that affect the degree of completeness of the revealed order in mechanisms with less than full support. We first discuss the denseness of support. Consider the following discrete version of our example:

Example. (cont.) Instead of a continuous distribution, let factual consumption be given by $\bar{C}_1 = (n+1)\varepsilon$, where n follows a Poisson process. Parameter $\varepsilon > 0$ determines how dense the support grid is.

[11] One could also define the lower bound for equivalent variation as

$$EV_{c,c'}^- \equiv \sum_{t=1}^{T} E \int_{C_t}^{C_t^+} v_t^-(y)dy - \sum_{t=1}^{T} E \int_{C_t^-}^{C_t} v_t^+(y)dy + C_0' - C_0. \tag{9}$$

With some work, however, one can show that $EV_{c,c'}^- = -EV_{c',c}^+$ and condition $EV_{c',c}^- > 0$ is equivalent to $EV_{c,c'}^+ < 0$. It follows that the lower bound condition is redundant.

The index bounds, given by step functions, are depicted in Figure 2. For $\varepsilon = 1$ the value of the inferred statistic is $EV^+_{c,c'} = 0.2 > 0$, and option prices do not allow for ranking available alternatives. The discrepancy between statistic $EV^+_{c,c'}$ and the actual value of equivalent variation is geometrically represented by the area between bound v_1^+ and index $v_1 = 1/y$. With a finer grid this difference becomes smaller and it vanishes completely as ε goes to zero. Thus, $EV^+_{c,c'}$ converges to $EV_{c,c'}$, and, for a sufficiently fine grid (e.g., $\varepsilon < 0.25$), the observed statistic $EV^+_{c,c'}$ is negative, revealing relation $c \succ_* c'$ even with discrete consumption support.

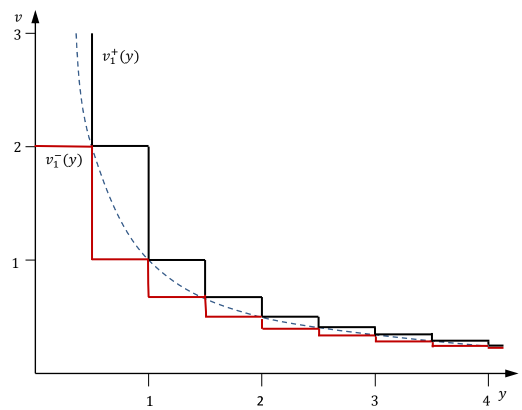

Figure 2: Surplus bounds

This observation generalizes to any finite collection of policies $\mathcal{P}^F \subset \mathcal{P}$, in which policies are non-indifferent (i.e., for any pair $c, c' \in \mathcal{P}^F$, one has $EV_{c,c'} \neq 0$) and for which consumption supports are bounded away from zero (i.e., for any $c \in \mathcal{P}^F$, set $S^c_t \cap \{0\} = \emptyset$ for all $t \geq 1$). Recall that for any two sets, $Y \subseteq Y' \subseteq \mathbb{R}_{++}$, set Y is said to be ε-dense in Y', if for any $y \in Y'$ there exists $y' \in Y$ such that the distance between the two points y and y' is less than ε (i.e., $|y - y'| < \varepsilon$).

Proposition 2. *There exists $\varepsilon > 0$ such that social preferences over set $\mathcal{P}^F$ are revealed if support $\bar{S}_t$ is ε-dense in $\mathbb{R}_+$ for any $t \geq 1$.*

For a sufficiently dense support preferences over finite collection of policies can be completely recovered. In general, a $\bar{C}$-spanning asset structure, although smaller than a complete one, may still require a continuum of securities. One implication of Proposition 2 is that social preferences over $\mathcal{P}^F$ are revealed in prices of a *countable* sub-collection of securities. Moreover, if in addition counterfactual policies have bounded supports, then factual supports are required to be dense only in a bounded[12] subset of $\mathbb{R}_+$, and, with discrete $\bar{c}$, social preferences can be revealed by prices of a finite set of securities.

[12] For any t, the set is given by the convex hull of set $\cup_{c \in \mathcal{P}^F} S^c_t$, which is bounded provided that each S^c_t is bounded as well.

Another factor that affects the completeness of the revealed order are *a priori* assumptions regarding agents' preferences. For example, the precautionary savings literature suggests that consumers tend to exhibit fear of catastrophically low consumption, which is captured by convexity of *marginal utility* of an agent. Suppose that the analyst believes that all agents are prudent; i.e., for any i and period t, the utility function satisfies $u_t^{i'''} \geq 0$. The next proposition shows that aggregate index v_t inherits the convexity property from the individual preferences.

Proposition 3. *For any t, function v_t satisfies $v_t'' \geq 0$.*

Convexity of index v_t improves the predictive power of statistic $EV^+_{c,c'}$ for any given grid. Consider again a discrete version of the example. Observe that in the example all agents are prudent.

Example. (cont.) Let grid value be $\varepsilon = 1$. For any point in the support indexed by $n = 0, 1, 2, ...$, let $o^n \equiv (n+1, v_1(n+1))$ be a corresponding point in the graph v_1. The collection of all points $\{o^n\}_{n=0}^{\infty}$ is the graph of restriction $v_1|_{\bar{S}_1}$, and hence it is observable. Next, consider a line passing through any two adjacent points o^n, o^{n+1}. According to Proposition 3, for any point in the interval $y \in [n+1, n+2]$ the line is above, and for $y \notin [n+1, n+2]$ it is be1low, index v_1. Thus, the line segments connecting points $\{o_n, o_{n+1}\}$ for $n = 0, 1, 2, ...$ and value ∞ for $y \leq 1$, jointly define an upper bound for index v_1 that is tighter than the step function v_1^+. Similarly, the parts of the line passing though $\{o_n, o_{n+1}\}$ that are outside interval $[n+1, n+2]$ are below the index. Thus, taking an upper envelope of such line segments for $n = 0, 1, 2, ...$ results in the lower bound. Figure 3 depicts the construction of tighter bounds. With new bounds, the value of statistic $EV^+_{c,c'} = -0.05$ and preference

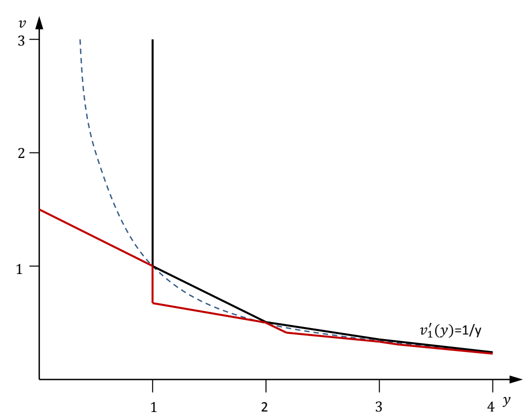

Figure 3: Surplus bounds with prudence

relation $c \succ^* c'$ is revealed, even with coarse grid $\varepsilon = 1$.

In general, for any pair of policies, statistic $EV^{+}_{c,c'}$ determined under assumption of prudent agents is always closer to the true value that without this assumption, and the revealed order is more complete. How much one can improve the completeness depends on agents' types of preferences. For example, in an allocative mechanism with quadratic utility functions, a non-parametric assumption $u_t^{i'''} \geq 0$ for all i allows for inference of a complete social order as long as the aggregate consumption distribution is not degenerate.

5 Discussion

In this paper we demonstrate that with some assumptions regarding the distribution of aggregate consumption, social preferences can be non-parametrically recovered from the prices of aggregate derivatives in dynamic allocative mechanisms with non-stationary heterogenous preferences. The central assumption that facilitates money-metric comparisons is that of quasilinear preferences. Outside this class, for some policies $c, c' \in \mathcal{P}$, equivalent variations $EV_{c,c'}$ and $EV_{c',c}$ may have the same signs, and the implied social preferences need not be rational. In Weretka (2015), in the context of financial markets, we provide a foundation for the assumption of quasilinear preferences. In particular, we consider an infinite horizon economy with I agents and assume a fairly general class of preferences. We show that money-metric preferences are rational and are revealed by the normative method in this paper, provided that policies affect consumption flows within a finite time horizon and agents are sufficiently patient. Intuitively, for patient consumers, the equilibrium effects of temporary policies on the marginal utility of money are negligible as the latter is determined by the entire lifetime consumption process. Thus, we give conditions under which the quasilinear approximation assumed in this paper is justified.[13]

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[13]This is a formalization of the Marshallian conjecture about negligible income effects in the context of financial markets. In *Principles of Economics,* Marshall justifies the assumption of quasilinear utility that underlines money metric analyses by arguing that "expenditure on any one thing, as, for instance, tea, is only a small part of his whole expenditure" (Marshall (1920), p. 842). We demonstrate this conjecture in the context of an infinite-horizon model with heterogenous agents.

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Appendix

Before giving the proofs for the results from the paper, we first introduce some notation and state auxiliary results. We say that function $g_t : \mathbb{R}_{++} \to \mathbb{R}_{++}$ is *standard* if it is continuously differentiable, satisfies $g_t' < 0$ and its limits are given by

$$\lim_{y\to 0} g_t(y) = \infty \text{ and } \lim_{y\to\infty} g(y) = 0.$$

Any standard function is a bijection, and the family of standard functions is closed under inversion (by the Inverse Function Theorem) and finite summation. Moreover, for any collection of standard functions $\{g_t\}_{t\geq 1}$, preferences given by $U^i(c^i) = c_0^i + E\sum_{t=1}^{T}\int_0^{c_t^i} g_t(y)dy$ satisfy the assumptions of Section 2.1.

Let $\{u_t^i\}_i$ be a collection of I twice continuously differentiable utility functions satisfying assumptions of strict monotonicity, strict convexity and Inada conditions (as in Section 2.1). Consider the following problem:

$$u_t(y) = \max_{\{y^i\}_i \in Y(y)} \sum_i u_t^i\left(y^i\right)/\lambda^i, \tag{10}$$

where $Y(\cdot)$ is a feasible allocation correspondence which, for any $y \in \mathbb{R}_{++}$, gives

$$Y(y) \equiv \{\{y^i\}_i \in \mathbb{R}_+^I | \sum_i y^i \leq y\}.$$

Let $v_t \equiv u_t'$ be the derivative of the value function of program (10).

Lemma 2. *Optimization program* (10) *satisfies:*

1. *There exist I functions $y_t^i : \mathbb{R}_{++} \to \mathbb{R}_{++}$ that give unique solutions for any $y \in \mathbb{R}_{++}$;*

2. *Index $v_t : \mathbb{R}_{++} \to \mathbb{R}_{++}$ is a standard function;*

3. Marginal rates of substitution for all i coincide, i.e., $u_t^{i\prime}(y^i(y))/\lambda^i = v_t(y)$.

Proof of Lemma 2: For any $y \in \mathbb{R}_{++}$, set $Y(y)$ is non-empty, convex and compact, while the objective function in (10) is continuous and strictly concave. Thus, the solution to problem (10) exists (Maximum Theorem) and it is unique. Therefore, the argmax functions $\{y_t^i(\cdot)\}_i$ are well defined (property 1). Under Inada conditions, non-negativity constraints are not binding. From Kuhn-Tucker Theorem, for any $y \in \mathbb{R}_{++}$ there exists a scalar $v_t \in \mathbb{R}$ such that a solution to problem (10) solves the unconstrained optimization problem

$$\max_{\{y^i\}_i \in Y(y)} \sum_i u_t^i(y^i)/\lambda^i - v_t \cdot \left(\sum_i y^i - y\right). \tag{11}$$

The first-order (necessary and sufficient) condition implies $\lambda^i v_t = u_t^{i\prime}(y^i)$ for each i. Derivative $u_t^{i\prime}(\cdot)$ is a standard function, and therefore its inverse $\tilde{y}_t^i(v_t) \equiv (u_t^{i\prime})^{-1}(\lambda^i v_t)$ and the sum of inverse functions for all agents, $\sum_i \tilde{y}_t^i(v_t) : \mathbb{R}_{++} \to \mathbb{R}_{++}$, are also standard. Function $v_t(y)$ that assigns corresponding Lagrangian multiplier v_t to each $y \in \mathbb{R}_{++}$ is implicitly defined by equation $\sum_i \tilde{y}_t^i(v_t) = y$, and thus is an inverse of a standard function and is standard as well (property 2). Finally, the first order optimization condition for problem (11) implies equality $u_t^{i\prime}(y_t^i(y))/\lambda^i = v_t(y)$ for any i (property 3). □

Lemma 3. *For any profile $c \in \mathcal{P}$, there exists a stochastic process $\zeta^c \in \mathcal{C}_{++}$ which satisfies $\zeta_t^c \overset{a.s.}{=} u_t^{\prime i}(c_t^i)/\lambda^i$ for any i and $t \geq 1$.*

Proof of Lemma 3: We first argue by contradiction that for any policy $c \in \mathcal{P}$, in each period $t \geq 1$, the profile of random variables $\{c_t^i\}_i$ solves

$$\max_{\{\hat{c}_t^i\}} E \sum_{i=1}^{I} u_t^i(\hat{c}_t^i)/\lambda^i : \sum_i \hat{c}_t^i \leq C_t \text{ and } \hat{c}_t^i \geq 0 \text{ for all } i. \tag{12}$$

where $C_t \equiv \sum_i c_t^i$. Suppose not. There exist non-negative random variables $\{c_t^{i\prime}\}_{i,t\geq 1}$ such that $\sum_i c_t^{i\prime} \leq C_t$ for all $t \geq 1$ and

$$\sum_{t\geq 1} E \sum_{i=1}^{I} u_t^i(c_t^{i\prime})/\lambda^i > \sum_{t\geq 1} E \sum_{i=1}^{I} u_t^i(c_t^i)/\lambda^i. \tag{13}$$

For any $i \geq 2$ define

$$c_0^{i\prime} \equiv E \sum_{t=1}^{T} u_t^i(c_t^i)/\lambda^i - E \sum_{t=1}^{T} u_t^i(c_t^{i\prime})/\lambda^i + c_0^i, \tag{14}$$

and let $c_0^{1\prime} \equiv C_0 - \sum_{i\geq 2} c_0^{i\prime}$. Equation (14) implies $U^i(c') = U^i(c)$ for all $i \geq 2$. For agent $i = 1$, equation (13) can be rewritten as

$$
\begin{aligned}
E\sum_{t\geq 1} u_t^1\left(c_t^{1\prime}\right)/\lambda^1 - E\sum_{t\geq 1} u_t^1\left(c_t^1\right)/\lambda^1 &> \sum_{i=2}^{I} E\sum_{t\geq 1}\left[u_t^i\left(c_t^i\right)/\lambda^i - u_t^i\left(c_t^{i\prime}\right)/\lambda^i\right] \\
&= \sum_{i\geq 2} c_0^{i\prime} - \sum_{i\geq 2} c_0^i = c_0^1 - c_0^{1\prime}.
\end{aligned}
$$

Rearranging terms gives $U^1(c') > U^1(c)$, and hence c is not efficient, which is a contradiction.

In problem (12) constraints are independent across states, and the value of program (12) cannot exceed state-by-state solution $\{y_t^i(C_t)\}_i$, where functions $\{y_t^i(\cdot)\}_i$ are argmax functions as defined in Lemma 2 (property 1). By the same lemma (property 3), for this solution marginal rates of substitution coincide for all agents, $u_t^{i\prime}(y_t^i(C_t))/\lambda^i = v_t(C_t)$ in each state. By the uniqueness of the solution for any y, a profile of non-negative random variables $\{c_t^i\}_i$, in state ω for which $\{c_{t,\omega}^i\}_i \neq \{y_t^i(C_{t,\omega})\}_i$, necessarily gives $\sum_i (\lambda^i)^{-1} u^i\left(c_{t,\omega}^i\right) < \sum_i (\lambda^i)^{-1} u^i(y_t^i(C_{t,\omega}))$. It follows that $\{c_t^i\}_i$ solves (12) if and only if the probability measure of states for which $\{c_t^i\}_i$ and $\{y_t^i(C_t)\}_i$ diverge is zero. Define stochastic process $\zeta^c = \{\zeta_t^c\}_{t\geq 0}$ as $\zeta_1^c \equiv 1$ and $\zeta_t^c \equiv v_t(C_t)$ for $t \geq 1$. From the former observations it follows that for any policy $c \in \mathcal{P}$, the corresponding process satisfies $\zeta_t^c \overset{a.s.}{=} u_t^{\prime i}(c_t^i)/\lambda^i$ for all i and $t \geq 1$. □

Proof of Lemma 1: Consider policy c and let ζ^c be the corresponding implicit prices. Let $U^{i\prime} \in \mathbb{R}$ be an arbitrary level of utility. By allocative efficiency and Lemma 3, one has $\zeta_t^c \overset{a.s.}{=} u_t^{i\prime}(c_t^i)/\lambda^i$ for all i and $t \geq 1$. This, along with condition $c_0^{i\prime} \equiv [U^{i\prime} - U^i(c^i)]/\lambda^i + c_0^i$ give necessary and sufficient conditions for cost minimization in the problem

$$
\min_{\hat{c}^i \in \mathcal{C}_{++}} \hat{c}_0^i + \sum_t E\zeta_t^c \hat{c}_t^i : U^i\left(\hat{c}^i\right) \geq U^{i\prime},
$$

Observe that for prices ζ^c cost minimizing consumption c_t^i in periods $t \geq 1$ is independent from $U^{i\prime}$. The expenditure function is given by $e^i(\zeta^c, U^{i\prime}) = c_0^{i\prime} + \sum_t E\zeta_t^c c_t^i$. Plugging this into the equivalent variation formula (1) gives

$$
EV_{c,c'}^i = \left[U^i\left(c^{i\prime}\right) - U^i\left(c^i\right)\right]/\lambda^i = E\sum_{t\geq 1}\left[\frac{u_t^i\left(c_t^{i\prime}\right)}{\lambda^i} - \frac{u_t^i\left(c_t^i\right)}{\lambda^i}\right] + c_0^{i\prime} - c_0^i.
$$

Aggregate equivalent variation $EV_{c,c'} \equiv \sum_i EV^i_{c,c'}$ is then given by

$$
\begin{aligned}
EV_{c,c'} &= E\sum_{t\geq 1}\sum_i \left[\frac{u^i_t\left(c^{i\prime}_t\right)}{\lambda^i} - \frac{u^i_t\left(c^i_t\right)}{\lambda^i}\right] + \sum_i \left[c^{i\prime}_0 - c^i_0\right] = \\
&= E\sum_{t\geq 1}\left[u_t\left(C'_t\right) - u_t\left(C_t\right)\right] + C'_0 - C_0,
\end{aligned}
\tag{15}
$$

where in the second inequality we used the fact that, by Lemma 3, policy $c \in \mathcal{P}$ almost surely solves problem (10) subject to the aggregate consumption constraint for any $t \geq 1$. The surplus function defined as $S(c) \equiv \sum_i c^i_0 + E\sum_{t\geq 1} u_t\left(\sum_i c^i_t\right)$ satisfies, for any pair of policies, $EV_{c,c'} = S(c') - S\left(c\right)$. Thus, inequality $EV_{c,c'} \leq 0$ holds if and only if $S\left(c\right) \geq S\left(c'\right)$ and surplus function $S : \mathcal{P} \to \mathbb{R}$ represents social preferences. It follows that preferences $\succeq_*$ are complete and transitive. Finally, suppose policy c weakly dominates c' in Pareto sense. Then $U^i(c^i) \geq U^i(c^{i\prime})$ for each i and, by monotonicity of the expenditure function in $U^{i\prime}$, one has $EV^i_{c,c'} \leq 0$. This implies that $EV_{c,c'} \leq 0$ and hence $c \succeq_* c'$, i.e., social preferences are Paretian. □

Proof of Theorem 1: We first give two lemmas and then prove the theorem.

Lemma 4. *The factual pricing kernel in period zero is* $\zeta^{\bar{c}}_t \equiv \left\{v_t(\bar{C}_t)\right\}^T_{t=1}$.

Proof of Lemma 4: According to Assumption 1 factual consumption $\bar{c}$ is efficient, and autarky is an equilibrium in financial markets. For any cash flow $x \in \mathcal{C}$, the first-order optimality condition and no-trade condition jointly imply that

$$
p_x = x_0 + \sum_{t\geq 1} E\frac{u^{i\prime}_t(\bar{c}^i_t)}{\lambda^i}x_t = x_0 + E\sum_{t\geq 1} v_t(\bar{C}_t)x_t, \tag{16}
$$

where the second equality follows from property 3 in Lemma 2, and the fact that the policy almost surely solves problem (10) for any $t \geq 1$ given aggregate consumption. □

Lemma 5. *For any $y > 0$, value $v_t(y)$ can be inferred from prices of a $\bar{C}$-spanning collection of assets if $y \in \bar{S}_t$.*

Proof of Lemma 5: For any $Y \subset \mathbb{R}$ let $\Omega_Y \equiv \left\{\omega \in \Omega | \bar{W}_{t,\omega} \in Y\right\}$. For factual consumption in period t, aggregate consumption support is defined as the closure $\bar{S}_t \equiv cl(\tilde{S}_t)$, where set $\tilde{S}_t$ is given by

$$
\tilde{S}_t \equiv \left\{y \in \mathbb{R} | \text{ for any } \varepsilon > 0, \text{ probability } \pi(\Omega_{(y-\varepsilon,y+\varepsilon)}) > 0\right\}.
$$

Consider an atom, i.e., a value $y \in \mathbb{R}$ for which $\pi(\Omega_{\{y\}}) > 0$. From a $\bar{W}$-spanning collection of assets, one can construct a portfolio with a dividend process x that pays $x_{t,\omega} = 1$ in period t and states $\omega \in \Omega_{\{y\}}$, and zero otherwise. Let p_x be the observable price. By Lemma 4, the price is given by

$$p_x = E x_t v_t(\bar{W}_t) = v_t\,(y)\,[\bar{F}_t\,(y) - \lim_{y' \uparrow y} \bar{F}_t\,(y')],$$

where $\bar{F}_t$ is the cdf of factual aggregate consumption. Since $\bar{F}_t\,(y) - \lim_{y' \uparrow y} \bar{F}_t\,(y') > 0$, ratio

$$v_t\,(y) = \frac{p_x}{\bar{F}_t\,(y) - \lim_{y' \uparrow y} \bar{F}_t\,(y')},$$

is well defined. Price p_x and cdf $\bar{F}_t$ are known to the analyst, and hence index $v_t\,(y)$ is identified.

Next, consider any $y \in \mathbb{R}$ for which cdf $\bar{F}_t$ is differentiable, and hence density $\bar{f}_t(y) \equiv \bar{F}_t'(y)$ is well defined. For any $\Delta > 0$, construct a portfolio with a dividend cash flow x that pays $x_{t,\omega} = 1/\Delta$ in period t and states $\omega \in \Omega_{(y-\Delta,y]}$, and zero otherwise. The probability measure of this set is $\pi(\Omega_{(y-\Delta,y]}) = \bar{F}_t(y) - \bar{F}_t(y-\Delta)$. Let $p_x(\Delta)$ denote the observed market price. Since for all $\omega \in \Omega_{(y-\Delta,y]}$ inequality $v_t(y-\Delta) \geq v_t(\bar{C}_{t,\omega}) \geq v_t(y)$ holds, by Lemma 4 the price of the cash flow x satisfies inequality

$$\frac{\pi(\Omega_{(y-\Delta,y]})}{\Delta} v_t(y-\Delta) \geq p_x(\Delta) \geq \frac{\pi(\Omega_{(y-\Delta,y]})}{\Delta} v_t(y).$$

By assumption, $\bar{F}_t$ is differentiable at y, and limit $\lim_{\Delta \to 0} \frac{\pi(\Omega_{(y-\Delta,y]})}{\Delta} = \bar{f}_t(y)$ is well defined and the limit price is

$$\lim_{\Delta \to 0} p_x(\Delta) = p_x(0) = \bar{f}_t(y) v_t(y). \tag{17}$$

Equation (17) shows that value $v_t(y) = p_x(0)/\bar{f}_t(y)$ is identified if density is non-zero, $\bar{f}_t(y) \neq 0$.

By assumption, the cdf of a random vector $\{\vec{c}_t^{\,i}\}_i$ is differentiable except for a countable set of realizations, and so is the cdf of aggregate consumption $\bar{C}_t = \sum_i \vec{c}_t^{\,i}$. It follows that density $\bar{f}_t(y) \equiv \bar{F}'\,(y)$ is well defined for all y except a countable set. Consider any $y \in \tilde{S}_t$. We next argue that for any $\varepsilon > 0$, there exists $y' \in (y-\varepsilon, y+\varepsilon)$ for which y' is either an atom or for which density is non-zero $\bar{f}_t(y') > 0$, and so $v_t(y')$ is identified. Suppose not. Interval $(y-\varepsilon, y+\varepsilon)$ can be partitioned into a countable collection of points for which $\bar{F}_t$ is not-differentiable, each with zero mass (no atoms), and a countable collection of open subintervals between these points, each satisfying $\bar{f}_t(y) = 0$ for all y in the subinterval and with zero mass. Thus, interval $(y-\varepsilon, y+\varepsilon)$ admits a countable partition where each partition element has zero probability mass, implying that $\pi(\Omega_{(y-\varepsilon,y+\varepsilon)}) = 0$, a contradiction to $y \in \tilde{S}_t$.

From the previous observations it follows that for any $y \in \tilde{S}_t$ there exits a sequence $\{y_n\}_n$ that converges to y, such that values $\{v_t(y_n)\}_{n=0}^{\infty}$ are identified. By continuity of v_t the sequence converges to limit $v_t(y)$ which can be inferred by an analyst, and value $v_t(y)$ is identified. Finally support $\bar{S}_t = cl(\tilde{S}_t)$ is defined as the union of all limit points for all convergent sequences in $\tilde{S}_t$, and the analogous identification argument extends for the entire support $\bar{S}_t$. □

Before concluding the proof of the theorem, we make several observations. Let $\{v_t\}_{t\geq 1}$ be data generating indices. From Lemma 5 it follows that restrictions $\{v_t|_{\bar{S}_t}\}_{t\geq 1}$ are observable by the analyst (can be inferred from prices of $\bar{W}$-spanning securities). By equation (16) restrictions $\{v_t|_{\bar{S}_t}\}_{t\geq 1}$ are also sufficient for factual prices of arbitrary cash flows, and hence prices of assets beyond $\bar{C}$-spanning securities are redundant in normative inference. Let $\{v_t^+, v_t^-\}_{t\geq 1}$ be bounds derived from $\{v_t|_{\bar{S}_t}\}_{t\geq 1}$ and let $\{\tilde{v}_t\}_{t\geq 1}$ be a collection of arbitrary standard functions satisfying $v_t^+ \geq \tilde{v}_t \geq v_t^-$ for all $t \geq 1$. Since on the support $\tilde{v}_t|_{\bar{S}_t} = v_t^+|_{\bar{S}_t} = v_t|_{\bar{S}_t}$, indices $\{\tilde{v}_t\}_{t\geq 1}$ give rise to the same pricing kernel, a single-agent mechanism with preferences $U^1(c^1) = c_0^1 + E\sum_{t=1}^{T}\int_0^{c_t^1} \tilde{v}_t(y)\,dy$ rationalizes the factual prices of assets. We next prove the sufficiency and the necessity of the full support condition.

(if) Suppose $\bar{S}_t = \mathbb{R}_+$ for all $t \geq 1$. By Lemma 5, index v_t can be inferred for the entire domain $\mathbb{R}_{++}$ for all $t \geq 1$. For any $c, c' \in \mathcal{P}$, equivalent variation (15) can be written as a sum of expected integrals of observable index v_t and formula (2) expresses a unique value of $EV_{c,c'}$ in terms of observables. Thus, social preferences are revealed for the universal set, $\mathcal{P}$.

(only if) Fix data generating $\{v_t\}_{t\geq 1}$, and suppose for some $t^* \geq 1$ one has $\bar{S}_{t^*} \neq \mathbb{R}_+$. Complement $(\bar{S}_{t^*})^{Com}$ is open in $\mathbb{R}_+$ and hence $(\bar{S}_{t^*})^{Com} \neq \{0\}$. Therefore, there exists a $y > 0$ and $\varepsilon > 0$ such that $(y-\varepsilon, y+\varepsilon) \subset (\bar{S}_{t^*})^{Com} \cap \mathbb{R}_{++}$.

Consider weakly decreasing functions $\tilde{v}_{t^*}^{RA} : \mathbb{R}_{++} \to \bar{\mathbb{R}}_+$ and $\tilde{v}_{t^*}^{RN} : \mathbb{R}_{++} \to \bar{\mathbb{R}}_+$ defined as follows. For all $y' \leq y - \varepsilon$, the functions coincide with upper bound $\tilde{v}_{t^*}^{RA}(y') = \tilde{v}_{t^*}^{RN}(y') \equiv v_{t^*}^+(y')$, while for $y' \geq y + \varepsilon$ let $\tilde{v}_{t^*}^{RA}(y') = \tilde{v}_{t^*}^{RN}(y') \equiv v_{t^*}^-(y')$. On the interval $(y-\varepsilon, y+\varepsilon)$ function $\tilde{v}_{t^*}^{RA}(y')$ is defined as

$$\tilde{v}_{t^*}^{RA}(y') \equiv v_{t^*}^+(y) + \frac{v_{t^*}^-(y) - v_{t^*}^+(y)}{2\varepsilon}(y' - y + \varepsilon).$$

The index is strictly decreasing on the interval and therefore the function represents (locally) risk-averse preferences. Index $v_{t^*}^{RN}$ on the considered interval exhibits constant marginal utility

$$\tilde{v}_{t^*}^{RN}(y') \equiv \frac{v_{t^*}^-(y) + v_{t^*}^+(y)}{2}.$$

and associated preferences are (locally) risk-neutral.

Consider policies $c, c' \in \mathcal{P}$ that differ only in terms of consumption in period t^* and period zero. For policy c, aggregate consumption in t^* is deterministic, $\bar{C}_{t^*} = \tilde{y}$, and for policy c' consumption is a mean preserving spread; i.e., it takes two values $\tilde{y} - \varepsilon$ and $\tilde{y} + \varepsilon$, each with probability $\frac{1}{2}$. Note that $c, c' \in \mathcal{P}$. Let

$$L \equiv \frac{1}{2} \left(\int_{\tilde{y}-\varepsilon}^{\tilde{y}} \tilde{v}_{t^*}^{RA}(y)dy - \int_{\tilde{y}}^{\tilde{y}+\varepsilon} \tilde{v}_{t^*}^{RA}(y)dy \right),$$

be the loss of surplus due to higher consumption variance under policy c' in period t^*, assuming risk-averse preferences. For policy c period zero consumption is then $\bar{C}_0 = 0$, while for policy c' it is $C'_0 = L/2$.

For $t \neq t^*$ let the two indices coincide with the data generating one, i.e., $\tilde{v}_t^{RA} = \tilde{v}_t^{RN} = v_t$. Since C'_0 compensates for only half of the loss, an agent with utility $U^{RA}(c^i) = c_0^i + E \sum_{t=1}^{T} \int_0^{c_t^i} \tilde{v}_t^{RA}(y)\, dy$ prefers a safe policy, i.e., $U^{RA}(C) > U^{RA}(C')$, while for risk neutral utility $U^{RN}(c) = c_0 + E \sum_{t=1}^{T} \int_0^{c_t} \tilde{v}_t^{RA}(y)\, dy$ the inequality is reversed, $U^{RA}(C) < U^{RA}(C')$. Observe that utility functions $\tilde{v}_{t^*}^{RA}$ and $\tilde{v}_{t^*}^{RA}$ are not continuously differentiable and hence are not standard. However, each of the two indices can be arbitrarily closely approximated by standard functions, such that $v_t^+ \geq \tilde{v}_t \geq v_t^-$, which preserve the inequalities in the utilities. A single-agent mechanism with such approximations rationalizes observed factual prices (for any t, indices $\tilde{v}_t$ satisfy $v_t^+ \geq \tilde{v}_t \geq v_t^-$), and in one mechanism the social ranking is $c \succ c'$, while in the other it is $c' \succ c$. Thus, prices of securities cannot reveal social preferences for set $\mathcal{P}$ given $\{v_t\}_{t \geq 1}$. □

Proof of Proposition 1: By strict monotonicity of $v_t(y)$ in y, one has $v_t^+ \geq v_t \geq v_t^- \geq 0$ for all $t \geq 1$, and hence $EV_{c,c'}^+ \geq EV_{c,c}$. Thus $EV_{c,c'}^+ \leq 0$ implies $EV_{c,c'} \leq 0$ and hence $c \succeq_* c'$. □

Proof of Proposition 2: Fix a finite collection of policies $\mathcal{P}^F$. Suppose for each $t \geq 1$ set $\bar{S}_t$ is ε-dense in $\mathbb{R}_{++}$ where $0 < \varepsilon < \min_{c \in \mathcal{P}^F, t \geq 1} \inf S_t^c$. Since each S_t^c is bounded away from zero, such ε exists. For any t, define recursively a countable subset of $\bar{S}_t$, denoted by $\{y_t^n\}_{n=0}^{\infty}$ as follows. For $n = 0$, choose $y_t^0 \in \bar{S}_t$ for which $0 < y_t^0 \leq \varepsilon$. Given $y_t^n \in \bar{S}_t$, let $y_t^{n+1} \in [y_t^n + \varepsilon, y_t^n + 3\varepsilon]$. By ε-denseness of $\bar{S}_t$, such collection $\{y_t^n\}_{n=0}^{\infty}$ exists. Moreover, by construction, for all elements in the collection the distance between any two elements is no smaller than ε, while the distance between any two adjacent elements is no larger than 3ε. Consider observable statistic $EV_{c,c'}^-$ defined in Footnote 11. For any pair of policies $EV_{c,c'}^- \leq EV_{c,c'}$ and hence $EV_{c,c'}^+ - EV_{c,c'} \leq EV_{c,c'}^+ - EV_{c,c'}^-$. The difference between the two statistics is given by

$$EV^{+}_{c,c'} - EV^{-}_{c,c'} = \sum_{t=1}^{T} E \int\limits_{\min(C_t, C'_t)}^{\max(C_t, C'_t)} \left(v^{+}_t(y) - v^{-}_t(y)\right) dy. \tag{18}$$

Since index v^{+}_t is non-increasing and v^{-}_t is non-decreasing in the support size, the difference $EV^{+}_{c,c'} - EV^{-}_{c,c'}$ is no larger than the analogous difference derived from a subset of the support, $\{y^n_t\}_{n=0}^{\infty} \subset \bar{S}_t$. For the latter subset, the index bounds v^{+}_t, v^{-}_t are given by step functions and, in period t and state ω, the difference in money-metric welfare can be written as a sum of "rectangular" areas,

$$\int\limits_{\min(C_t, C'_t)}^{\max(C_t, C'_t)} \left(v^{+}_t(y) - v^{-}_t(y)\right) dy \leq \sum_{n=0}^{\infty} \left(y^{n+1}_t - y^n\right) \times \left(v_t\left(y^n_t\right) - v_t\left(y^{n+1}_t\right)\right). \tag{19}$$

By construction, $(y_{n+1} - y_n) \leq 3\varepsilon$ and also $\sum_{n=0}^{\infty} v_t\left(y^n_t\right) - v_t\left(y^{n+1}_t\right) = v_t\left(y^0_t\right)$. Thus, the inequality can be written as

$$\int\limits_{\min(C_t, C'_t)}^{\max(C_t, C'_t)} \left(v^{+}_t(y) - v^{-}_t(y)\right) dy \leq 3\varepsilon \sum_{n=0}^{\infty} \left(v_t\left(y^{n+1}_t\right) - v_t\left(y^n_t\right)\right) \leq 3\varepsilon v_t\left(y^0_t\right). \tag{20}$$

Define positive scalar $\alpha \equiv \max_{t \geq 1} |v_t(y^0_t)|/3T < \infty$. Using this and (18) gives

$$EV^{+}_{c,c'} - EV^{-}_{c,c'} \leq 3\varepsilon \sum_{t} v_t\left(y^0_t\right) \leq \varepsilon\alpha. \tag{21}$$

Fix $\varepsilon = \frac{1}{2} \min_{c,c' \in \mathcal{P}^F} |EV_{c,c'}| / \alpha$. Consider any pair $c, c' \in \mathcal{P}^F$. Observe that

$$EV^{+}_{c,c'} - EV_{c,c'} \leq EV^{+}_{c,c'} - EV^{-}_{c,c'} \leq \frac{1}{2} \min_{c,c' \in \mathcal{P}^F} |EV_{c,c'}|,$$

and hence $EV^{+}_{c,c'} \leq EV_{c,c'} + \frac{1}{2}|EV_{c,c'}|$ and it follows that inequality $EV_{c,c'} < 0$ implies $EV^{+}_{c,c'} < 0$. Since policies in $\mathcal{P}^F$ are non-indifferent, $EV_{c,c'} \neq 0$, either $EV^{+}_{c,c'} < 0$ or $EV^{+}_{c',c} < 0$ holds and the preference relation for pair c, c' is revealed. Since this is true for all pairs of policies in $\mathcal{P}^F$, social preferences are revealed provided that supports $\bar{S}_t$ are $\varepsilon-$dense. $\square$

Proof of Proposition 3: We use the well-known fact that an inverse of a strictly decreasing and convex bijection is convex, and hence a subset of convex standard functions is closed under inversion and summation. Consider definitions used in the proof of Lemma **2**. Observe that function $\tilde{y}^i(v_t)$ is the inverse of a strictly decreasing and convex (standard) function $u_t^{i\prime}(\cdot)$ and hence it is strictly decreasing and convex. Thus, the sum $\sum_t \tilde{y}_t^i(v_t)$ is also strictly decreasing and convex. Finally function $v_t(y)$ is an inverse of $\sum_t \tilde{y}_t^i(v_t)$, and hence it is convex. $\square$