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Real Exchange Rate Risk and FDI flows: stylized facts and theory

Alexander McQuoid, Jacek Rothert, and Katherine Smith

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Alexander McQuoid
U.S. Naval Academy

Jacek Rothert
U.S. Naval Academy
and FAME|GRAPE

Katherine Smith
U.S. Naval Academy

Abstract

We document a robust negative relationship between bilateral RER volatility and bilateral FDI flows in the European Union. We then extend the standard international business cycle model to allow for domestic and foreign ownership of physical capital stock to be less than perfect substitutes. This allows the model to have meaningful predictions about the behavior of gross FDI flows. We characterize the conditions under which lower RER volatility coincides with larger bilateral FDI flows. We also show, both theoretically, and using numerical simulations, that the magnitude of the relationship between the RER volatility and FDI flows depends crucially on one parameter: the elasticity of substitution between domestic and foreign ownership of capital stock used in production. Our results suggest the existence of a new channel through which a reduction in RER volatility can be welfare improving: more efficient allocation of capital across countries (capital diversity).

Keywords:
FDI, real exchange rates, international financial integration, exchange rate risk

JEL Classification
E, F

Corresponding author
Jacek Rothert, jacek.rothert@gmail.com

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Foundation of Admirers and Mavens of Economics
ull. Mazowiecka 11/14
00-052 Warszaw
Poland

W | grape.org.pl
E | grape@grape.org.pl
TT | GRAPE_ORG
FB | GRAPE.ORG
PH | +48 799 012 202

1 Introduction

Free movement of capital is one of the pillars of the European economic integration, along with free movement of goods, services, and labor. Foreign Direct Investment (FDI) flows as a share of total investment expenditures in the European Union averages almost 30% - roughly double the ratio observed in the rest of the world. Interestingly, the share of FDI flows in total investment expenditures in the European Union exploded within two years prior to the introduction of the Euro, and remained persistently higher within the Eurozone.

In this note, we document the empirical patterns about the behavior of gross FDI flows around the introduction of the common currency in particular, and during periods of declines in real exchange rate (RER) risk in general. We then characterize, theoretically, the conditions under which a decline in the RER risk would coincide with an increase in gross FDI flows and discuss how the strength of the relationship between RER risk and gross FDI flows depends on the degree to which the domestic and foreign capital are complements vs. substitutes.

Empirically, we find robust evidence that gross FDI flows between country pairs are higher when those countries face a smaller bilateral RER risk. We document this pattern using annual data on bilateral FDI flows within the European Union - a unique environment that is as close to free international capital mobility as possible, while providing data on cross-border flows and positions. Using a staggered difference in differences analysis we find first that forming a currency union is associated with a permanent increase in gross bilateral FDI flows. Our analysis exploits the fact that the initial creation of the EMU included eleven countries, with the additional eight members joining gradually between 2001 and 2023. This staggered expansion of the EMU offers a good deal of variation in the data across countries and over time. Additionally, we show that a decline in RER risk in general is associated with an increase in gross bilateral FDI flows.

We then investigate the relationship between gross FDI and RER risk in the workhorse international business cycle model of Backus et al. (1994) - a foundational framework in open economy macroeconomics with well defined real exchange rates. We model gross FDI flows in a manner pioneered in McQuiod et al. (2022) — we relax the implicit assumption of an infinite elasticity of substitution between domestic and foreign ownership of physical capital stock. That single modification allows the model to have well defined FDI stocks and flows. Within our framework we characterize the conditions under which a decline in RER risk would be associated with an increase in gross bilateral FDI flows.

We first show theoretically that in the absence of inter-temporal shocks, a decline in RER risk coincides with an increase in gross bilateral FDI flows if and only if the correlation between aggregate consumption and the real exchange rate is negative. Our theoretical results also indicate that the strength of (any) relationship between RER risk and gross FDI flows depend on a key parameter — the elasticity of substitution between the domestic and foreign ownership of the local physical capital stock. In the presence of inter-temporal shocks (either supply or demand), we use numerical simulations, because the shocks have direct impact on the marginal product of capital and/or on the stochastic discount factor. We show that an increase in the cross-country correlation of supply shocks leads to both a decline in the RER risk and an increase in bilateral FDI flows for a large space of reasonable parameter values. We confirm our theoretical conjecture that the strength of the negative relationship between RER risk and gross FDI flows depends on the elasticity of substitution between the domestic and foreign ownership of the local physical capital stock - the larger that elasticity is, the stronger the relationship. Hence, the empirical magnitude of the relationship between RER risk and gross FDI flows can help identify the elasticity of substitution between domestic and foreign ownership of capital stock.

2 FDI and RER risk: stylized facts

We start by documenting a number of stylized facts about the relationship between the RER risk and FDI flows. We focus on the European Union (EU) for two reasons. First it is hard (if not impossible) to find any region that is closer to free international capital mobility. Second, the EU features substantial variation in both short-term and long-term bilateral RER volatility, both over time and across country pairs.

2.1 Data

Our sample of countries includes 27 current members of the European Union, as well as Iceland, Norway, Switzerland, and the United Kingdom.[1] Our data on FDI flows, nominal and real exchange rates, aggregate consumer prices, and other macroeconomic indicators are drawn from a variety of sources.

Bilateral real exchange rates comes from the IMF's International Financial Statistics (IFS) database. We use quarterly data from 1980Q1 through 2017Q4. We construct bilateral real exchange rates for each country pair using the nominal exchange rate multiplied by the ratio of CPI's.

[1] UK was a member of the EU during the sample period studied. While Iceland, Norway, and Switzerland have never been part of the EU, they form the European Free Trade Association (jointly with Liechtenstein, which we omit due to its small size and a tax haven status). As such, these countries have agreements with the EU on free movement of goods and services, people, and capital.

The bilateral FDI flows are drawn from the OECD and are measured in current prices in millions of U.S. dollars. This sample covers annual data from 1986 until 2016.

Additionally, we use quarterly data on aggregate consumption and investment expenditures, and Gross Domestic Product for each country in our sample. This data is drawn from the IFS covering 1980Q1 through 2017Q4. All variables are measured in domestic currency, and for our analysis, we focus on the ratio of consumption and investment to GDP.

2.2 RER risk in the EU

The main source of variation in RER risk in the EU is the creation of the European Monetary Union (EMU), as illustrated in Figure 2. The figure shows the rolling 12 quarter standard deviation of the RER between Germany and seven European countries, 3 of whom adopted the euro along with Germany in 1999 Q1 (France, Spain, and Italy), while the other four did not (UK, Norway, Switzerland, and Sweden).

While the RER SD varied notably across these 7 partner countries in 1991, by 2014, real exchange rate variability had bifurcated into two distinct groups, those that shared a common currency with Germany and those that did not. At the beginning of the sample, Germany had similarly high RER volatility with both Italy and Sweden, but by the end of the sample - after the adoption of the euro - the RER variability with Italy was almost non-existent.

RER variability with Sweden declined over the sample, but by far less than among euro partner countries. While RER volatility declined for most countries vis-a-vis Germany during the sample period, this was not true of all countries - RER volatility between Germany and Switzerland was higher at the end of the sample - and RER volatility fluctuations are more pronounced among non-euro countries.

This clear pattern observed between Germany and a selection of non-euro and euro partner countries is present among the larger set of European countries. Using quarterly data from 27 EU countries as well as Norway, Iceland, Switzerland, and the UK, we calculate the rolling standard deviation of the real exchange rate using 12-quarter and 20-quarter averages over a thirty-eight year period starting in the first quarter of 1980. Summary Statistics are reported in Table 1.

For the overall sample, the mean value of the standard deviation of the real exchange rate is 0.063 (20 quarter rolling average). If we look only at countries that will eventually join the euro, the 20 quarter rolling average is 0.045 compared to an average of 0.073 for all other country pairs. Among the euro country pairs, the 20 quarter rolling average prior to euro adoption is 0.056, and 0.013 after euro adoption, which is highly suggestive of the role euro adoption plays in reducing real exchange rate risk.

If we look at the original 11 euro adopters (those countries that adopted in 1999), the real exchange rate risk among these countries prior to adoption was already low (0.040), but it is notably lower after adoption (0.012). While the countries that adopted the euro first were likely those that had more stable, integrated economies, a more stark pattern emerges when later euro adopters are considered. For the 8 countries that eventually adopted the euro after 1999, real exchange risk is notably higher prior to adoption (0.062) compared to post adoption (0.016).

To provide a more formal analysis of the impact of euro adoption on real exchange rate risk, consider the results presented in Table 2. In columns (1) and (2), we report the difference-in-difference estimator for euro adoption. We find that adopting the euro is associated with a decline of 0.0127 for the 12 quarter rolling average of the standard deviation of the real exchange rate and a 0.0191 point decline for the 20 quarter rolling average. Given the overall means for these two statistics, this represents a 26% and 30% decline, respectively. The choice of the window for calculating the rolling standard deviation does not alter the qualitative findings, as can be seen in columns (3)-(6).

2.3 RER risk and FDI

We analyze the relationship between the RER risk and FDI by considering the differential experience for European countries who shared a single market, only some of whom adopted a shared currency. Our general empirical specification takes the following form:

$$\ln FDI_{ij,t} = \alpha_{ij} + \gamma_t + \delta \cdot \texttt{RERrisk}_{ij,t} + \\ + \beta_1 \ln\left(GDP_{i,t} + GDP_{j,t}\right) + \beta_2 \ln\left(Pop_{i,t} + Pop_{j,t}\right) + \epsilon_{ij,t}$$

We consider two measures of RER risk. First, given the results described in the previous subsection, we define $\texttt{RERrisk}_{ij,t}$ to be an indicator variable equal to 1 if the pair ij shares a common currency in year t, and 0 otherwise.[2] Second, we set $\texttt{RERrisk}_{ij,t}$ to be the 12-quarter or the 20-quarter rolling standard deviations of the realized values of RER, averaged over the calendar year.[3] The results are presented in Tables 3 and 4.

First, consider the difference in bilateral FDI flows between country pairs that do share a common currency compared to those that do not. It is important to note that euro adoption was

[2]This is formally a measure of the lack of RER risk, and the estimated coefficient δ will be interpreted accordingly.

[3]We calculate the rolling standard deviations using quarterly data, but take the average over the four quarters to aggregate up to an annual level to match the level of observation in the FDI data. Results are consistent using alternative window timing to construct the rolling standard deviations of realized values of the RER.

staggered across countries, with 11 countries adopting in 1999 and 8 additional countries adopting over the next 16 years. Since the euro treatment is achieved only when two countries have a common currency, our DD estimator is a staggered treatment indicator, starting in the year when both countries have adopted the euro.

The results presented in Table 3 show that euro adoption is associated with increased gross FDI flows. Relative to the baseline of pre-monetary union gross FDI flows, adoption of the euro is associated with a 25% increase in gross FDI flows (column (4)). The results are robust to alternative controls (columns (1)-(3)). The estimates are similar if we restrict the sample to just first wave adopters and those countries that never adopt the euro (column (5)). Further, if we restrict the sample to just the largest countries, the adoption of the euro is associated with a 20% increase in gross FDI flows. The results are suggestive that the adoption of the euro had a significant positive effect on FDI flows across euro partner countries.

These results are consistent with the previous literature that has evaluated the impact of the EMU on FDI flows. The seminal work of Petroulas (2007) estimates that the introduction of the euro resulted in an increase in inward FDI flows by 17%, while more recent studies find estimates between 10-30% (see Camarero et al. (2018), De Sousa and Lochard (2011), Dinga and Dingová (2011), Schiavo (2007). Our estimates are similar even though we have a longer time horizon post adoption, which includes globally volatile periods following the Great Recession and the Euro Financial Crisis.

Next, consider the relationship between real exchange rate volatility and gross bilateral FDI flows, using our two measures of real exchange rate volatility from Table 2, 12-quarter and 20-

quarter rolling standard deviations. The results based on those two measures are presented in Table 4. Column (1) reports the results for the 12-quarter rolling standard deviation of the real exchange rate, controlling for year and bilateral pair fixed effects. As the real exchange rate volatility increases, gross FDI flows decrease. To get a sense of the implied magnitudes, the coefficient implies that a doubling of the mean of the 12 quarter rolling RER SD would result in a 34% decline in gross FDI flows. The estimate in column (2) includes additional controls for GDP and population. Based on this specification, a doubling of the mean would result in a 17% percent decline in gross FDI flows. Columns (3) and (4) repeat the above analysis, but focus on the 20-quarter rolling SD of the real exchange rate. The estimated magnitudes are 46% and 21% respectively.

For robustness, we re-estimate the models on two separate sub-samples. First, we focus on the first group of euro adopters, who tended to be richer and more integrated economies. These bilateral pairs already had more stable real exchange rates and notable volumes of gross FDI flows. Nonetheless, these countries too benefited from the reduced real exchange rate risk associated with the (nearly) irrevocable adoption of the euro. Based on the average decline for the original adopters of the 12-quarter RER standard deviation reported in Table 1 of 0.028, the increase in gross FDI flows associated with the adoption of the euro for these countries was 9.1%. Using the 20-quarter measure instead, we estimated a 11% implied increase in gross FDI flows as a result of the decline in real exchange rate risk associated with euro adoption. Finally, we consider an alternative sample of large countries with similar quantitative results (see columns (7) and (8)).

To help put these RER risk results into context, the overall estimated impact of the adoption of the euro on FDI flows is roughly 20%. The adoption of the euro, however, has a multitude of economic effects that are likely to impact cross-border investment flows above and beyond real exchange rate risk. Our results here suggest that roughly half of the estimated net impact on FDI flows within the region comes through the specific mechanism of a reduction in real exchange rate risk.

3 Model

We study the behavior of FDI over the business cycle using using a classic international business cycle framework of Backus et al. (1994). There two countries: Home (A) and Foreign (B). The GDP in each country is produced using capital and labor.

In order to study FDI in that model, we need a notion of capital ownership, so that it makes a difference whether capital located physically in country A is owned by residents of the Home or the Foreign. We do so by defining a concept of effective capital stock, $\tilde{K}$, as follows:

$$\tilde{K} = \left[\omega k_A{}^{\frac{\theta-1}{\theta}} + (1-\omega) k_A^*{}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}},$$

where k_A is owned by residents of the Home country, and k_A^* is owned by residents of the Foreign country. Similar specification for $\tilde{K}^*$ reads: $\tilde{K}^* = \left[\omega k_B^*{}^{\frac{\theta-1}{\theta}} + (1-\omega) k_B{}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}$. ***This is our key innovation, and the key difference between the BKK framework and ours***.

The capital stocks located in country A and owned by either Home or Foreign residents evolve over time as follows:

$$k_A\left(s^t\right) = k_A\left(s^{t-1}\right) + x_A\left(s^t\right)$$

$$k_A^*\left(s^t\right) = k_A^*\left(s^{t-1}\right) + x_A^*\left(s^t\right)$$

where x_A are net purchases of country A capital goods made by domestic residents, and x_A^* are net purchases of country A capital goods made by foreign residents. Similar relationships hold for k_B and k_B^*.

New capital goods in each country are produced from the two tradeable goods, combined using a Cobb-Douglas function (Bems, 2008). The new capital goods can be purchased by either home or foreign residents:

$$x_A + x_A^* = H(a_x, b_x) \equiv {a_x}^{\omega_x} {b_x}^{1-\omega_x}$$

$$x_B + x_B^* = H^*(a_x^*, b_x^*) \equiv {b_x^*}^{\omega_x} {a_x^*}^{1-\omega_x},$$

where ω_x is the home bias in investment expenditures. The stand-in household's aggregate consumption in each country is a CES composite of the two tradeable goods:

$$C = G(a_c, b_c) \equiv \left[\omega_c {a_c}^{\frac{\eta-1}{\eta}} + (1-\omega_c){b_c}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$$

$$C^* = G^*(a_c^*, b_c^*) \equiv \left[\omega_c {b_c^*}^{\frac{\eta-1}{\eta}} + (1-\omega_c){a_c^*}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}},$$

where ω_c is a home bias in consumption expenditures and η is the elasticity of substitution between domestic and foreign tradeable goods in consumption.

The price of the Home country's aggregate good is normalized to 1. Given that, the price of good A is denoted with q^a, the price of good B is denoted with q^b, and the price of the Foreign country's aggregate good is the real exchange rate, defined as:

$$rx \equiv \min_{a^*,b^*} q^a \cdot a^* + q^b \cdot b^* \qquad \text{subject to: } 1 \geq G^*(a^*, b^*)$$

With the above definition, an increase in rx means depreciation (appreciation) in the Home (Foreign) country.

The resource constraints and the production technologies for the tradeable goods are given by:

$$a + a^* = Y = e^z \cdot \tilde{K}^\alpha L^{1-\alpha}$$

$$b + b^* = Y^* = e^{z^*} \cdot \tilde{K^*}^\alpha L^{*1-\alpha}$$

The country-specific productivities, z and z^* are stochastic. A stand in household maximizes life-time expected utility:

$$E_0 \sum_{t=1}^{\infty} \beta^{t-1} \psi_t U(C_t),$$

where ψ_t is a preference (demand) shock.

State of the world In each period t, the world is hit with shock $s_t \in \mathbb{S} \subset \mathbb{R}^N$, governed by the Markov process $\pi(s_{t+1}|s_t)$. A history of shocks up to time t is denoted with $s^t := (s_1, s_2, ..., s_t)$. The realization of s_t affects the values of stochastic parameters in the model: $s_t \equiv \left(z_t, z_t^*, \psi_t, \psi_t^*, \omega_t^c, \omega_t^{c,*}, \omega_t^x, \omega_t^{x,*}\right)$. We assume that the shocks to the demand- and the supply-side are orthogonal. Otherwise, they can be arbitrarily correlated within and between countries.

Utility maximization A stand-in household in each country is endowed with a unit of labor and solves:

$$\max \sum_{t=1}^{\infty} \beta^t \left[\sum_{s^t} \pi\left(s^t\right) \psi\left(s^t\right) U\left(C(s^t)\right) \right]$$

subject to:

$$C(s^t) + x_A\left(s^t\right) + rx\left(s^t\right) \cdot x_B\left(s^t\right) \leq w(s^t) + r_A\left(s^t\right) k_A\left(s^{t-1}\right) + r_B\left(s^t\right) k_B\left(s^{t-1}\right) \tag{3.1}$$

$$k_A\left(s^t\right) \leq (1-\delta) k_A\left(s^{t-1}\right) + x_A\left(s^t\right) \tag{3.2}$$

$$k_B\left(s^t\right) \leq (1-\delta) k_B\left(s^{t-1}\right) + x_B\left(s^t\right) \tag{3.3}$$

The household takes as given all prices as well as the aggregate allocations.

Definition 3.1 (Equilibrium)**.** Equilibrium is a tuple $\left[\mathbf{z}(\mathbf{s^t}), \mathbf{z}^*(\mathbf{s^t}), \mathbf{rx}(\mathbf{s^t}), \mathbf{q^a}(\mathbf{s^t}), \mathbf{q^b}(\mathbf{s^t})\right]$, where $\mathbf{z} := \left[(k_i, x_i, c, r_i)_{i=A,B}\,, y\right]$ such that

- given prices, allocations solve households' and firms' optimizations problems
- all markets clear.

Definition 3.2 (History of no-shocks)**.** A history of no shocks is defined as $\mathbf{S}^t = (s_1, s_2, ..., s_t)$ where $s_\tau = \mathbf{0}$, for all $\tau \leq t$.

Definition 3.3 (Steady-state and symmetric steady-state)**.** A steady-state allocation is the limit of the equilibrium allocation under history of no-shocks. A symmetric steady state is a steady-state allocation with $\mathbf{z} = \mathbf{z}^*$, $q^a = q^b$, and $rer = 1$.

It is important to point out that the steady-state allocation defined above is different from what is typically considered as a non-stochastic steady-state in DSGE models with policy functions approximated locally. The non-stochastic steady-state is equivalent to assuming that all shocks will have a zero-realization with probability one. Our focus is the exact opposite: we want to understand the impact of the future risk on today's allocations.

3.1 Characterization

In state s^t, the inter-temporal Euler conditions for the domestically (k_A) and foreign-located (k_B) capital are:

$$U'\left(C\left(s^t\right)\right)=\sum_{s^{t+1}}\pi\left(s^{t+1}|s^t\right)\frac{\psi(s^{t+1})}{\psi(s^t)}U'\left(C\left(s^{t+1}\right)\right)\left[1-\delta+r_A\left(s^{t+1}\right)\right] \tag{3.4}$$

$$p\left(s^t\right)\cdot U'\left(C\left(s^t\right)\right)=\sum_{s^{t+1}}\pi\left(s^{t+1}|s^t\right)\frac{\psi(s^{t+1})}{\psi(s^t)}U'\left(C\left(s^{t+1}\right)\right)\left[(1-\delta)rx\left(s^{t+1}\right)+r_B\left(s^{t+1}\right)\right] \tag{3.5}$$

Combining the two we get:

$$E_t\left[U'(C_{t+1})\psi_{t+1}(1-\delta+r_{A,t+1})\right]=\frac{1}{p_t}E_t\left[U'(C_{t+1})\psi_{t+1}\left(rx_{t+1}\cdot(1-\delta)+r_{B,t+1}\right)\right], \tag{3.6}$$

where the rental rates are given by:

$$r_{A,t+1}=q^a_{t+1}\cdot z_{t+1}\cdot\omega\left(\frac{\tilde{K}_t}{k_{A,t}}\right)^{\frac{1}{\theta}}\alpha\tilde{K}_t^{\alpha-1} \tag{3.7}$$

$$r_{B,t+1}=q^b_{t+1}\cdot z^*_{t+1}\cdot(1-\omega)\left(\frac{\tilde{K}^*_t}{k_{B,t}}\right)^{\frac{1}{\theta}}\alpha\tilde{K}^{*\,\alpha-1}_t \tag{3.8}$$

Proposition 3.4 summarizez our theoretical results that link the RER risk with FDI flows.

Proposition 3.4. *Suppose $Var(\epsilon_z)=Var(\epsilon_z^*)=Var(\epsilon_\psi)=Var(\epsilon_\psi^*)=0$ and $\delta=1$. Then, in a symmetric steady-state, a decrease in $Var(rx_{t+1})$ will increase $\frac{k_{B,t}}{k_{A,t}}$ if and only if $COV\left(C_{t+1},rx_{t+1}\right)\geq 0$.*

Proof. With $\delta=1$, and with $Var(\epsilon_z)=Var(\epsilon_z^*)=Var(\epsilon_\psi)=Var(\epsilon_\psi^*)=0$, then (3.6) in a symmetric steady-state reduces to:

$$E_t\left[U'(C_{t+1})\left(q^a_{t+1}\cdot\omega\left(\frac{\tilde{K}_t}{k_{A,t}}\right)^{\frac{1}{\theta}}\right)\right]=E_t\left[U'(C_{t+1})\left(q^b_{t+1}\cdot(1-\omega)\left(\frac{\tilde{K}^*_t}{k_{B,t}}\right)^{\frac{1}{\theta}}\right)\right].$$

which, using $\tilde{K}_t = \tilde{K}_t^*$ and the fact that $E(xy) = E(x)E(y) + COV(x, y)$, can be written as:

$$\frac{\omega}{1-\omega}\left(\frac{k_{B,t}}{k_{A,t}}\right)^{\frac{1}{\theta}} = \frac{E_t\left[U'(C_{t+1})\right] \cdot E_t\left[q^b_{t+1}\right] + corr\left(U'(C_{t+1}), q^b_{t+1}\right) \cdot \sqrt{Var\left(U'(C_{t+1})\right) Var(q^b_{t+1})}}{E_t\left[U'(C_{t+1})\right] \cdot E_t\left[q^a_{t+1}\right] + corr\left(U'(C_{t+1}), q^a_{t+1}\right) \cdot \sqrt{Var\left(U'(C_{t+1})\right) Var(q^a_{t+1})}} \tag{3.9}$$

Since $E_t\left[q^a_{t+1}\right] = E_t\left[q^b_{t+1}\right]$ and $Var(q^a_{t+1}) = Var(q^b_{t+1})$, the effect that an increase in $Var(rx)$ has on $\frac{k^*_{B,t}}{k_{A,t}}$ will depend on $corr\left(U'(C_{t+1}), q^b_{t+1}\right)$ and $corr\left(U'(C_{t+1}), q^a_{t+1}\right)$. Since $corr(q^a, q^b) = -1$ it then follows that an increase in $Var(rx)$ will coincide with a decline in $\frac{k_{B,t}}{k_{A,t}}$ if and only if $corr\left(U'(C_{t+1}), q^b_{t+1}\right) \leq 0 \iff corr\left(C_{t+1}, rx_{t+1}\right) \leq 0$, because $rx \propto \frac{q^a}{q^b}$. □

In words, if the only source of variation in the real exchange rate are intra-temporal demand shocks (those that do not affect the trade-off between today's and future aggregate consumption), then the reduction in the volatility of the real exchange rate will be associated with the increase in the bilateral FDI stocks (and flows) if and only if domestic consumption and real exchange rate are negatively correlated.

It is tempting to say that the strength of the relationship between $\frac{k^*_B}{k_A}$ and $Var(rx)$ depends on θ, since we have $\left(\frac{k_{B,t}}{k_{A,t}}\right)^{\frac{1}{\theta}}$ on the left-hand side of 3.9. However, changing the value of θ means changing the steady-state of the model, which means that the terms on the right-hand side, evaluated at the stochastic steady-state, will be different. In the next section we will evaluate numerically how the strength of the relationship between RER variance and the magnitude of gross FDI flows depends on the elasticity of substitution θ.

4 Numerical examples

We now turn to numerical simulations to get a better sense how the elasticity of substitution θ affects the relationship between RER volatility and gross FDI flows in the model. Since in the previous sections we offered some theoretical results under the assumption of no supply shocks, in this section we present numerical simulations from the model parametrization where supply shocks are the only sources of fluctuations. The volatility of the real exchange rate is an endogenous object in the model, and its reduction can be achieved by a variety of ways. We consider one of them: an increase in the synchronization of shocks.

The simulations are based on the third-order approximation of the decisions rules around the non-stochastic steady-state. We use third-order approximation so that the decision rules depend on the variance and covariance of shocks.

Figure 1 presents the results from an experiment where supply shocks are the main drivers of fluctuations, and the decline in the volatility of RER happens via their increased synchronization. The figure considers two values of the elasticity of substitution between domestic and foreign goods in aggregate consumption: 0.9 in the left column, corresponding to the value in Heathcote and Perri (2002)[4], and 1.5 in the right column, the original value used in Backus et al. (1994).

The key message is clear: the magnitude of the relationship between average level of FDI flows and the volatility of the RER is a function of the elasticity of substitution between the domestic and foreign ownership. When θ_K is higher, the elasticity of FDI flows w.r.t. RER risk is larger (the downward sloping line in the top panels in Figure 1 indicates the stronger response). If θ_K is high then the reduction in $\frac{k_A^*}{k_A}$ has little impact on marginal products of capital and on the relative real returns, but household's income is now more tied to the price of the good it cares about. As θ_K decreases, moving capital back home has a bigger impact on returns to capital (reducing marginal

[4] Corsetti et al. (2008) and Rothert (2020) both offer an even lower estimate for that parameter, but they both have a somewhat different model - the former includes a non-traded sector, while the latter has a different parametrization with one of the countries being a small, emerging economy. The model in Heathcote and Perri (2002) is much closer to ours.

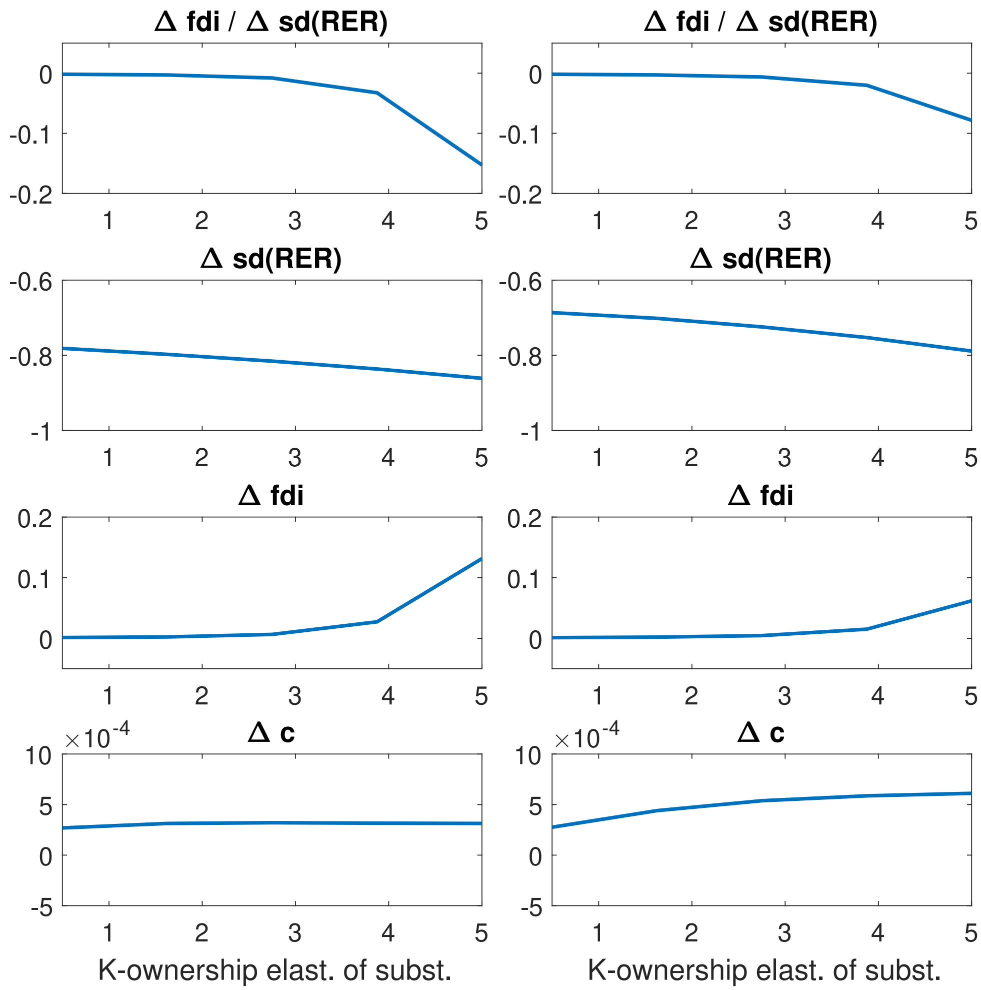

Figure 1: Higher synchronization - TFP shocks only

NOTES: Left column - $\theta_C = 0.9$; Right column - $\theta_C = 1.5$. Reduction in RER volatility is a result of the change in the correlation of TFP shocks - from 0 to 1.

product of capital owned domestically, and rising marginal product of capital owned abroad). This in turn reduces the incentive to change the allocation of capital, making FDI flows less responsive to changes in RER risk. Hence, ***the elasticity of FDI flows w.r.t. the volatility of the real exchange rate can help us identify the elasticity of substitution between the domestic and foreign ownership of capital stock***.

5 Conclusions

Empirically, a reduction in RER volatility is associated with larger bilateral FDI flows in a sample of the EU countries. Theoretically, a reduction in RER volatility is associated with larger bilateral FDI flows as long as aggregate consumption tends to be larger when the real exchange rate depreciates. Importantly, the elasticity of FDI flows w.r.t. RER volatility depends crucially on the elasticity of substitution between domestic and foreign ownership of capital stock. Hence, the elasticity of FDI flows w.r.t. RER volatility can be used to estimate the elasticity of substitution between domestic and foreign ownership of capital stock in the aggregate production function.

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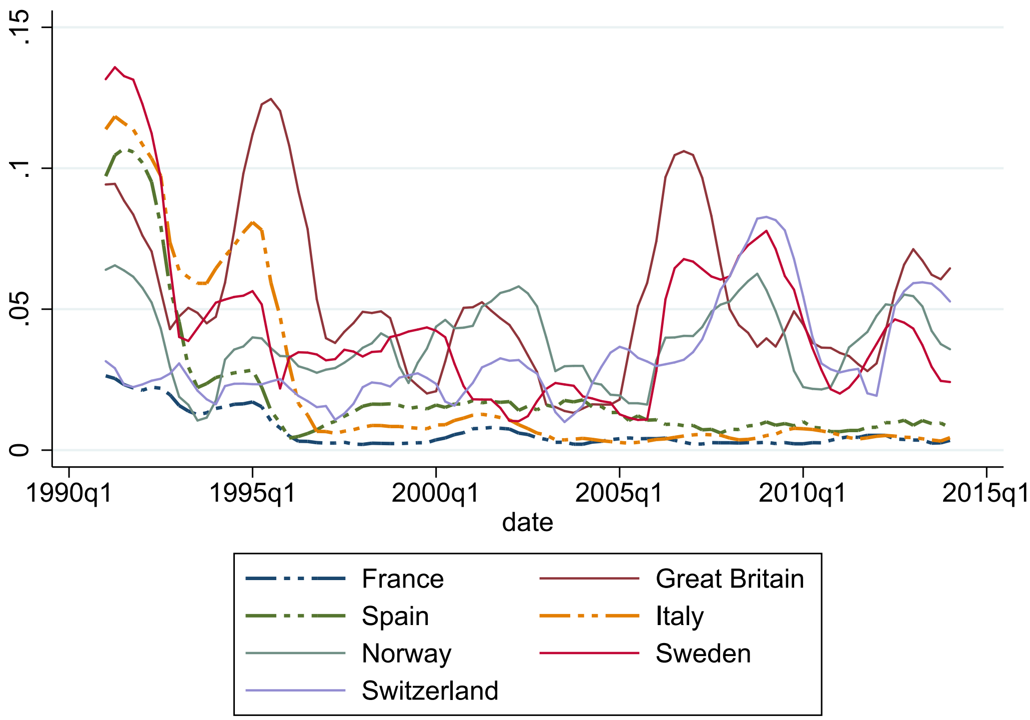

Figure 2: SD of German Bilateral Real Exchange Rates (12 quarter rolling average)

Table 1: Summary Statistics - Euro Adoption and Real Exchange Rate Risk

	Mean 20 quarter rolling average	N	Mean 12 quarter rolling average	N
Overall	0.063	46,843	0.049	50,563
Non-euro Pair	0.073	29,318	0.058	31,670
Euro Country Pairs	0.045	17,525	0.034	18,893
Pre-adoption	0.056	13,129	0.044	13,392
Post-adoption	0.013	4,396	0.010	5,501
Original 11	0.028	6,655	0.021	7,095
Pre-Adoption	0.040	3,740	0.033	3,740
Post-adoption	0.012	2,915	0.009	3,355
Later Euro Adopters	0.056	10,870	0.042	11,798
Pre-adoption	0.062	9,389	0.049	9,652
Post-adoption	0.016	1,481	0.011	2,146

Notes: Sample means are reported for both 12 quarter and 20 quarter forward rolling averages of the standard deviation of the real exchange rate for bilateral pairs. There are 31 countries in the sample (27 EU countries plus Iceland, Norway, Switzerland, and the UK) covering 38 years from 1980 to 2017, drawn from the IFS dataset. A Non-euro Pair is defined as any bilateral pair of countries where at least one country never joins the euro. A Euro Country Pair is all bilateral pairs where both countries eventually join the euro. The Original 11 group is defined as the 11 countries that adopted the euro currency in 1999. Later Euro Adopters are the 8 countries that adopt the euro at any point after 1999.

Table 2: Euro Adoption and Real Exchange Rate Risk

	(1)	(2)	(3)	(4)	(5)	(6)
Euro Adoption Pair	-0.0127	-0.0191	-0.0166	-0.0267	-0.0147	-0.0220
	(0.000501)***	(0.000591)***	(0.000575)***	(0.000734)***	(0.000532) ***	(0.000635) ***
Euro Country Pair	-0.0198	-0.0235	-0.0171	-0.0171	-0.0186	-0.0212
	(0.000568)***	(0.000658) ***	(0.000637) ***	(0.000793) ***	(0.000596) ***	(0.000709) ***
Observations	50563	46843	50563	46843	50563	46843
R^2	0.252	0.218	0.255	0.226	0.254	0.221
Quarter Fixed Effects	Y	Y	Y	Y	Y	Y
Window Length (Quarters)	12	20	12	20	12	20
Window Timing	Forward	Forward	Backward	Backward	Middle	Middle

Notes: The variable of interest is a difference-in-difference indicator that turns on when both countries in the bilateral pair have adopted the euro, and is zero otherwise. The outcome variable is alternative measures of real exchange rate risk defined as a rolling average standard deviation of the real exchange rate for each bilateral pair. Forward is calculated using the next 12 (or 20) quarters, while Backward is calculated using the last 12 (or 20) quarters. Middle uses the previous 6 (10) and next 6 (10) quarters to calculate the standard deviation. Euro Country Pair is a dummy equal to 1 for all observations if the country pair eventually joins the monetary union (0 otherwise). Euro Adoption Pair is an indicator equal to 1 for all observations for which the country pair has adopted the euro, and 0 otherwise. The full sample includes 31 countries and 38 years worth of quarterly data. Robust standard errors in parentheses. Significance: * 10 percent; ** 5 percent; *** 1 percent.

Table 3: Euro Adoption and Bilateral FDI Flows

ln (Gross FDI Flows)	(1)	(2)	(3)	(4)	(5)	(6)
Euro Adoption	1.701	0.754	0.680	0.247	0.251	0.200
	(0.093)***	(0.091)***	(0.096)***	(0.093)***	(0.094)***	(0.095)**
ln (Gross GDP)		1.134	1.173	1.690	1.627	1.647
		(0.0291)***	(0.0310)***	(0.127)***	(0.132)***	(0.128)***
ln (Gross Population)		-0.947	-1.012	-3.281	-3.748	-3.865
		(0.063)***	(0.068)***	(1.462)**	(1.543)**	(1.521)**
Observations	3248	3248	3248	3248	2936	3094
Year FE	No	No	Yes	Yes	Yes	Yes
Bilateral Pair FE	No	No	No	Yes	Yes	Yes
Sample	Full	Full	Full	Full	First Wave Adopters	Large Countries

Notes: The variable of interest is a difference-in-difference indicator that turns on when both countries in the bilateral pair have adopted the euro, and is zero otherwise. The outcome variable is the natural log of the sum of inward and outward FDI flows between bilateral pair in a given year. ln (Gross GDP) (Gross Population) is the natural log of the gross sum of GDP (population) in each of the bilateral pair countries in that year. The full sample includes 27 countries (26 EU countries (missing Croatia) plus the UK) and 31 years (1986-2016). The First Wave Adopters sub-sample includes all non-euro countries as well as those countries that adopted the euro in 1999 and 2001. The Large Countries sub-sample restricts the sample to the 16 largest countries in the sample (excludes: Bulgaria, Cyprus, Czech Republic, Estonia, Hungary, Latvia, Lithuania, Luxembourg, Malta, Slovakia, and Slovenia). Significance: * 10 percent; ** 5 percent; *** 1 percent.

Table 4: Real Exchange Risk and Bilateral FDI Flows

ln (Gross FDI Flows)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Real Exchange Rate SD 12 Quarter Rolling Average	-6.802 (1.130)***	-3.444 (1.037)***			-3.828 (1.202)***		-3.203 (1.036)***	
Real Exchange Rate SD 20 Quarter Rolling Average			-6.696 (1.228)***	-3.238 (1.195)***		-4.151 (1.449)***		-2.947 (1.195)***
ln (Gross GDP)		1.768 (0.231) ***		1.720 (0.233)***	1.720 (0.243)***	1.642 (0.244)***	1.717 (0.234)***	1.674 (0.237)***
ln (Gross Population)		-3.080 (2.345)		-2.845 (2.365)	-3.663 (2.493)	-3.033 (2.560)	-2.656 (2.425)	-2.441 (2.454)
Observations	3044	2612	2759	2612	2332	2332	2515	2515
Bilateral Pair FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Sample	Full	Full	Full	Full	First Wave Adopters	First Wave Adopters	Large Countries	Large Countries

Notes: The outcome variable is the natural log of the sum of inward and outward FDI flows between bilateral pair countries in a given year. ln (Gross GDP) (Gross Population) is the natural log of the gross sum of GDP (population) in each of the bilateral pair countries in that year. The First Wave Adopters sub-sample includes all non-euro countries as well as those countries that adopted the euro in 1999 and 2001. The Large Countries sub-sample restricts the sample to the 16 largest countries in the sample (excludes: Bulgaria, Cyprus, Czech Republic, Estonia, Hungary, Latvia, Lithuania, Luxembourg, Malta, Slovakia, and Slovenia). Significance: * 10 percent; ** 5 percent; *** 1 percent.

Table 5: Euro Adoption and Investment

Investment / GDP	(1)	(2)	(3)	(4)	(5)	(6)
Euro Adoption	-1.66	-0.91	-1.39	-1.98	-1.49	-1.77
	(0.292)***	(0.39)**	(0.32)***	(0.30)***	(0.42)***	(0.33)**
Price of Oil				-0.07	-0.06	-0.07
				(0.05)	(0.05)	(0.05)
Observations	3180	2528	2892	3060	2408	2772
Quarter FE	Yes	Yes	Yes	Yes	Yes	Yes
Country FE	Yes	Yes	Yes	Yes	Yes	Yes
Sample	All	First Wave Adopters	Exclude Late Adopters	All	First Wave Adopters	Exclude Late Adopters
Countries	31	24	28	31	24	28

Notes: The variable of interest is a difference-in-difference indicator that turns on when the euro is adopted, and is zero otherwise. The outcome variable is investment spending as a percentage of GDP. The full sample includes quarterly data for 31 countries from 1980 to 2018. The First Wave Adopters subsample includes all non-euro countries as well as those countries that adopted the euro in 1999 and 2001. Late Adopters (Latvia, Lithuania, and Estonia) are excluded in a subsample as post-euro observations are limited. Significance: * 10 percent; ** 5 percent; *** 1 percent.

Table 6: Euro Adoption and Consumption

Consumption / GDP	(1)	(2)	(3)	(4)	(5)	(6)
Euro Adoption	0.684	0.953	0.889	0.607	0.825	0.807
	(0.212)***	(0.292)***	(0.24)***	(0.22)***	(0.31)***	(0.25)***
Price of Oil				-0.08	-0.08	-0.08
				(0.03)**	(0.03)**	(0.03)**
Observations	3236	2584	2948	3076	2424	2788
Quarter FE	Yes	Yes	Yes	Yes	Yes	Yes
Country FE	Yes	Yes	Yes	Yes	Yes	Yes
Sample	All	First Wave Adopters	Exclude Late Adopters	All	First Wave Adopters	Exclude Late Adopters
Countries	31	24	28	31	24	28

Notes: The variable of interest is a difference-in-difference indicator that turns on when the euro is adopted, and is zero otherwise. The outcome variable is consumption spending as a percentage of GDP. The full sample includes quarterly data for 31 countries from 1980 to 2018. The First Wave Adopters sub-sample includes all non-euro countries as well as those countries that adopted the euro in 1999 and 2001. Late Adopters (Latvia, Lithuania, and Estonia) are excluded in a sub-sample as post-euro observations are limited. Significance: * 10 percent; ** 5 percent; *** 1 percent.