

Pareto-improving Pricing: Why 3 is better than 2

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Motivation

Waiting (due to congestion) is common in resource allocation:

- Internet traffic;
- vehicular traffic;
- security lines at airports;
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Equity-efficiency trade-off associated with **pricing priority**:

- Gains from allocating priority to the most time-pressed agents...
- ... but drawback of making things worse for poorer agents.

Motivation

The Lexus Lanes – and why they won't work

Instead of rewarding carpools and getting people out of private cars, we are rewarding wealth and encouraging more people to drive. How does this make sense?

By

Zelda Bronstein

October 19, 2017



Share



WEALTH CONCENTRATION

HOT lanes make my blood boil: Fast tracks for the rich on the rise

ARTICLES

OCTOBER 31, 2012

by Salvatore Babones

Motivation

MONEY

Why must we pay to have a slightly less miserable time at the airport?

TSA PreCheck, Clear, and how the airport splits travelers into haves and have-nots.

by **Emily Stewart**

May 10, 2022, 7:00 AM CDT



Travelers wait in a long queue at the security checkpoint of Orlando International Airport the day before the Thanksgiving holiday on November 24, 2021. Paul Hennessy/SOPA Images/LightRocket via Getty Images



Emily Stewart covered business and economics for Vox and wrote the newsletter *The Big Squeeze*, examining the ways ordinary people are being squeezed under capitalism. Before joining Vox, she worked for *TheStreet*.

Motivation

 METROPOLIS

Important People Are Noticing How Terrible CLEAR Is for Airports

A profit-seeking entity has no place in a federally mandated process. Nothing hazy about that.

BY DAVID ZIPPER

DEC 05, 2023 • 10:00 AM



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#2. When the trade-off between dispersion and mean is not too severe, **Pareto improvement with 3** priority tiers exists.

(Seed of a) literature review

- Possibility of Pareto improvements in priority service:
Chao and Wilson (1987), Gershkov and Schweinzer (2010), Hall (2018),
Gershkov and Winter (2022),...
- Market-design approach to congestion pricing:
Ostrovsky and Schwarz (2018), Ostrovsky and Yang (2024),...
- Inequality-aware Market Design:
Weitzman (1977), Condorelli (2013), Dworczak ^① Kominers ^① Akbarpour
(2021), Kang (2023, 2024), Kang and Watt (2025),...
- Recent empirical papers on congestion:
Hanna, Kreindler, Olken (2017), Kreindler (2024), Cook and Li (2024),...

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- Types r are drawn from a continuous distribution with cdf G and strictly positive pdf g on $[\underline{r}, \bar{r}]$.
- Think of t as being measured in utils; we can accommodate the case when $t = c(\tau)$, where c is an increasing function and τ is physical wait time.

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- Constant $\bar{U}$ pinned down by budget balance.

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- Then, the set of feasible distributions is

$$\mathcal{F} = \left\{ F \in \Delta(\mathbb{R}_+) : F \in \text{MPC}(F_m) \text{ for some } m \in \Delta^{N-1} \right\}.$$

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Unpriced benchmark (laissez-faire allocation):

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- The unique mechanism corresponding to F_m charges everyone a price of zero and assigns wait time $w(1/N)$ to everyone.

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- Relative price pinned down by: $-\frac{4}{5}w\left(\frac{2}{5}\right) - p_S = -\frac{4}{5}w\left(\frac{1}{5}\right) - p_F$.
- Budget balance: $\frac{1}{5}p_F + \frac{4}{5}p_S = 0$.

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↪ Giving everyone the same wait time minimizes average wait time.

↪ (This follows from the previous axiom if $\mathcal{F}$ is compact.)

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Assumption (Smoothness)

For every $\epsilon > 0$ small, there exist $F \in \mathcal{F}$ and $q \in (0, 1)$ such that

$$F^{-1}(q + \sqrt{\epsilon}) < t_0 \text{ and } \frac{\mathbb{E}_F[t] - t_0}{q(t_0 - \mathbb{E}_F[t | F(t) \leq q])} < \epsilon.$$

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$\rightsquigarrow$ The trade-off between dispersion and average wait time is not too severe (true in the lane-pricing example if $w(\cdot)$ is differentiable).

Assumptions on $\mathcal{F}$

We will sometimes refer to the opposite case:

Assumption (No-friction)

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$\rightsquigarrow$ Note that "No-friction" implies "Smoothness."

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We say that $F \in \mathcal{F}$ offers n (priority) tiers if $|\text{supp}(F)| = n$.

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Theorem

Assume Randomization, Friction, and Smoothness.

There exists a (strict) Pareto improvement over the unpriced benchmark that uses 3 priority tiers.

It is not possible to (weakly) Pareto improve over the unpriced benchmark with 2 priority tiers.

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Discussion:

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- In applications, we often see priority systems with 2 priority tiers, e.g., fast lane+slow lane, single fast-track option (e.g., Clear).
- The theorem implies that these priority systems must make some agents worse off.
- “Friction” results in a tradeoff between average wait time and dispersion.
- Intuitively, 2 priority tiers is insufficient to resolve this tradeoff while improving for all agents.

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- In the lane-pricing application, there exists a deterministic Pareto-improving mechanism if there are three or more lanes.
- If there are only two lanes, no deterministic mechanism can be a Pareto improvement.
- However, even with two lanes, Pareto improvement is possible with a stochastic mechanism (that offers three options).

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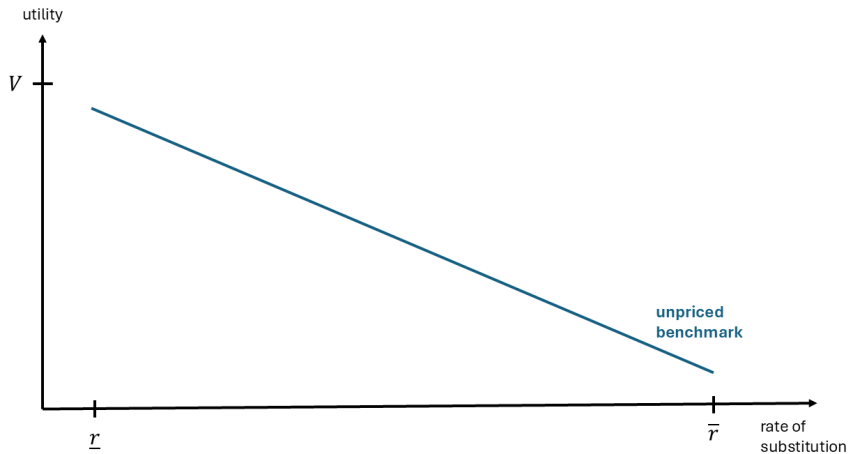
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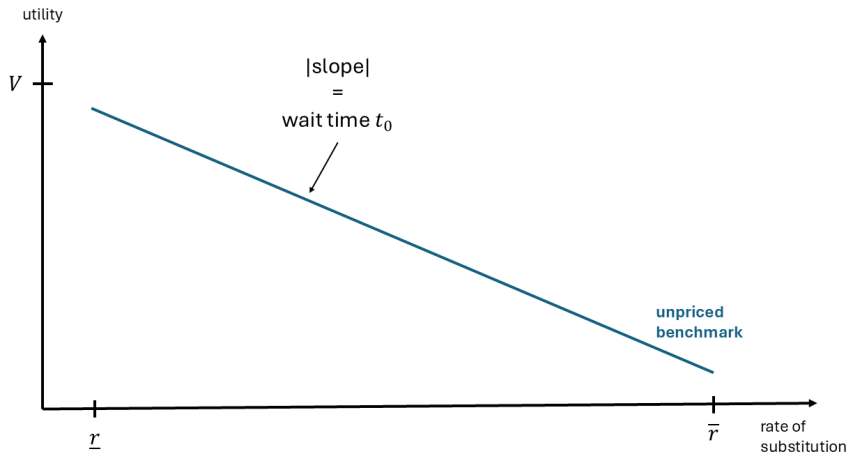
Proof

Proof of main result

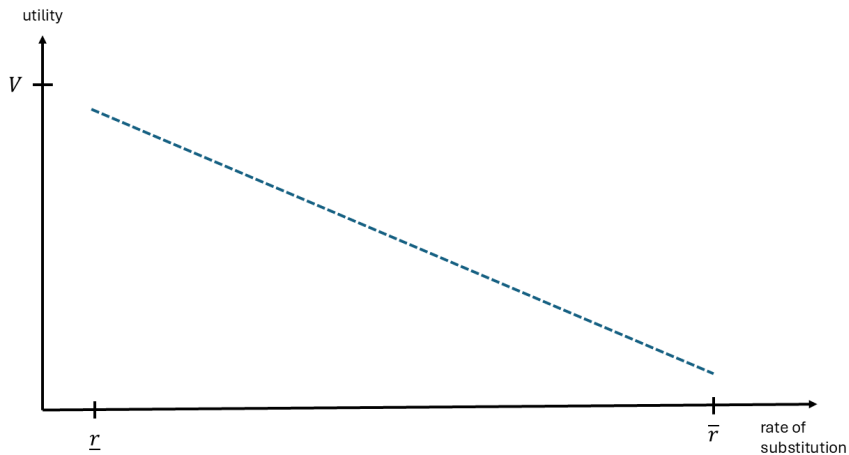
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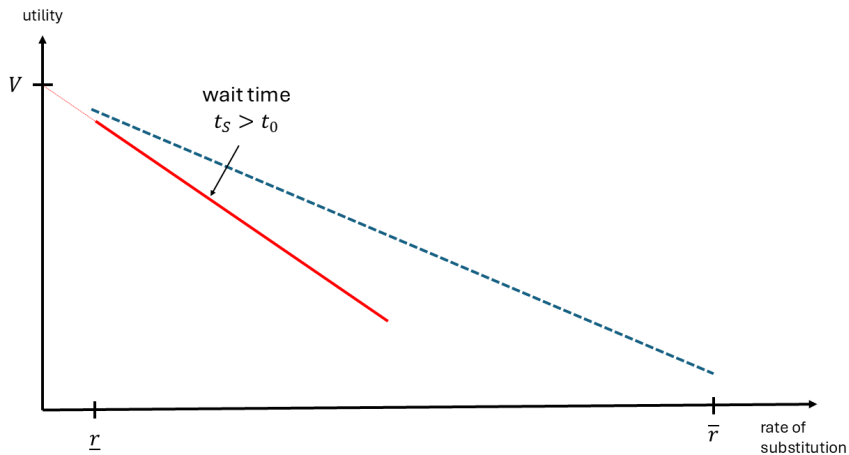
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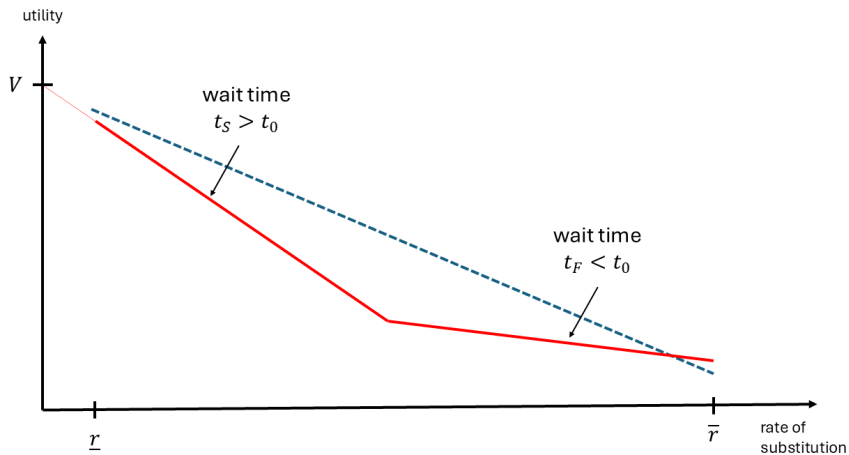
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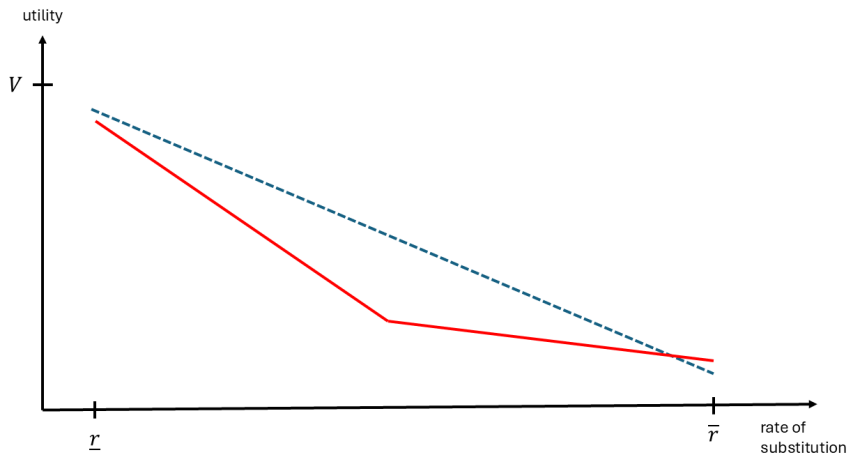
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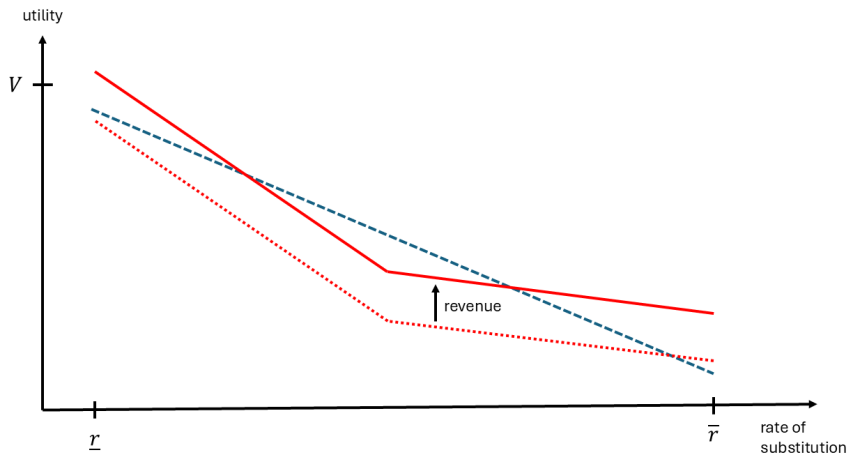
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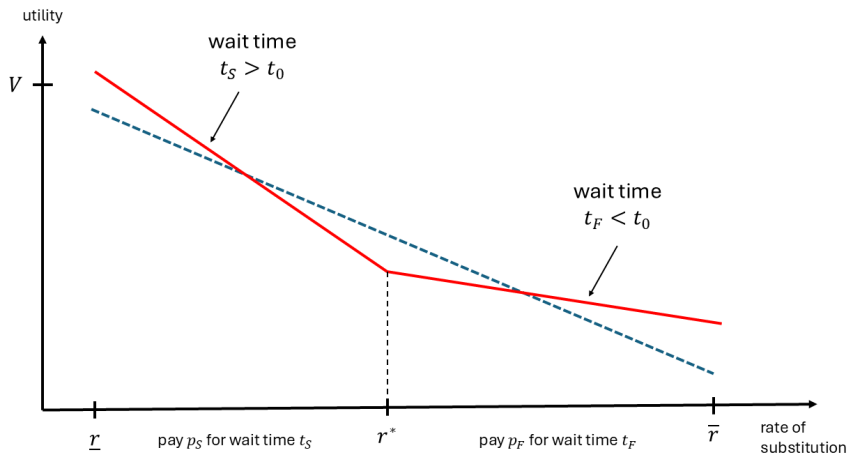
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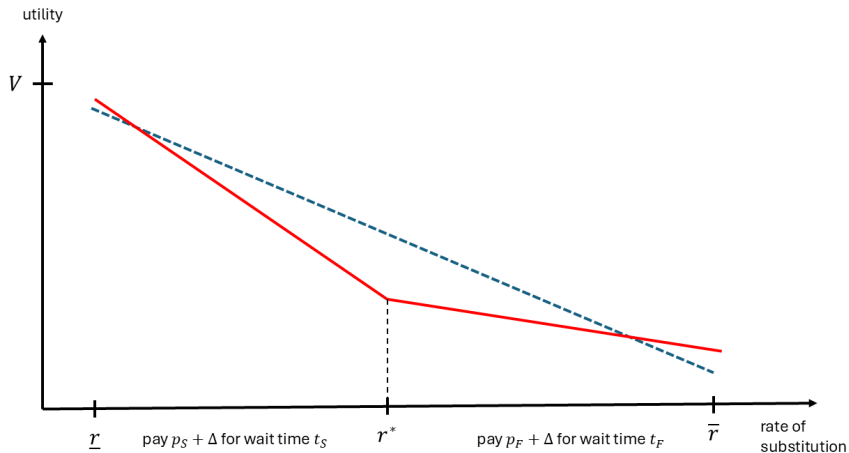
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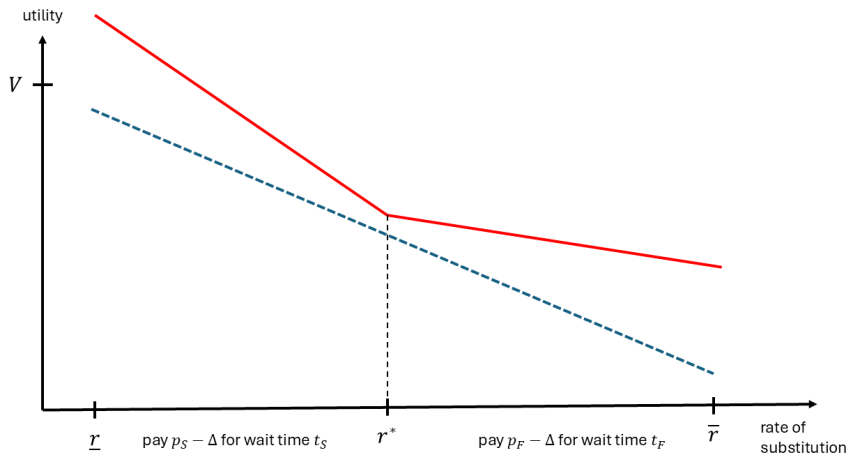
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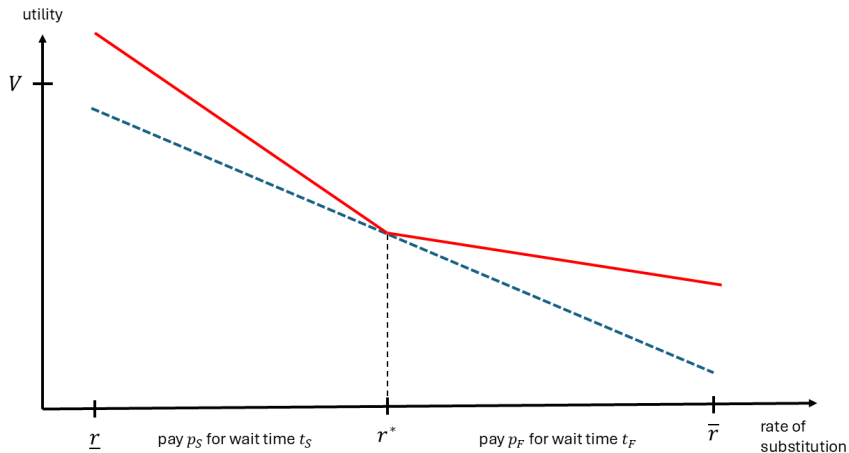
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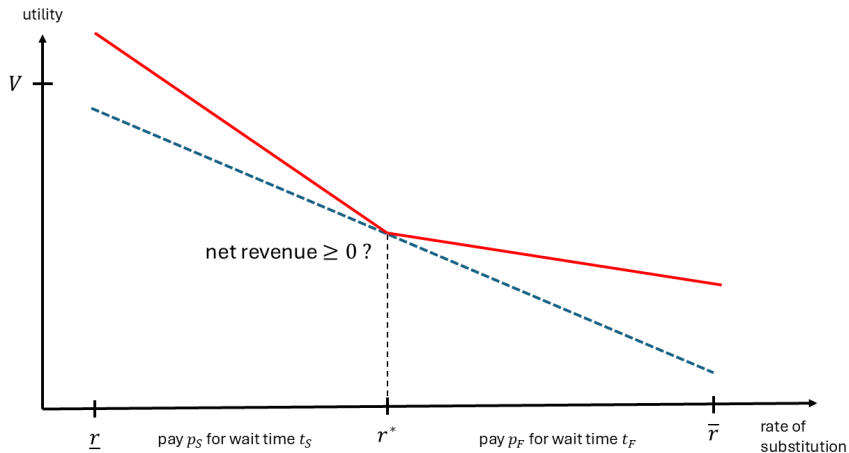
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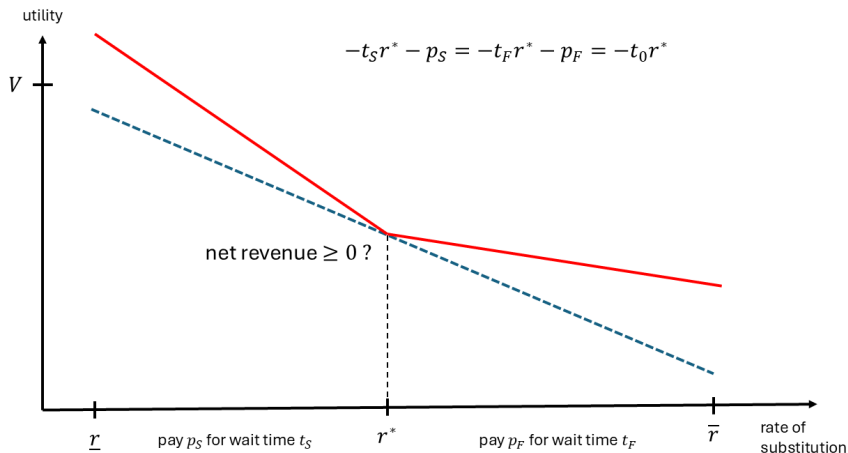
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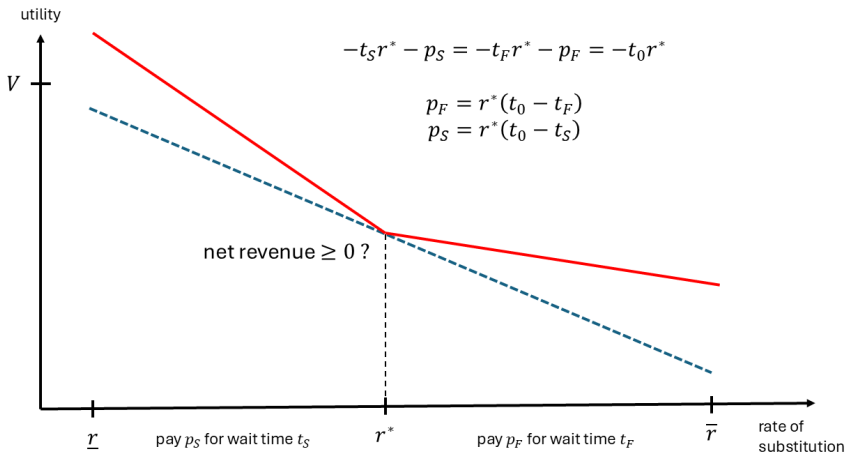
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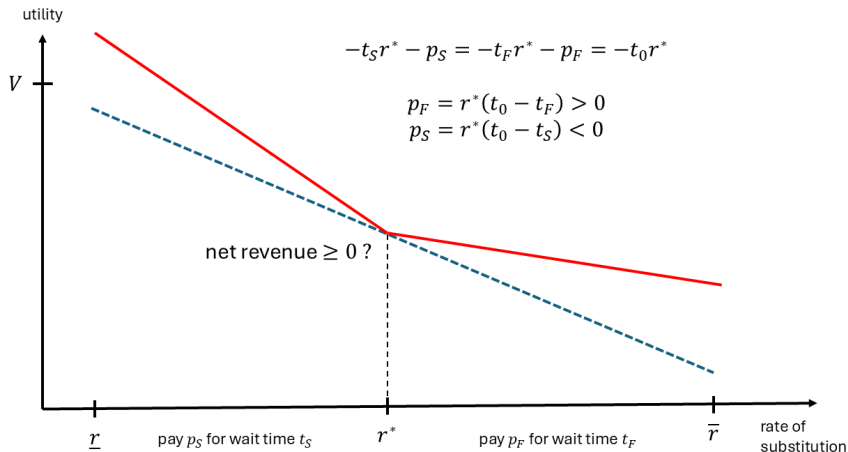
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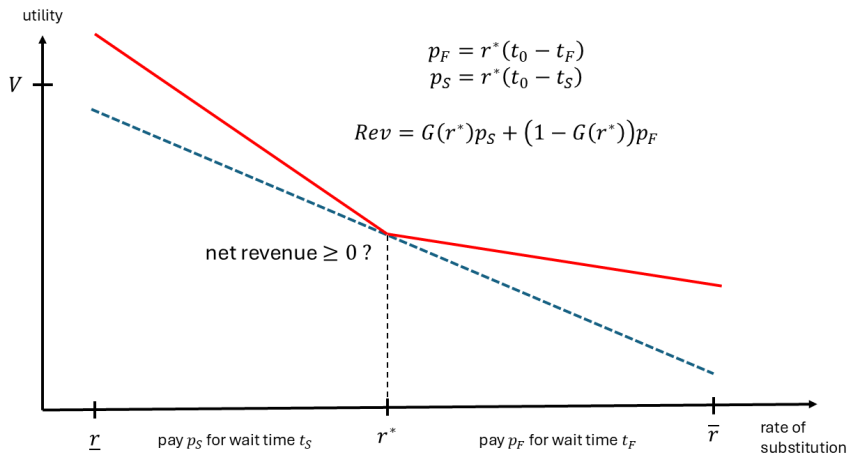
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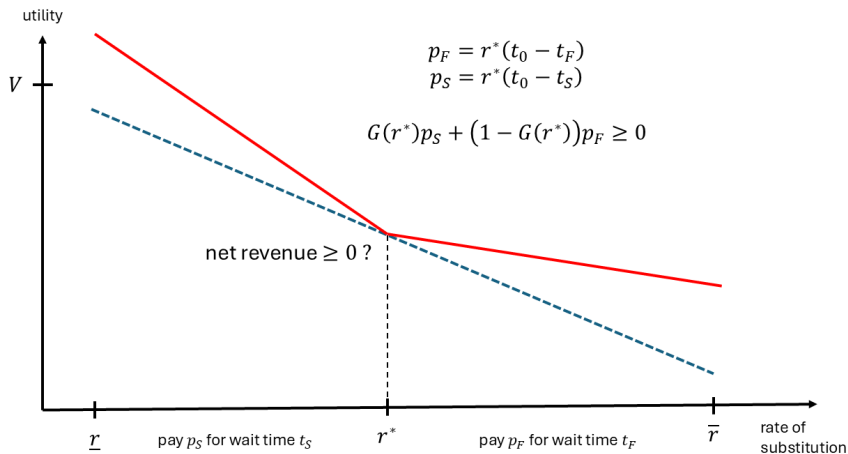
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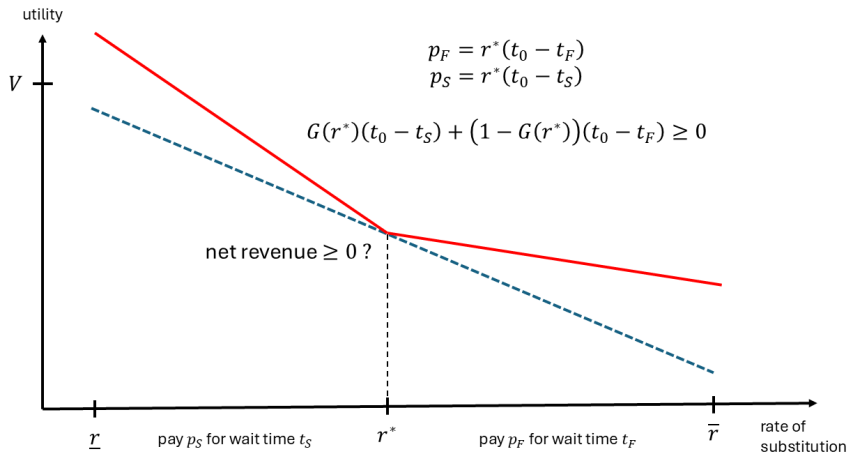
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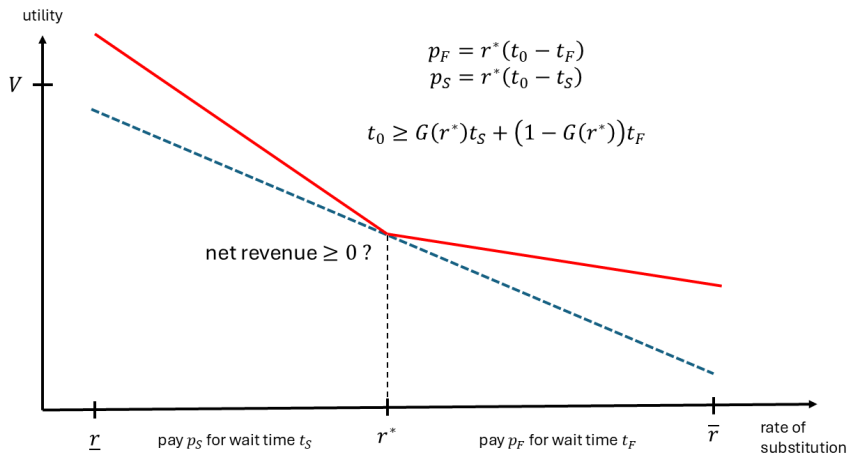
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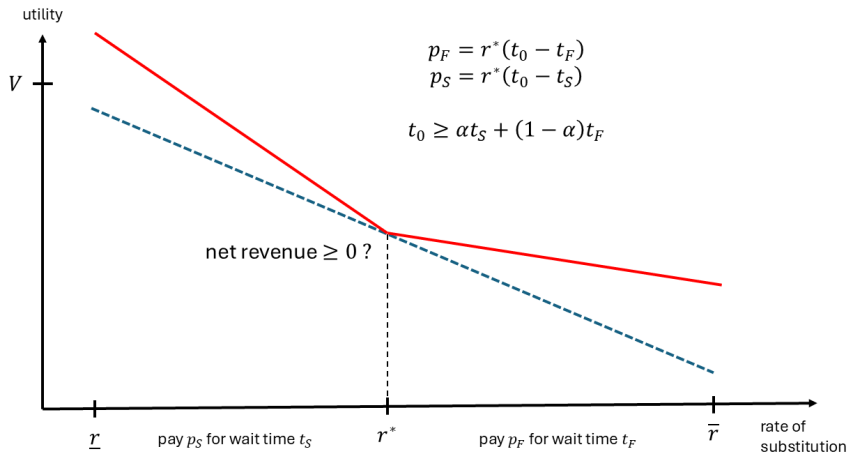
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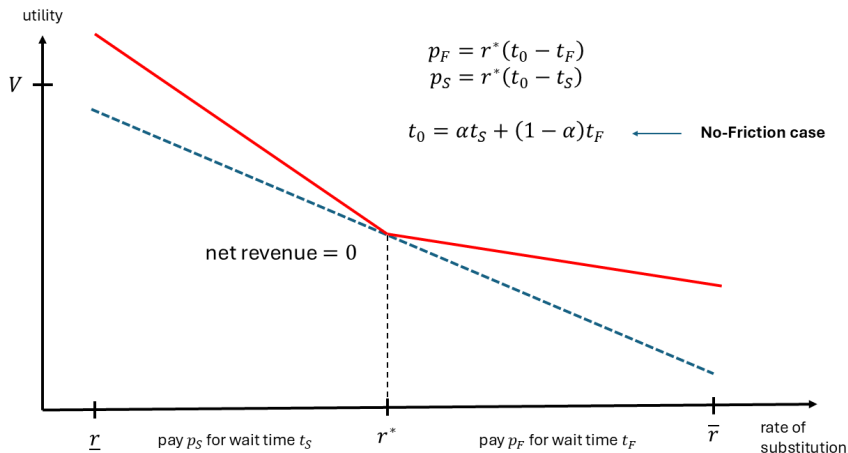
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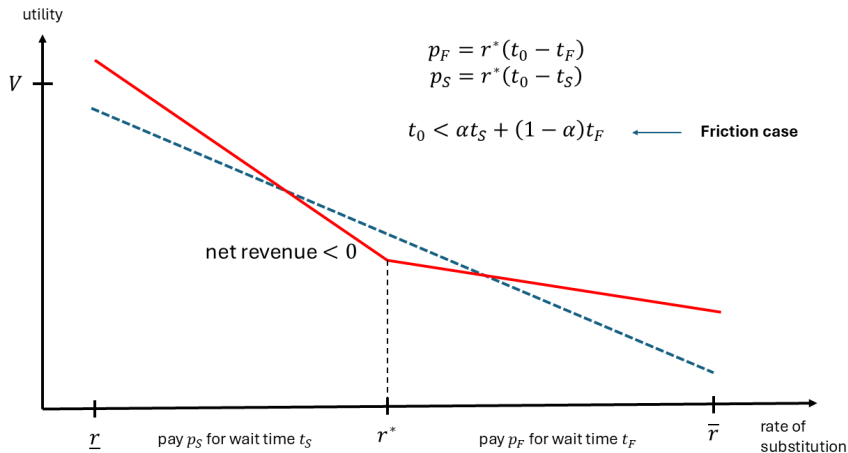
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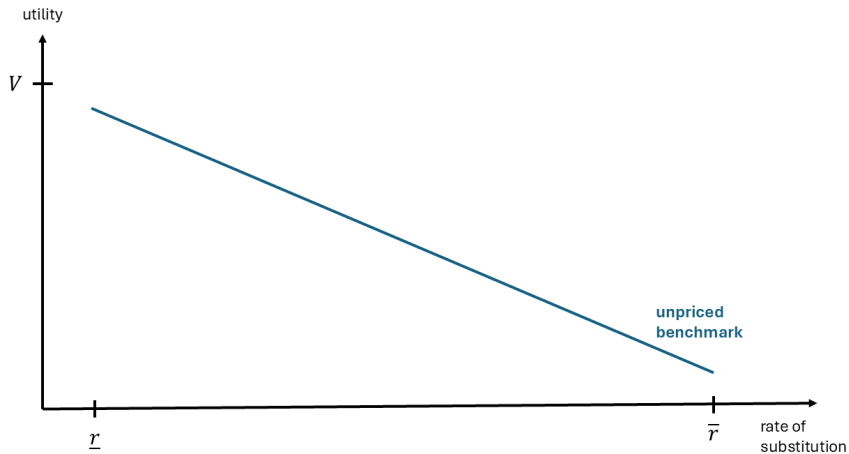
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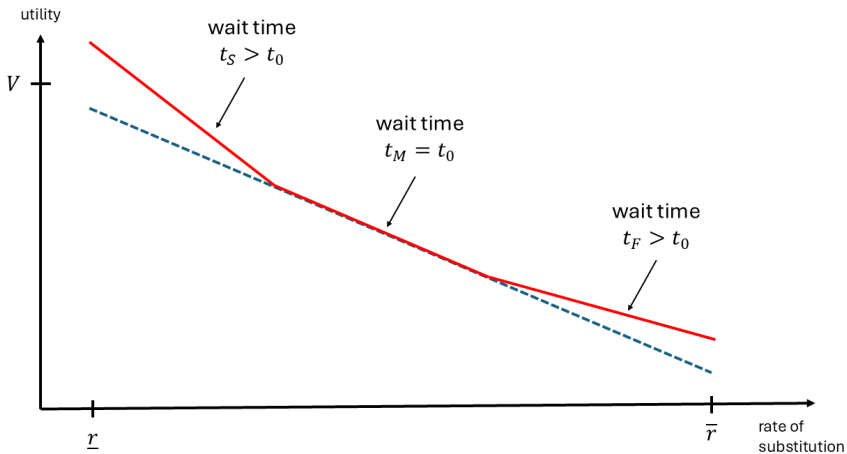
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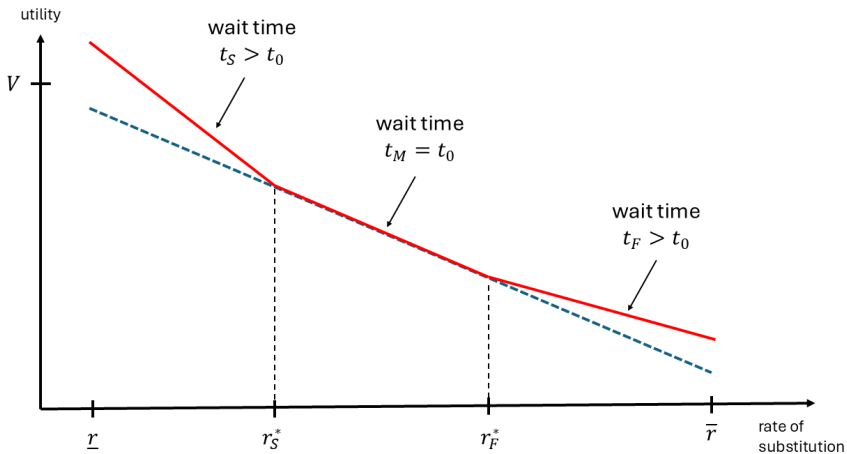
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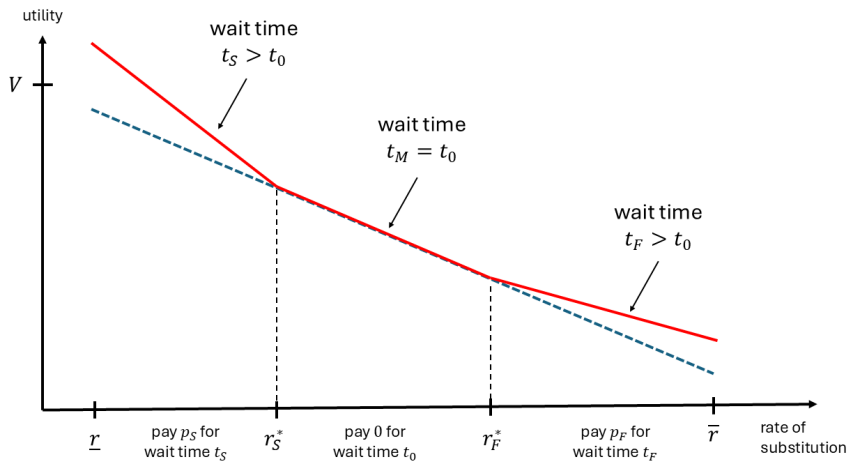
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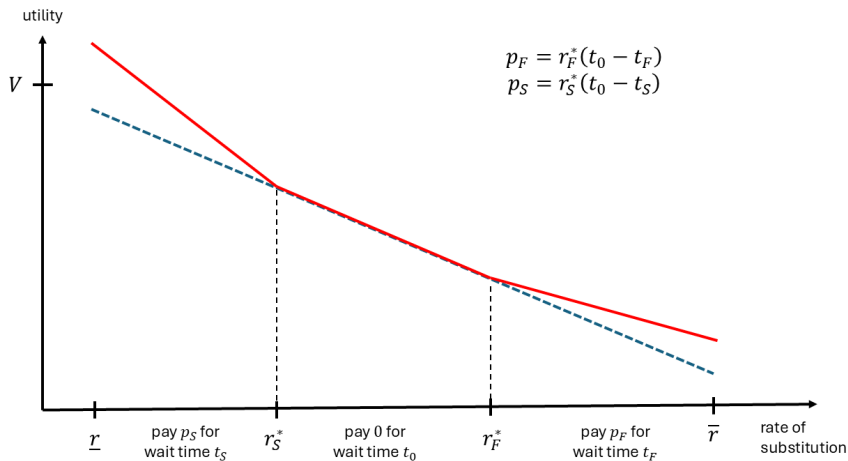
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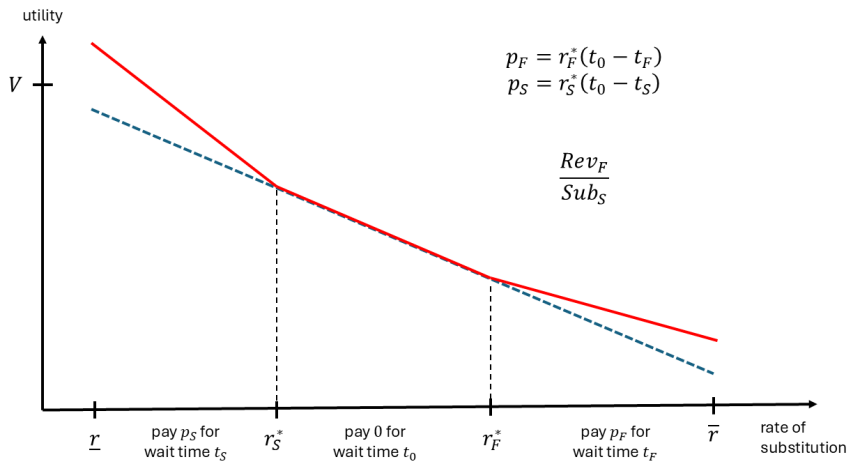
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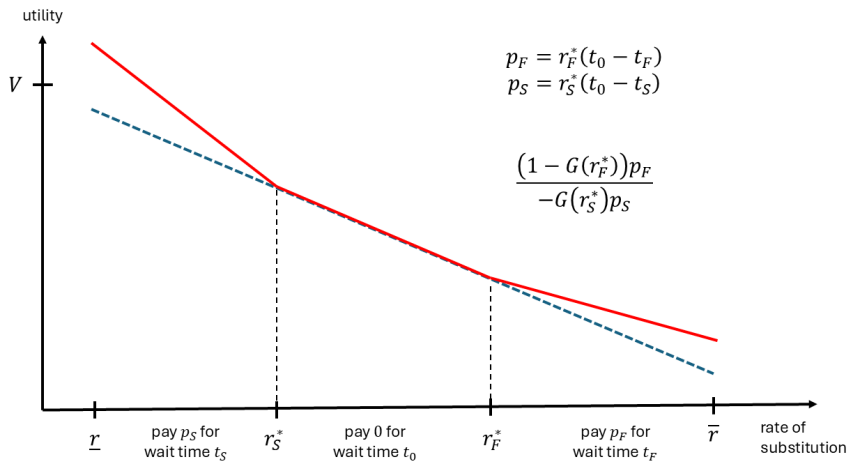
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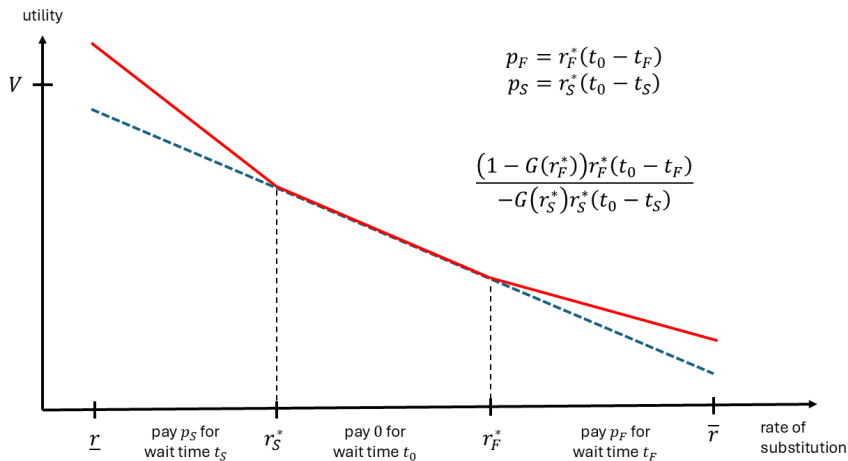
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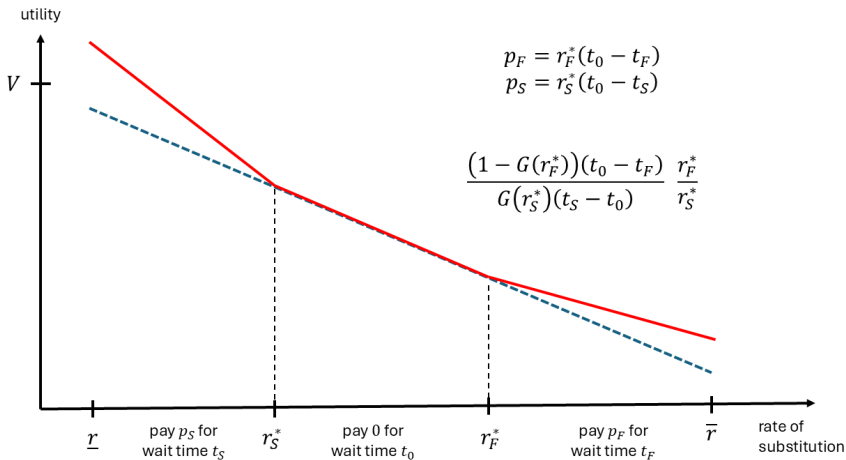
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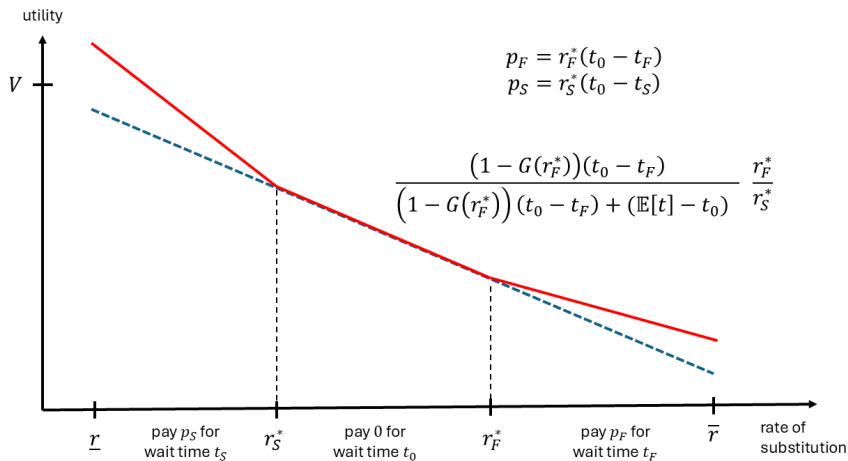
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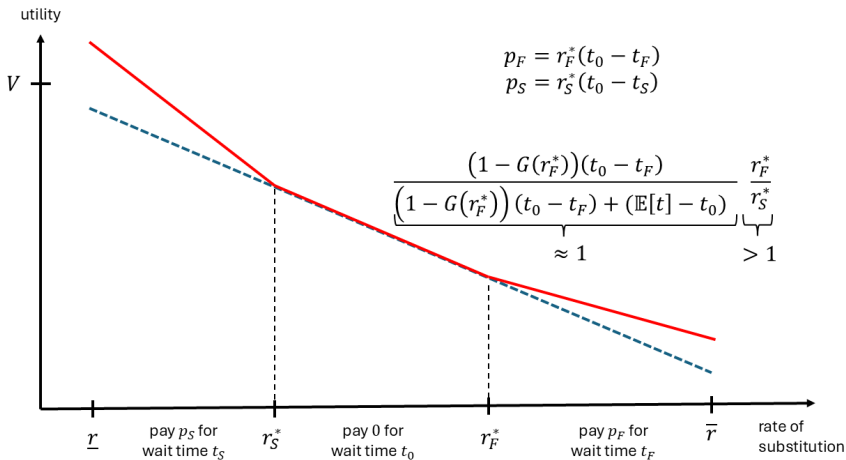
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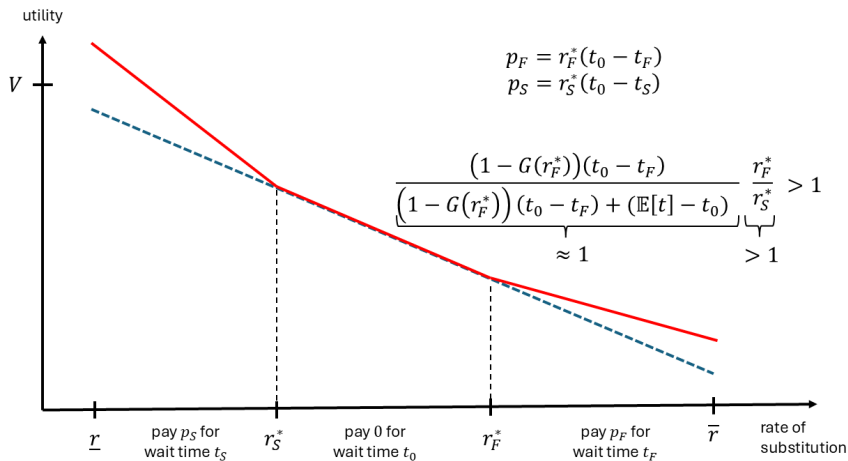
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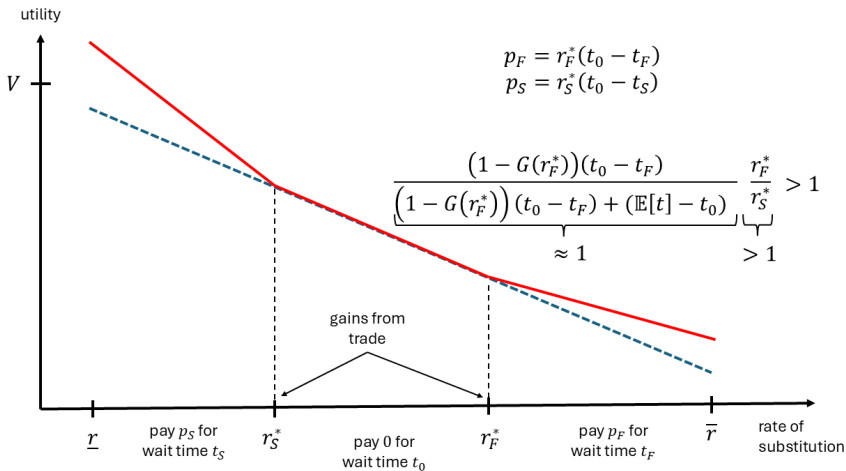


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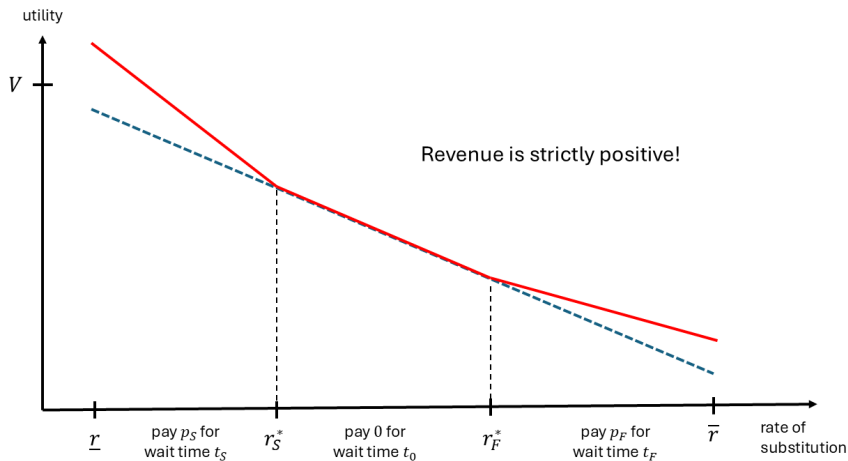


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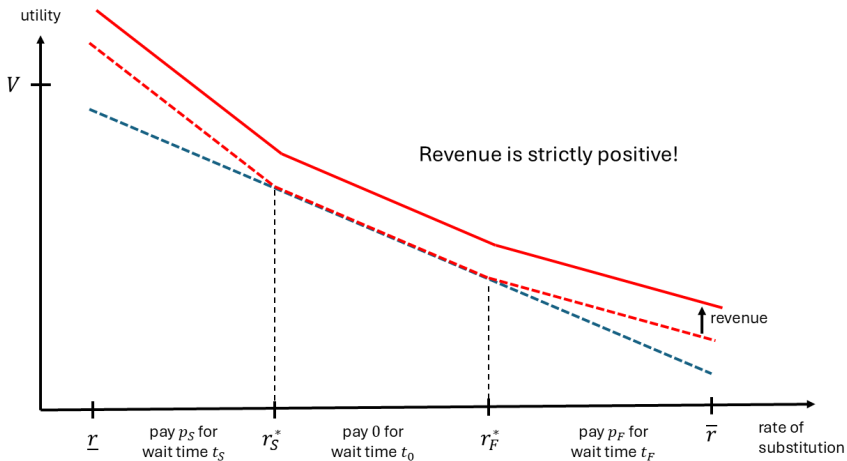




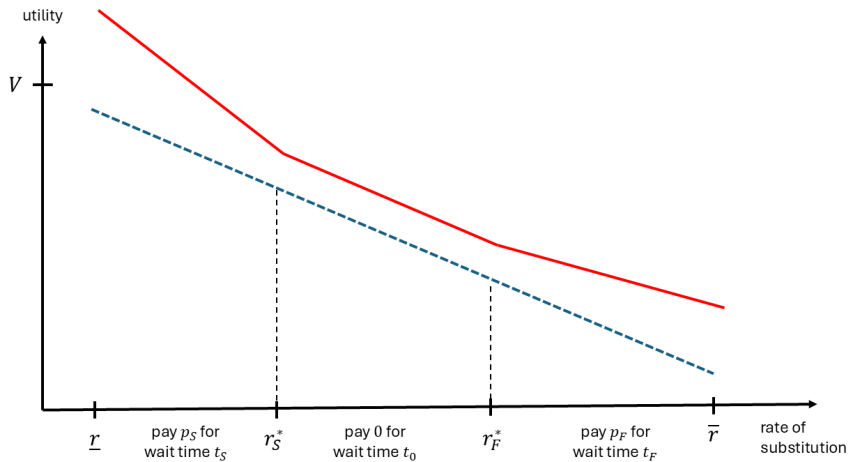
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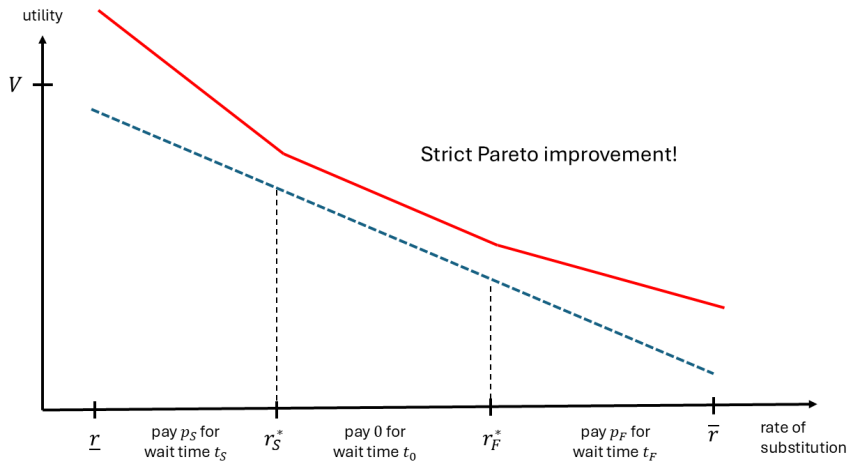
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$$3 > 2$$