

Optimal Redistribution via Income Taxation and Market Design

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Inequality-aware Market Design

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- **Housing** (rent control; public housing; LIHTC in the US)
- **Health care** (public health care in Europe, Medicare and Medicaid in the US)
- **Food** (food stamps in the US; in-kind food transfer programs (rice, wheat,...) in developing countries)
- **Energy** (electricity in virtually all European countries at the moment; kerosene in India; energy subsidies in Latin America;...)
- **Covid-19 vaccines**
- **Road access**
- ...

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Desideratum: Understand optimal design of such policies.

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II Welfare Theorem implies that markets are a superior allocation mechanism even under redistributive preferences.

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How to design *individual* markets in the presence of socioeconomic *inequality* and *private information*?

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- **Inequality-aware Market Design (IMD)**:
How to design *individual* markets in the presence of socioeconomic *inequality* and *private information*?
- Market **distortions** (price control, rationing, taxes/ subsidies) may be part of the **optimal** mechanism.

Example

Example

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Based on ideas in: Dworczak [Ⓡ] Kominers [Ⓡ] Akbarpour (2021)

Shout-out to: **Weitzman** (1977), **Spence** (1977), **Nichols and Zeckhauser** (1982), **Condorelli** (2013)

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$$\log(w) + u \cdot x,$$

where x is the probability of having a house.

Example

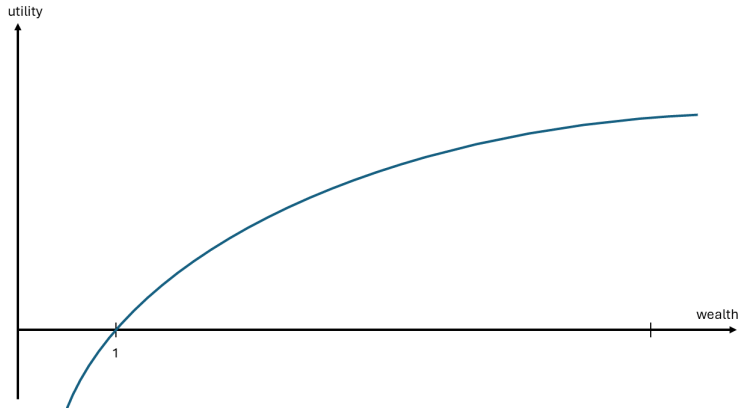
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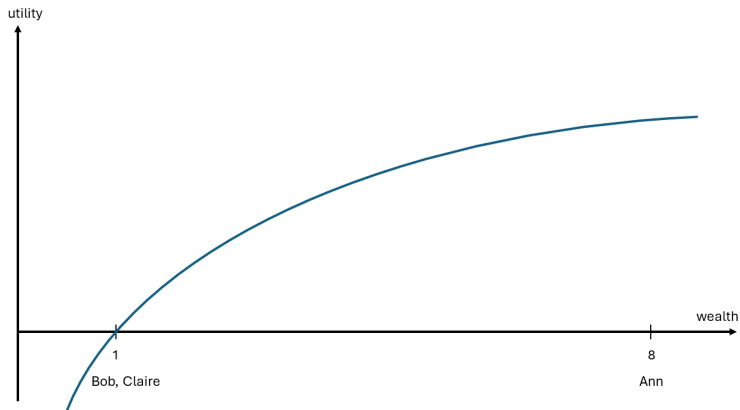
where x is the probability of having a house.

- Social planner aims to maximize the sum of utilities.

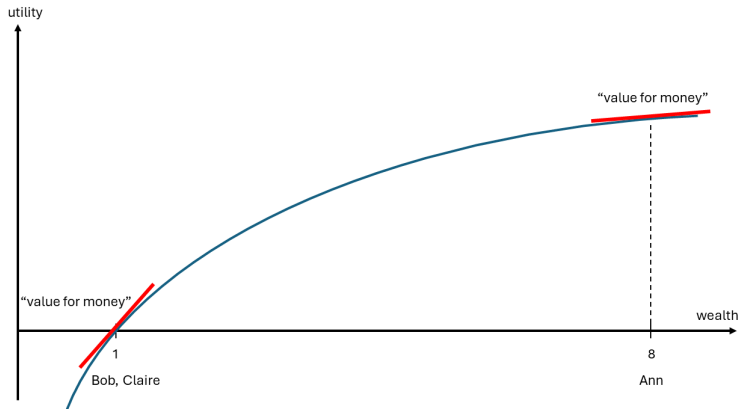
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	utility for good	value for money	
Ann	6	$1/8$	
Bob	18	1	
Claire	24	1	

Example

	utility for good	value for money	willingness to pay
Ann	6	$1/8$	48
Bob	18	1	18
Claire	24	1	24

Example

	utility for good	value for money	willingness to pay
Ann	6	1/8	48
Bob	18	1	18
Claire	24	1	24

Equivalent representation of the problem:

	util. good	val. money	willing. pay	welfare weight
Ann	48	1	48	1/8
Bob	18	1	18	1
Claire	24	1	24	1

Example

	val. good	val. money	WTP	utility
Ann	6	1/8	48	
Bob	18	1	18	
Claire	24	1	24	

Mechanism #1: **Market equilibrium** (+ lump-sum transfers)

Example

	val. good	val. money	WTP	utility
Ann	6	1/8	48	
Bob	18	1	18	
Claire	24	1	24	

Mechanism #1: **Market equilibrium** (+ lump-sum transfers)

Market-clearing price: 24

House owners: Ann & Claire

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	val. good	val. money	WTP	utility
Ann	6	1/8	48	
Bob	18	1	18	
Claire	24	1	24	

Mechanism #1: **Market equilibrium** (+ lump-sum transfers)

Market-clearing price: 24

House owners: Ann & Claire

Lump-sum transfer: 16

Example

	val. good	val. money	WTP	utility
Ann	6	1/8	48	$6 - 1/8 \cdot (24 - 16)$
Bob	18	1	18	
Claire	24	1	24	

Mechanism #1: **Market equilibrium** (+ lump-sum transfers)

Market-clearing price: 24

House owners: Ann & Claire

Lump-sum transfer: 16

Example

	val. good	val. money	WTP	utility
Ann	6	1/8	48	5
Bob	18	1	18	$0 + 1 \cdot 16$
Claire	24	1	24	

Mechanism #1: **Market equilibrium** (+ lump-sum transfers)

Market-clearing price: 24

House owners: Ann & Claire

Lump-sum transfer: 16

Example

	val. good	val. money	WTP	utility
Ann	6	1/8	48	5
Bob	18	1	18	16
Claire	24	1	24	$24 - 1 \cdot (24 - 16)$

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	val. good	val. money	WTP	utility
Ann	6	1/8	48	5
Bob	18	1	18	16
Claire	24	1	24	16

Mechanism #1: **Market equilibrium** (+ lump-sum transfers)

Market-clearing price: 24

House owners: Ann & Claire

Lump-sum transfer: 16

Total utility: 37

Example

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Ann	6	1/8	48	
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	val. good	val. money	WTP	utility
Ann	6	1/8	48	
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Mechanism #2: **Random allocation**

Example

	val. good	val. money	WTP	utility
Ann	6	1/8	48	
Bob	18	1	18	
Claire	24	1	24	

Mechanism #2: **Random allocation**

Price: 0

House owners: $\frac{2}{3}$ Ann & $\frac{2}{3}$ Bob & $\frac{2}{3}$ Claire

Example

	val. good	val. money	WTP	utility
Ann	6	1/8	48	
Bob	18	1	18	
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Mechanism #2: **Random allocation**

Price: 0

House owners: $\frac{2}{3}$ Ann & $\frac{2}{3}$ Bob & $\frac{2}{3}$ Claire

Lump-sum transfer: 0

Example

	val. good	val. money	WTP	utility
Ann	6	1/8	48	$2/3 \cdot 6$
Bob	18	1	18	
Claire	24	1	24	

Mechanism #2: **Random allocation**

Price: 0

House owners: $\frac{2}{3}$ Ann & $\frac{2}{3}$ Bob & $\frac{2}{3}$ Claire

Lump-sum transfer: 0

Example

	val. good	val. money	WTP	utility
Ann	6	1/8	48	4
Bob	18	1	18	$2/3 \cdot 18$
Claire	24	1	24	

Mechanism #2: **Random allocation**

Price: 0

House owners: $\frac{2}{3}$ Ann & $\frac{2}{3}$ Bob & $\frac{2}{3}$ Claire

Lump-sum transfer: 0

Example

	val. good	val. money	WTP	utility
Ann	6	1/8	48	4
Bob	18	1	18	12
Claire	24	1	24	$2/3 \cdot 24$

Mechanism #2: **Random allocation**

Price: 0

House owners: $\frac{2}{3}$ Ann & $\frac{2}{3}$ Bob & $\frac{2}{3}$ Claire

Lump-sum transfer: 0

Example

	val. good	val. money	WTP	utility
Ann	6	1/8	48	4
Bob	18	1	18	12
Claire	24	1	24	16

Mechanism #2: **Random allocation**

Price: 0

House owners: $\frac{2}{3}$ Ann & $\frac{2}{3}$ Bob & $\frac{2}{3}$ Claire

Lump-sum transfer: 0

Total utility: 32 (< 37)

Example

	val. good	val. money	WTP	utility
Ann	6	1/8	48	4
Bob	18	1	18	12
Claire	24	1	24	16

Mechanism #2: **Random allocation**

Price: 0

House owners: $\frac{2}{3}$ Ann & $\frac{2}{3}$ Bob & $\frac{2}{3}$ Claire

Lump-sum transfer: 0

Among all mechanisms charging a single price p , $p = 24$ is optimal.

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Mechanism #3: Allocate 1st unit by **lottery**, sell 2nd unit in **market**

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	val. good	val. money	WTP	utility
Ann	6	1/8	48	
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Market-clearing price for 2nd unit: 24

(Ann's indifference: $6 - 1/8 \cdot 24 = 1/2 \cdot 6$)

House owners: Ann & 50% Claire & 50% Bob

Example

	val. good	val. money	WTP	utility
Ann	6	1/8	48	
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House owners: Ann & 50% Claire & 50% Bob

Lump-sum transfer: 8

Example

	val. good	val. money	WTP	utility
Ann	6	1/8	48	$6 - 1/8 \cdot (24 - 8)$
Bob	18	1	18	
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Example

	val. good	val. money	WTP	utility
Ann	6	1/8	48	4
Bob	18	1	18	$1/2 \cdot 18 + 1 \cdot 8$
Claire	24	1	24	

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Example

	val. good	val. money	WTP	utility
Ann	6	1/8	48	4
Bob	18	1	18	17
Claire	24	1	24	20

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Total utility: 41 (> 37)

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This is in fact the **optimal mechanism!** (subject to IC constraints)

Example

	val. money	WTP	utility—market	utility—hybrid
Ann	1/8	48	5	4
Bob	1	18	16	17
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Example

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Intuition:

- Values for money (welfare weights) are **negatively correlated** with willingness to pay.

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- Values for money (welfare weights) are **negatively correlated** with willingness to pay.
- Market **behavior** can be **used to screen**: Choosing the lottery reveals a high value for money.

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Intuition:

- Values for money (welfare weights) are **negatively correlated** with willingness to pay.
- Market **behavior** can be **used to screen**: Choosing the lottery reveals a high value for money.
- Designer can increase the utility of agents with high values for money by **reducing the price of the rationed option**.

Inequality-aware Market Design (IMD)

- Redistribution in **simple (one- and two-sided) markets**:
DKA (2021), Reuter and Groh (2020), ADK (2024), Tokarski KAD (2023) (energy), ...
- Problems with **allocative externalities**:
Kang (2023), A Budish DK (2024) (Covid-19 vaccines), Pai and Strack (2025), ...
- Optimality of **costly screening** and allocating bads:
Kang and Zheng (2020), Yang DA (2024), D (2025), ...
- Interaction of redistributive mechanisms and **private markets**:
Kang (2023), Kang and Watt (2024a, 2024b), ...
- **Information-design** interventions:
Doval and Smolin (2024), Augias [Ⓘ] Ghersengorin [Ⓘ] Barreto (2025), ...

IMD and Income Taxation

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Redistribution should be accomplished entirely via income taxation.

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- Market-design perspective: **Taste heterogeneity is crucial.**

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 $\Leftarrow$ It is known that **AS theorem fails in this case!**

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 - Attempt to build a bridge to the public finance literature.

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- **This paper:**
 - Attempt to build a bridge to the public finance literature.
 - Use modern tools from mechanism design to study the **interaction between IMD and income taxation.**

IMD and Income Taxation

Result #1: Under separability and multidimensional heterogeneity, if

1. taste type does not affect the marginal utility of income;
2. welfare weights depend only on the ability type;
3. taste type and ability type are statistically independent,

then **redistribution** should be effected **entirely via income taxes**.

IMD and Income Taxation

Result #1: Under separability and multidimensional heterogeneity, if Assumptions 1, 2, 3 hold, then **redistribution** should be effected **entirely via income taxes**.

Result #2: If any of the assumptions 1, 2, or 3 are relaxed, then **redistributive market design also plays a role**.

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Result #2: If any of the assumptions 1, 2, or 3 are relaxed, then **redistributive market design also plays a role**.

1. We provide **closed-form solutions** in a linear model with two goods, binary ability type, and continuous taste type.
2. MD approach yields new intuitions about the optimal combination of market-level interventions and income taxation and uncovers a strong role of heterogeneity in tastes.

Literature Review

- **Suboptimality of goods market distortions:**

Diamond and Mirrlees (1971), Atkinson and Stiglitz (1976), Gauthier and Laroque (2009), Doligalski et al. (2025), ...

- **Taste heterogeneity and failure of Atkinson-Stiglitz theorem:**

Cremer and Gavhari (1995, 1998, 2002), Saez (2002), Kaplow (2008), Golosov et al. (2013), Gauthier and Henriët (2018), Allcott et al. (2019), Scheuer and Slemrod (2020), Schulz (2021), Hellwig and Werquin (2023), Ferey et al. (2024), Gerritsen et al. (2025), Gaubert et al. (2025), Ahlvik et al. (2025),...

- **Multi-dimensional heterogeneity in public finance:**

Cremer et al. (2003), Blomquist and Christiansen (2008), Diamond and Spinnewijn (2011), Piketty and Saez (2013), Scheuer and Wolitzky (2016), Saez and Stantcheva (2018), Golosov and Krasikov (2025), Bierbrauer et al. (2023), Boerma et al. (2025), ...

- **Related mechanism-design papers:** Jullien (2000), Che et al. (2013),

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- There is a unit mass of agents characterized by two-dimensional types: (t, a) , where t is “**taste**” and a is “**ability**.”

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 - **numeraire** consumption good $c \in \mathbb{R}$,
 - **earnings** (pre-tax) $z \in \mathbb{R}$ (determined by labor supply).
- Each agent's utility function is

$$U((c, x, z), (t, a)) = u(c) + v(x; t) - w(z; a),$$

where the functions u , v , w are increasing.

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- Goods x can be produced at **fixed marginal costs** $k \in \mathbb{R}_+^L$.

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- The planner **maximizes social surplus**

$$\int \lambda(t, a) U(t, a) dF(t, a)$$

subject to **incentive compatibility** and the **resource constraint**:

$$\int [z(t, a) - c(t, a) - k \cdot x(t, a)] dF(t, a) \geq G.$$

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subject to **incentive compatibility** and the **resource constraint**:

$$\int [z(t, a) - c(t, a) - k \cdot x(t, a)] dF(t, a) \geq G.$$

- We assume existence of solutions and strong duality.

When Interventions in Markets are Redundant

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Useful concept:

- Let m denote *disposable income* (evaluated at prices equal to marginal costs): $m = c + k \cdot x$.

When Interventions in Markets are Redundant

Useful concept:

- Let m denote *disposable income* (evaluated at prices equal to marginal costs): $m = c + k \cdot x$.
- Let $V(m, t)$ denote utility type t gets from disposable income m :

$$V(m, t) = \max_{x \in \mathbb{R}_+^L} \{u(m - k \cdot x) + v(x; t)\}.$$

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- Define the **marginal utility of disposable income** (whenever exists) as $\partial V(m, t)/\partial m$.

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Theorem

Suppose that:

- 1. Taste does not affect the marginal utility of income:
 $u(c) = c, c \in \mathbb{R};$*
- 2. Welfare weights depend only on the ability type:
 $\lambda(t, a) = \bar{\lambda}(a), \forall(t, a);$*
- 3. Ability and taste types are statistically independent:
 $F(t, a) = F_t(t)F_a(a), \forall(t, a).$*

*Any **optimal mechanism** induces an **efficient choice of goods** (by pricing them at marginal costs), and can be implemented with an **income tax alone**.*

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4. Hence, the solution to the relaxed problem is **implementable in the original problem** (by pricing goods at marginal cost).

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- Let $F_i(t)$ be the CDF of taste t conditional on ability type i , supported on $[0, \bar{t}]$, for $i \in \{L, H\}$.
- **Linear utility function** is:

$$u(c) + t \cdot x - \frac{1}{a} \cdot z \quad \text{where} \quad u(c) = \begin{cases} c & c \geq \underline{c}, \\ -\infty & c < \underline{c}, \end{cases}$$

where $\underline{c}$ is a “**subsistence**” level.

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Assumption 1 fails (subsistence constraint has bite):

$$\mu_H \left(1 - \frac{1}{a_H} \right) < G + \underline{c} + k.$$

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Suppose $\frac{1-F(t)}{f(t)}$ is non-increasing and $\frac{F(t)}{f(t)}$ is non-decreasing.

There exists an optimal mechanism in which:

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 - low-ability agents with sufficiently high taste choose the **subsidized low-quality option**;
 - high-ability agents with sufficiently high taste choose the **taxed high-quality option**.
- High-quality option with tax $\implies$ more surplus can be extracted from (some) high-ability agents.

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High-level intuition:

Without Assumption 1, purchasing the good implies **higher marginal utility of income** $\implies$ planner values redistribution towards low-ability agents with high t .

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Contrast to original Atkinson-Stiglitz logic (example: **healthcare**)

- AS: More able agents consume more healthcare **only** because they have more income; better to redistribute income without distorting the allocation of healthcare.
- Our setting: Demand for healthcare stems from **both** income (ability) and need (taste); subsidies for low-quality healthcare improve redistribution because they **endogenously target low-ability agents who have a particularly high marginal utility of income**.

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2. *Income is taxed at a rate $\tau < 1 - 1/a_H$ (high-ability agents work); the good is sold at a **lower price to low-ability agents** than to high-ability agents.*

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 - The optimal mechanism can be implemented as a **means-tested subsidy for the good**.

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- Mechanism 1 is optimal iff the motive to redistribute across ability types is sufficiently strong.

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- More research at the intersection of market design and public finance is needed!

Appendix

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 - $\Lambda_L(t) \geq \Lambda_H(t)$, for all t (weaker than $\lambda_L(t) \geq \lambda_H(t)$ for all t).

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 - $(\bar{\lambda}_L - 1)h_L(t) \geq (\bar{\lambda}_H - 1)h_H(t)$, for all t .

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If the designer is primarily concerned with **redistribution across ability** types, it is optimal to redistribute as much as possible through the income tax.

Otherwise, it may be optimal to limit the role of income taxes to make income useful for **price discrimination** in market design:

⇒ income tax and market design are to some extent **substitutes**.

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Robustness of results to concavity in the utility function u :

- Suppose that u is a strictly increasing, strictly concave, continuously differentiable function.

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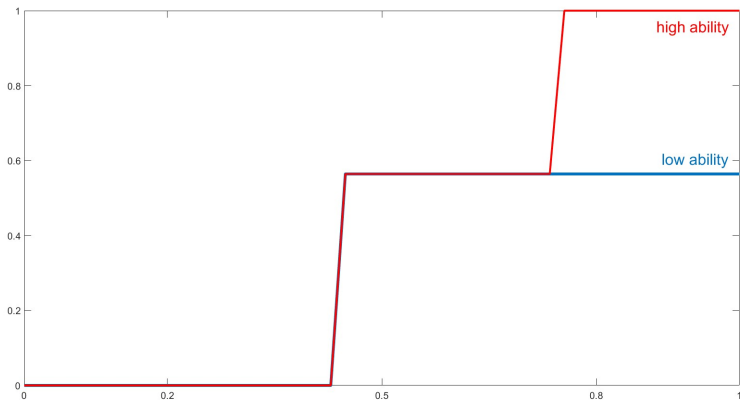
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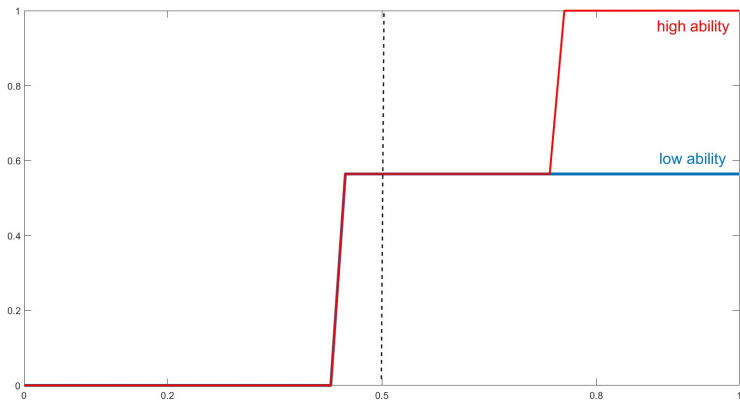
- Suppose that u is a strictly increasing, strictly concave, continuously differentiable function.
- We prove that:
 - Any optimal mechanism distorts the goods allocation away from efficiency;
 - The consumption of low-ability agents is distorted upwards;
 - The consumption of high-ability agents with high enough taste type is distorted downwards.

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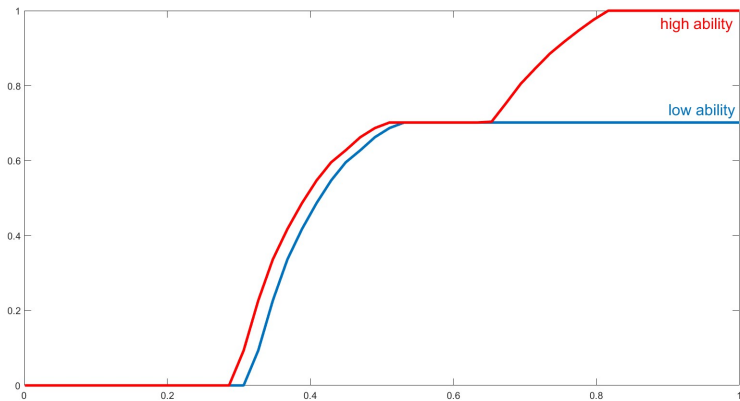
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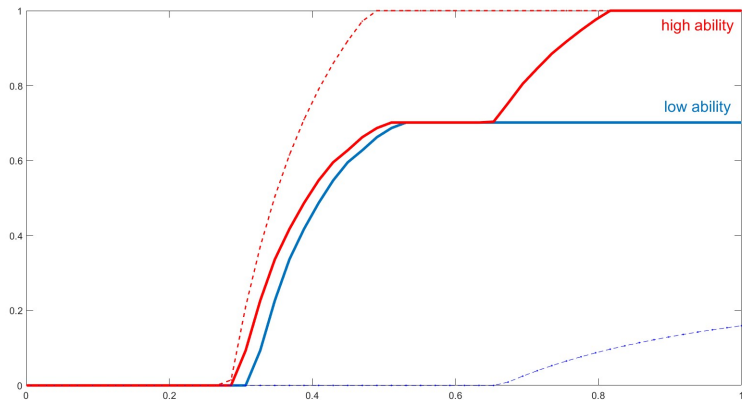
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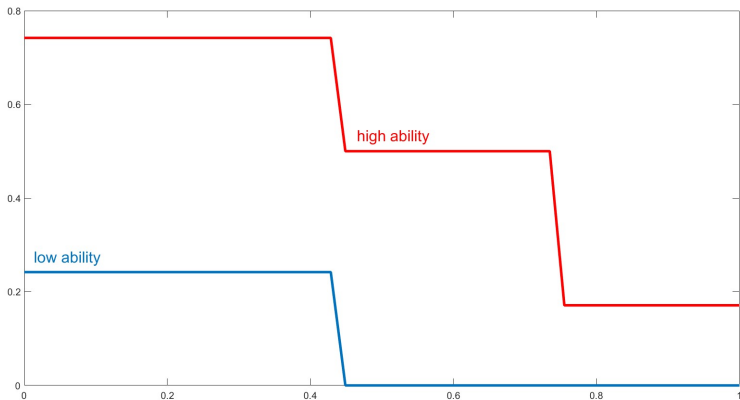
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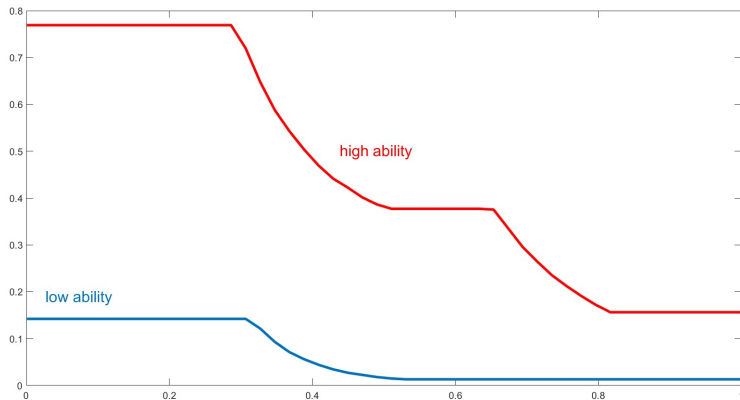
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Numeraire consumption in optimal mechanism

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Theorem 1 under weaker assumptions

Goods $x \in \mathcal{X} = \prod_{i=1}^L [\underline{x}_i, \infty)$; earnings $z \in \mathbb{R}_+$. No explicit numeraire. Utility $U(x, z, t, a)$ is continuous in (x, z) and locally non-satiated in x .

Assumption A0. The ability type does not affect the marginal rate of substitution between goods: $U(x, z, t, a) \equiv \mathcal{U}(v(x, z, t), z, a)$, for some functions $\mathcal{U}$ and v , with $\mathcal{U}$ strictly increasing in the first argument.

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Let $V(m, z, t, a) := \max_x \{U(x, z, t, a) : x \cdot k \leq m\}$.

Assumption A1'. The indirect utility function is additively separable in allocation (m, z) and tastes t :

$$V(m, z, t, a) \equiv \mathcal{W}(m, z, a) + \mathcal{W}_0(t, a),$$

for some functions $\mathcal{W}$ and $\mathcal{W}_0$.

Theorem 1 under weaker assumptions

A mechanism (x, z) is *admissible* if it is feasible and satisfies $U(x(t, a), z(t, a), t, a) \geq U(\underline{x}, z(t, a), t, a)$ for all (t, a) .

Theorem 1'. Suppose that Assumptions A0, A1', A2, and A3 hold. Then, for any admissible mechanism, there exists a feasible mechanism that (weakly) improves the planner's objective, induces an efficient allocation of goods, and can be implemented with a competitive goods market and a (potentially stochastic) income tax.

Theorem 1 under weaker assumptions

A1': $V(m, z, t, a) \equiv \mathcal{W}(m, z, a) + \mathcal{W}_0(t, a)$.

This is both an **ordinal** property of agents' preferences (if imposed for a *range* of k) and a **cardinal** property of social preferences:

- **Ordinal**: Engel curves of any two taste types, keeping z and a constant, either never cross, or perfectly overlay:

$$\frac{\partial x_i^*(m, z, t, a)}{\partial m} = \alpha_i(m, z, a) + \beta_i(m, z, a)x_i^*(m, z, t, a),$$

for some functions α_i and β_i that *do not* depend on t (satisfied by quasi-linear, homothetic, Stone-Geary preferences).

- **Cardinal**: A1' requires a **knife-edge** cardinal representation (akin to choosing a particular set of welfare weights).