

Optimal Redistribution via Income Taxation and Market Design

Paweł Doligalski (r)

University of Bristol

Mohammad Akbarpour (r)

Stanford University

Piotr Dworczak^{*}

Northwestern University; GRAPE

Scott Kominers

Harvard University; a16z

Seminar, University of Mannheim
October 30, 2025

^{*}Co-funded by the European Union (ERC, IMD-101040122). Views and opinions expressed are those of the authors only and do not necessarily reflect those of the European Union or the European Research Council.  

Introduction

How to allocate scarce resources?

Introduction

Use markets.

Use markets.

(Smith, 1776)

Use markets.

(Smith, 1776)

I Welfare Theorem

How to allocate scarce resources

**How to allocate scarce resources
in the presence of inequality?**

Introduction

Still markets.

Still **markets**.

II Welfare Theorem

How to allocate scarce resources

How to allocate scarce resources in the presence of *inequality*

**How to allocate scarce resources
in the presence of **inequality**
and **incomplete information**?**

Example

Example

Example

Example

Based on ideas in: Dworczak [Ⓡ] Kominers [Ⓡ] Akbarpour (2021)

Shout-out to: **Weitzman** (1977), **Spence** (1977), **Nichols and Zeckhauser** (1982), **Condorelli** (2013)

Example

- There are 3 agents: Ann, Bob, and Claire.

Example

- There are 3 agents: Ann, Bob, and Claire.
- There are 2 homogeneous goods (e.g., houses) to allocate.

Example

- There are 3 agents: Ann, Bob, and Claire.
- There are 2 homogeneous goods (e.g., houses) to allocate.
- Agents have the following utilities u for a house:
Ann: 6, Bob: 18, Claire: 24.

Example

- There are 3 agents: Ann, Bob, and Claire.
- There are 2 homogeneous goods (e.g., houses) to allocate.
- Agents have the following utilities u for a house:
Ann: 6, Bob: 18, Claire: 24.
- However, these agents also differ in wealth w :
Ann: 8, Bob: 1, Claire: 1.

Example

- There are 3 agents: Ann, Bob, and Claire.
- There are 2 homogeneous goods (e.g., houses) to allocate.
- Agents have the following utilities u for a house:
Ann: 6, Bob: 18, Claire: 24.
- However, these agents also differ in wealth w :
Ann: 8, Bob: 1, Claire: 1.
- All agents have the same utility function

$$\log(w) + u \cdot x,$$

where x is the probability of having a house.

Example

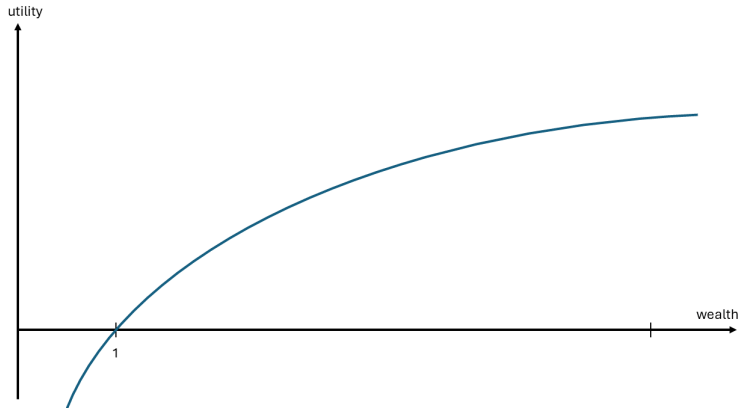
- There are 3 agents: Ann, Bob, and Claire.
- There are 2 homogeneous goods (e.g., houses) to allocate.
- Agents have the following utilities u for a house:
Ann: 6, Bob: 18, Claire: 24.
- However, these agents also differ in wealth w :
Ann: 8, Bob: 1, Claire: 1.
- All agents have the same utility function

$$\log(w) + u \cdot x,$$

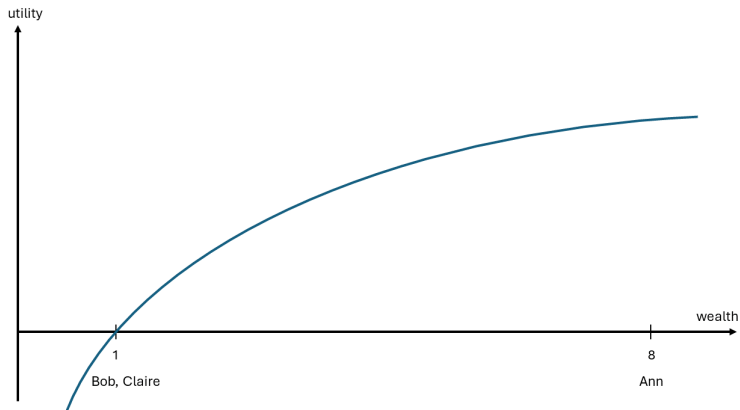
where x is the probability of having a house.

- Social planner aims to maximize the sum of utilities.

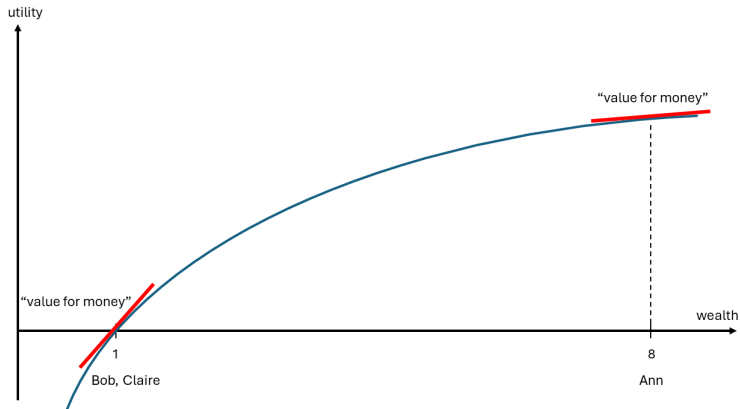
Example



Example



Example



Example

	utility for good	value for money	
Ann	6	$1/8$	
Bob	18	1	
Claire	24	1	

Example

	utility for good	value for money	willingness to pay
Ann	6	$1/8$	48
Bob	18	1	18
Claire	24	1	24

Example

	utility for good	value for money	willingness to pay
Ann	6	1/8	48
Bob	18	1	18
Claire	24	1	24

Equivalent representation of the problem:

	util. good	val. money	willing. pay	welfare weight
Ann	48	1	48	1/8
Bob	18	1	18	1
Claire	24	1	24	1

Example

	val. good	val. money	WTP	utility
Ann	6	1/8	48	
Bob	18	1	18	
Claire	24	1	24	

Mechanism #1: **Market equilibrium** (+ lump-sum transfers)

Example

	val. good	val. money	WTP	utility
Ann	6	1/8	48	
Bob	18	1	18	
Claire	24	1	24	

Mechanism #1: **Market equilibrium** (+ lump-sum transfers)

Market-clearing price: 24

House owners: Ann & Claire

Example

	val. good	val. money	WTP	utility
Ann	6	1/8	48	
Bob	18	1	18	
Claire	24	1	24	

Mechanism #1: **Market equilibrium** (+ lump-sum transfers)

Market-clearing price: 24

House owners: Ann & Claire

Lump-sum transfer: 16

Example

	val. good	val. money	WTP	utility
Ann	6	1/8	48	$6 - 1/8 \cdot (24 - 16)$
Bob	18	1	18	
Claire	24	1	24	

Mechanism #1: **Market equilibrium** (+ lump-sum transfers)

Market-clearing price: 24

House owners: Ann & Claire

Lump-sum transfer: 16

Example

	val. good	val. money	WTP	utility
Ann	6	1/8	48	5
Bob	18	1	18	$0 + 1 \cdot 16$
Claire	24	1	24	

Mechanism #1: **Market equilibrium** (+ lump-sum transfers)

Market-clearing price: 24

House owners: Ann & Claire

Lump-sum transfer: 16

Example

	val. good	val. money	WTP	utility
Ann	6	1/8	48	5
Bob	18	1	18	16
Claire	24	1	24	$24 - 1 \cdot (24 - 16)$

Mechanism #1: **Market equilibrium** (+ lump-sum transfers)

Market-clearing price: 24

House owners: Ann & Claire

Lump-sum transfer: 16

Example

	val. good	val. money	WTP	utility
Ann	6	1/8	48	5
Bob	18	1	18	16
Claire	24	1	24	16

Mechanism #1: **Market equilibrium** (+ lump-sum transfers)

Market-clearing price: 24

House owners: Ann & Claire

Lump-sum transfer: 16

Total utility: 37

Example

	val. good	val. money	WTP	utility
Ann	6	1/8	48	
Bob	18	1	18	
Claire	24	1	24	

Example

	val. good	val. money	WTP	utility
Ann	6	1/8	48	
Bob	18	1	18	
Claire	24	1	24	

Mechanism #2: **Random allocation**

Example

	val. good	val. money	WTP	utility
Ann	6	1/8	48	
Bob	18	1	18	
Claire	24	1	24	

Mechanism #2: **Random allocation**

Price: 0

House owners: $\frac{2}{3}$ Ann & $\frac{2}{3}$ Bob & $\frac{2}{3}$ Claire

Example

	val. good	val. money	WTP	utility
Ann	6	1/8	48	
Bob	18	1	18	
Claire	24	1	24	

Mechanism #2: **Random allocation**

Price: 0

House owners: $\frac{2}{3}$ Ann & $\frac{2}{3}$ Bob & $\frac{2}{3}$ Claire

Lump-sum transfer: 0

Example

	val. good	val. money	WTP	utility
Ann	6	1/8	48	$2/3 \cdot 6$
Bob	18	1	18	
Claire	24	1	24	

Mechanism #2: **Random allocation**

Price: 0

House owners: $\frac{2}{3}$ Ann & $\frac{2}{3}$ Bob & $\frac{2}{3}$ Claire

Lump-sum transfer: 0

Example

	val. good	val. money	WTP	utility
Ann	6	1/8	48	4
Bob	18	1	18	$2/3 \cdot 18$
Claire	24	1	24	

Mechanism #2: **Random allocation**

Price: 0

House owners: $\frac{2}{3}$ Ann & $\frac{2}{3}$ Bob & $\frac{2}{3}$ Claire

Lump-sum transfer: 0

Example

	val. good	val. money	WTP	utility
Ann	6	1/8	48	4
Bob	18	1	18	12
Claire	24	1	24	$2/3 \cdot 24$

Mechanism #2: **Random allocation**

Price: 0

House owners: $\frac{2}{3}$ Ann & $\frac{2}{3}$ Bob & $\frac{2}{3}$ Claire

Lump-sum transfer: 0

Example

	val. good	val. money	WTP	utility
Ann	6	1/8	48	4
Bob	18	1	18	12
Claire	24	1	24	16

Mechanism #2: **Random allocation**

Price: 0

House owners: $\frac{2}{3}$ Ann & $\frac{2}{3}$ Bob & $\frac{2}{3}$ Claire

Lump-sum transfer: 0

Total utility: 32 (< 37)

Example

	val. good	val. money	WTP	utility
Ann	6	1/8	48	4
Bob	18	1	18	12
Claire	24	1	24	16

Mechanism #2: **Random allocation**

Price: 0

House owners: $\frac{2}{3}$ Ann & $\frac{2}{3}$ Bob & $\frac{2}{3}$ Claire

Lump-sum transfer: 0

Among all mechanisms charging a single price p , $p = 24$ is optimal.

Example

	val. good	val. money	WTP	utility
Ann	6	1/8	48	
Bob	18	1	18	
Claire	24	1	24	

Example

	val. good	val. money	WTP	utility
Ann	6	1/8	48	
Bob	18	1	18	
Claire	24	1	24	

Mechanism #3: Allocate 1st unit by **lottery**, sell 2nd unit in **market**

Example

	val. good	val. money	WTP	utility
Ann	6	1/8	48	
Bob	18	1	18	
Claire	24	1	24	

Mechanism #3: Allocate 1st unit by **lottery**, sell 2nd unit in **market**

Market-clearing price for 2nd unit: 24

(Ann's indifference: $6 - 1/8 \cdot 24 = 1/2 \cdot 6$)

House owners: Ann & 50% Claire & 50% Bob

Example

	val. good	val. money	WTP	utility
Ann	6	1/8	48	
Bob	18	1	18	
Claire	24	1	24	

Mechanism #3: Allocate 1st unit by **lottery**, sell 2nd unit in **market**

Market-clearing price for 2nd unit: 24

(Ann's indifference: $6 - 1/8 \cdot 24 = 1/2 \cdot 6$)

House owners: Ann & 50% Claire & 50% Bob

Lump-sum transfer: 8

Example

	val. good	val. money	WTP	utility
Ann	6	1/8	48	$6 - 1/8 \cdot (24 - 8)$
Bob	18	1	18	
Claire	24	1	24	

Mechanism #3: Allocate 1st unit by **lottery**, sell 2nd unit in **market**

Market-clearing price for 2nd unit: 24

(Ann's indifference: $6 - 1/8 \cdot \mathbf{24} = 1/2 \cdot 6$)

House owners: Ann & 50% Claire & 50% Bob

Lump-sum transfer: 8

Example

	val. good	val. money	WTP	utility
Ann	6	1/8	48	4
Bob	18	1	18	$1/2 \cdot 18 + 1 \cdot 8$
Claire	24	1	24	

Mechanism #3: Allocate 1st unit by **lottery**, sell 2nd unit in **market**

Market-clearing price for 2nd unit: 24

(Ann's indifference: $6 - 1/8 \cdot 24 = 1/2 \cdot 6$)

House owners: Ann & 50% Claire & 50% Bob

Lump-sum transfer: 8

Example

	val. good	val. money	WTP	utility
Ann	6	1/8	48	4
Bob	18	1	18	17
Claire	24	1	24	$1/2 \cdot 24 + 1 \cdot 8$

Mechanism #3: Allocate 1st unit by **lottery**, sell 2nd unit in **market**

Market-clearing price for 2nd unit: 24

(Ann's indifference: $6 - 1/8 \cdot 24 = 1/2 \cdot 6$)

House owners: Ann & 50% Claire & 50% Bob

Lump-sum transfer: 8

Example

	val. good	val. money	WTP	utility
Ann	6	1/8	48	4
Bob	18	1	18	17
Claire	24	1	24	20

Mechanism #3: Allocate 1st unit by **lottery**, sell 2nd unit in **market**

Market-clearing price for 2nd unit: 24

(Ann's indifference: $6 - 1/8 \cdot 24 = 1/2 \cdot 6$)

House owners: Ann & 50% Claire & 50% Bob

Lump-sum transfer: 8

Total utility: 41 (> 37)

Example

	val. good	val. money	WTP	utility
Ann	6	1/8	48	4
Bob	18	1	18	17
Claire	24	1	24	20

Mechanism #3: Allocate 1st unit by **lottery**, sell 2nd unit in **market**

Market-clearing price for 2nd unit: 24

(Ann's indifference: $6 - 1/8 \cdot 24 = 1/2 \cdot 6$)

House owners: Ann & 50% Claire & 50% Bob

Lump-sum transfer: 8

This is in fact the **optimal mechanism!** (subject to IC constraints)

Example

	val. money	WTP	utility—market	utility—hybrid
Ann	1/8	48	5	4
Bob	1	18	16	17
Claire	1	24	16	20

Example

	val. money	WTP	utility—market	utility—hybrid
Ann	1/8	48	5	4
Bob	1	18	16	17
Claire	1	24	16	20

Intuition:

- Values for money (welfare weights) are **negatively correlated** with willingness to pay.

Example

	val. money	WTP	utility—market	utility—hybrid
Ann	1/8	48	5	4
Bob	1	18	16	17
Claire	1	24	16	20

Intuition:

- Values for money (welfare weights) are **negatively correlated** with willingness to pay.
- Market **behavior** can be **used to screen**: Choosing the lottery reveals a high value for money.

Example

	val. money	WTP	utility—market	utility—hybrid
Ann	1/8	48	5	4
Bob	1	18	16	17
Claire	1	24	16	20

Intuition:

- Values for money (welfare weights) are **negatively correlated** with willingness to pay.
- Market **behavior** can be **used to screen**: Choosing the lottery reveals a high value for money.
- Designer can increase the utility of agents with high values for money by **reducing the price of the rationed option**.

Example

	val. money	WTP	utility—market	utility—hybrid
Ann	1/8	48	5	4
Bob	1	18	16	17
Claire	1	24	16	20

Intuition:

- Values for money (welfare weights) are **negatively correlated** with willingness to pay.
- Market **behavior** can be **used to screen**: Choosing the lottery reveals a high value for money.
- Designer can increase the utility of agents with high values for money by **reducing the price of the rationed option**.
- Mathematically, rationing results from Myerson's (1981) **ironing**.

Inequality-aware Market Design (IMD)

- Market with **buyers and sellers** for an indivisible good:
 - DKA (2021), ...
- One-sided allocation with **different qualities and labels**:
 - ADK (2024), Tokarski KAD (2023) (energy pricing), ...
- Problems with **allocative externalities**:
 - Kang (2023), A Budish DK (2024) (Covid-19 vaccines), Pai and Strack (2025), ...
- Optimality of **costly screening** and allocating bads:
 - Kang and Zheng (2020), Yang DA (2024), D (2025), ...
- Interaction of redistributive mechanisms and **private markets**:
 - Kang (2023), Kang and Watt (2024a, 2024b), ...

Inequality-aware Market Design (IMD)

Policymakers frequently **distort allocation of goods and services**, at least in part due to **redistributive concerns**:

- **Housing** (rent control; public housing; LIHTC in the US)
- **Health care** (public health care in Europe, Medicare and Medicaid in the US)
- **Food** (food stamps in the US; in-kind food transfer programs (rice, wheat,...) in developing countries)
- **Energy** (electricity in virtually all European countries at the moment; kerosene in India; energy subsidies in Latin America;...)
- **Covid-19 vaccines** (allocated for free in most countries)
- **Road access** (congestion pricing is still rare in practice)
- ...

Question

**How to allocate scarce resources
in the presence of inequality
and incomplete information?**

How to allocate scarce resources in the presence of **inequality and **incomplete information**?**

I have cheated a bit...

IMD and Income Taxation

- We have looked at a **single market** (market-design approach).

IMD and Income Taxation

- We have looked at a **single market** (market-design approach).
- But what if the designer can **control other tools** to address inequality?

IMD and Income Taxation

- We have looked at a **single market** (market-design approach).
- But what if the designer can **control other tools** to address inequality?
- What if **income taxation** is available as a tool (the public-finance approach)?

IMD and Income Taxation

- We have looked at a **single market** (market-design approach).
- But what if the designer can **control other tools** to address inequality?
- What if **income taxation** is available as a tool (the public-finance approach)?
- **Main question for this project:**

IMD and Income Taxation

- We have looked at a **single market** (market-design approach).
- But what if the designer can **control other tools** to address inequality?
- What if **income taxation** is available as a tool (the public-finance approach)?
- **Main question for this project:**
 - When (if at all) **should we distort goods markets** to effect redistribution when income taxation is feasible?

IMD and Income Taxation

- We have looked at a **single market** (market-design approach).
- But what if the designer can **control other tools** to address inequality?
- What if **income taxation** is available as a tool (the public-finance approach)?
- **Main question for this project:**
 - When (if at all) **should we distort goods markets** to effect redistribution when income taxation is feasible?
 - What is the optimal **interaction between inequality-aware market design and income taxation?**

Quick review of public finance...

Quick review of public finance...

Atkinson and Stiglitz (1976): Redistribution should be accomplished entirely via income taxation

Quick review of public finance...

Atkinson and Stiglitz (1976): Redistribution should be accomplished entirely via income taxation when

- agents' utilities are **separable** between labor and consumption;
- agents **differ only in ability** to generate income.

Quick review of public finance...

Atkinson and Stiglitz (1976): Redistribution should be accomplished entirely via income taxation when

- agents' utilities are **separable** between labor and consumption;
- agents **differ only in ability** to generate income.

Market-design perspective: We should allow for **heterogeneity in consumption tastes!**

Quick review of public finance...

Atkinson and Stiglitz (1976): Redistribution should be accomplished entirely via income taxation when

- agents' utilities are **separable** between labor and consumption;
- agents **differ only in ability** to generate income.

Market-design perspective: We should allow for **heterogeneity in consumption tastes**! AS theorem may fail in this case:

Cremer and Gavhari (1995, 1998, 2002), Saez (2002), Diamond and Spinnewijn (2011), Golosov et al. (2013), Gauthier and Henriot (2018), Scheuer and Slemrod (2020), Schulz (2021), Hellwig and Werquin (2023), Ferey et al. (2024), Ahlvik et al. (2024), Gerritsen et al. (2025), Gaubert et al. (2025),...

Quick review of public finance...

Atkinson and Stiglitz (1976): Redistribution should be accomplished entirely via income taxation when

- agents' utilities are **separable** between labor and consumption;
- agents **differ only in ability** to generate income.

Market-design perspective: We should allow for **heterogeneity in consumption tastes**! AS theorem may fail in this case:

Cremer and Gavhari (1995, 1998, 2002), **Saez (2002)**, Diamond and Spinnewijn (2011), Golosov et al. (2013), Gauthier and Henriët (2018), Scheuer and Slemrod (2020), Schulz (2021), Hellwig and Werquin (2023), **Ferey, Lockwood, Taubinsky (2024)**, Ahlvik et al. (2024), Gerritsen et al. (2025), Gaubert et al. (2025),...

Our paper

Framework: Agents differ in **taste** and **ability**, choose **labor** and **consumption**, designer maximizes welfare over IC mechanisms.

Our paper

Framework: Agents differ in **taste** and **ability**, choose **labor** and **consumption**, designer maximizes welfare over IC mechanisms.

Result #1: Assuming separable utility functions, if

1. taste type does not affect the marginal utility of income;
2. welfare weights depend only on the ability type;
3. taste type and ability type are statistically independent,

Our paper

Framework: Agents differ in **taste** and **ability**, choose **labor** and **consumption**, designer maximizes welfare over IC mechanisms.

Result #1: Assuming separable utility functions, if

1. taste type does not affect the marginal utility of income;
2. welfare weights depend only on the ability type;
3. taste type and ability type are statistically independent,

then **redistribution** should be effected **entirely via income taxes**.

Our paper

Framework: Agents differ in **taste** and **ability**, choose **labor** and **consumption**, designer maximizes welfare over IC mechanisms.

Result #2: If any of the assumptions 1, 2, or 3 are relaxed, then **market design also plays a role.**

Our paper

Framework: Agents differ in **taste** and **ability**, choose **labor** and **consumption**, designer maximizes welfare over IC mechanisms.

Result #2: If any of the assumptions 1, 2, or 3 are relaxed, then **market design also plays a role.**

- We provide **analytical solutions** in a linear model with two goods, binary ability type, and continuous taste type.

Our paper

Framework: Agents differ in **taste** and **ability**, choose **labor** and **consumption**, designer maximizes welfare over IC mechanisms.

Result #2: If any of the assumptions 1, 2, or 3 are relaxed, then **market design also plays a role.**

- We provide **analytical solutions** in a linear model with two goods, binary ability type, and continuous taste type.
- This allows us to characterize the **interaction** of inequality-aware market design and income taxation.

Our paper

Framework: Agents differ in **taste** and **ability**, choose **labor** and **consumption**, designer maximizes welfare over IC mechanisms.

Result #2: If any of the assumptions 1, 2, or 3 are relaxed, then **market design also plays a role.**

- We provide **analytical solutions** in a linear model with two goods, binary ability type, and continuous taste type.
- This allows us to characterize the **interaction** of inequality-aware market design and income taxation.
- Take-away: Interventions in markets are **typically part of the optimal mechanism.**

Model

Model

Model

- There is a unit mass of agents characterized by two-dimensional types: (t, a) , where t is “**taste**” and a is “**ability**.”

Model

- There is a unit mass of agents characterized by two-dimensional types: (t, a) , where t is “**taste**” and a is “**ability**.”
- Social planner allocates:

Model

- There is a unit mass of agents characterized by two-dimensional types: (t, a) , where t is “**taste**” and a is “**ability**.”
- Social planner allocates:
 - vector of **goods** $x \in \mathbb{R}_+^L$ (e.g., health services),

Model

- There is a unit mass of agents characterized by two-dimensional types: (t, a) , where t is “**taste**” and a is “**ability**.”
- Social planner allocates:
 - vector of **goods** $x \in \mathbb{R}_+^L$ (e.g., health services),
 - **numeraire** consumption good $c \in \mathbb{R}$,

Model

- There is a unit mass of agents characterized by two-dimensional types: (t, a) , where t is “**taste**” and a is “**ability**.”
- Social planner allocates:
 - vector of **goods** $x \in \mathbb{R}_+^L$ (e.g., health services),
 - **numeraire** consumption good $c \in \mathbb{R}$,
 - **earnings** (pre-tax) $z \in \mathbb{R}$ (determined by labor supply).

Model

- There is a unit mass of agents characterized by two-dimensional types: (t, a) , where t is “**taste**” and a is “**ability**.”
- Social planner allocates:
 - vector of **goods** $x \in \mathbb{R}_+^L$ (e.g., health services),
 - **numeraire** consumption good $c \in \mathbb{R}$,
 - **earnings** (pre-tax) $z \in \mathbb{R}$ (determined by labor supply).
- Each agent's utility function is

$$U((c, x, z), (t, a)) = u(c) + v(x; t) - w(z; a),$$

where the functions u , v , w are increasing.

Model

- Goods x can be produced at **fixed marginal costs** $k \in \mathbb{R}_+^L$.

Model

- Goods x can be produced at **fixed marginal costs** $k \in \mathbb{R}_+^L$.
- Types (t, a) have a joint distribution $F(t, a)$ in the population.

Model

- Goods x can be produced at **fixed marginal costs** $k \in \mathbb{R}_+^L$.
- Types (t, a) have a joint distribution $F(t, a)$ in the population.
- The planner uses social welfare weights $\lambda(t, a) \geq 0$, with average weight normalized to 1.

Model

- Goods x can be produced at **fixed marginal costs** $k \in \mathbb{R}_+^L$.
- Types (t, a) have a joint distribution $F(t, a)$ in the population.
- The planner uses social welfare weights $\lambda(t, a) \geq 0$, with average weight normalized to 1.
- The planner **maximizes social surplus**

$$\int \lambda(t, a) U(t, a) dF(t, a)$$

subject to **incentive compatibility** and the **resource constraint**:

$$\int [z(t, a) - c(t, a) - k \cdot x(t, a)] dF(t, a) \geq G.$$

Model

- Goods x can be produced at **fixed marginal costs** $k \in \mathbb{R}_+^L$.
- Types (t, a) have a joint distribution $F(t, a)$ in the population.
- The planner uses social welfare weights $\lambda(t, a) \geq 0$, with average weight normalized to 1.
- The planner **maximizes social surplus**

$$\int \lambda(t, a) U(t, a) dF(t, a)$$

subject to **incentive compatibility** and the **resource constraint**:

$$\int [z(t, a) - c(t, a) - k \cdot x(t, a)] dF(t, a) \geq G.$$

- We assume existence of solutions and strong duality.

Atkinson-Stiglitz result

When Interventions in Markets are Redundant

Atkinson-Stiglitz result

Useful concept:

- Let m denote *disposable income* (evaluated at prices equal to marginal costs): $m = c + k \cdot x$.

Atkinson-Stiglitz result

Useful concept:

- Let m denote *disposable income* (evaluated at prices equal to marginal costs): $m = c + k \cdot x$.
- Let $V(m, t)$ denote utility type t gets from disposable income m :

$$V(m, t) = \max_{x \in \mathbb{R}_+^L} \{u(m - k \cdot x) + v(x; t)\}.$$

Atkinson-Stiglitz result

Useful concept:

- Let m denote *disposable income* (evaluated at prices equal to marginal costs): $m = c + k \cdot x$.
- Let $V(m, t)$ denote utility type t gets from disposable income m :

$$V(m, t) = \max_{x \in \mathbb{R}_+^L} \{u(m - k \cdot x) + v(x; t)\}.$$

- Define the **marginal utility of disposable income** (whenever exists) as $\partial V(m, t) / \partial m$.

Atkinson-Stiglitz result

Theorem

Suppose that:

1. *Taste does not affect the marginal utility of income:*
 $u(c) = c, c \in \mathbb{R};$
2. *Welfare weights depend only on the ability type:*
 $\lambda(t, a) = \bar{\lambda}(a), \forall(t, a);$
3. *Ability and taste types are statistically independent:*
 $F(t, a) = F_t(t)F_a(a), \forall(t, a).$

*Any **optimal mechanism** induces an **efficient choice of goods** (by pricing them at marginal costs), and can be implemented with an **income tax alone**.*

Atkinson-Stiglitz result

Idea of the proof:

1. Solve a relaxed problem **pretending that taste t is observed**.

Atkinson-Stiglitz result

Idea of the proof:

1. Solve a relaxed problem **pretending that taste t is observed**.
2. “Standard” AS theorem $\implies$ **no distortion** in goods markets (see Doligalski et al. (2024) for a mechanism-design approach).

Atkinson-Stiglitz result

Idea of the proof:

1. Solve a relaxed problem **pretending that taste t is observed**.
2. “Standard” AS theorem $\implies$ **no distortion** in goods markets (see Doligalski et al. (2024) for a mechanism-design approach).
3. At optimum, disposable income of each consumer, conditional on ability, does not depend on taste t :

Atkinson-Stiglitz result

Idea of the proof:

1. Solve a relaxed problem **pretending that taste t is observed**.
2. “Standard” AS theorem $\implies$ **no distortion** in goods markets (see Doligalski et al. (2024) for a mechanism-design approach).
3. At optimum, disposable income of each consumer, conditional on ability, does not depend on taste t :
 - Social preferences do not depend on taste t , either directly (assumption 1 & 2), or through correlation with ability a (assumption 3).

Atkinson-Stiglitz result

Idea of the proof:

1. Solve a relaxed problem **pretending that taste t is observed**.
2. “Standard” AS theorem $\implies$ **no distortion** in goods markets (see Doligalski et al. (2024) for a mechanism-design approach).
3. At optimum, disposable income of each consumer, conditional on ability, does not depend on taste t :
 - Social preferences do not depend on taste t , either directly (assumption 1 & 2), or through correlation with ability a (assumption 3).
 - Taste t does not affect incentives to work because marginal utility of disposable income is constant (assumption 1).

Atkinson-Stiglitz result

Idea of the proof:

1. Solve a relaxed problem **pretending that taste t is observed**.
2. “Standard” AS theorem $\implies$ **no distortion** in goods markets (see Doligalski et al. (2024) for a mechanism-design approach).
3. At optimum, disposable income of each consumer, conditional on ability, does not depend on taste t :
 - Social preferences do not depend on taste t , either directly (assumption 1 & 2), or through correlation with ability a (assumption 3).
 - Taste t does not affect incentives to work because marginal utility of disposable income is constant (assumption 1).
4. Hence, the solution to the relaxed problem is **implementable in the original problem** (by pricing goods at marginal cost).

Interaction of Income Taxation and Market Design

Interaction of Income Taxation and Market Design

Framework

- We make further simplifying assumptions.

Framework

- We make further simplifying assumptions.
- **Single indivisible good**, $x \in [0, 1]$ (can be interpreted as bounded quality); earnings $z \in [0, \bar{z}]$.

Framework

- We make further simplifying assumptions.
- **Single indivisible good**, $x \in [0, 1]$ (can be interpreted as bounded quality); earnings $z \in [0, \bar{z}]$.
- **Ability** type a is **binary**: $a \in \{a_L, a_H\}$, where $a_L \approx 0$, and $a_H > 1$ (assume that $\bar{\lambda}_H \leq \bar{\lambda}_L$).

Framework

- We make further simplifying assumptions.
- **Single indivisible good**, $x \in [0, 1]$ (can be interpreted as bounded quality); earnings $z \in [0, \bar{z}]$.
- **Ability** type a is **binary**: $a \in \{a_L, a_H\}$, where $a_L \approx 0$, and $a_H > 1$ (assume that $\bar{\lambda}_H \leq \bar{\lambda}_L$).
- Let $F_i(t)$ be the CDF of taste t conditional on ability type i , supported on $[0, \bar{t}]$, for $i \in \{L, H\}$.

Framework

- We make further simplifying assumptions.
- **Single indivisible good**, $x \in [0, 1]$ (can be interpreted as bounded quality); earnings $z \in [0, \bar{z}]$.
- **Ability** type a is **binary**: $a \in \{a_L, a_H\}$, where $a_L \approx 0$, and $a_H > 1$ (assume that $\bar{\lambda}_H \leq \bar{\lambda}_L$).
- Let $F_i(t)$ be the CDF of taste t conditional on ability type i , supported on $[0, \bar{t}]$, for $i \in \{L, H\}$.
- **Linear utility function** is:

$$u(c) + t \cdot x - \frac{1}{a} \cdot z \quad \text{where} \quad u(c) = \begin{cases} c & c \geq \underline{c}, \\ -\infty & c < \underline{c}, \end{cases}$$

where $\underline{c}$ is a “**subsistence**” level.

Relaxing Assumption 1

Relaxing Assumption 1

Relaxing Assumption 1

Theorem

Suppose that:

1. *Taste does not affect the marginal utility of income:*
 $u(c) = c, c \in \mathbb{R};$
2. *Welfare weights depend only on the ability type:*
 $\lambda(t, a) = \bar{\lambda}(a), \forall(t, a);$
3. *Ability and taste types are statistically independent:*
 $F(t, a) = F_t(t)F_a(a), \forall(t, a).$

...

Relaxing Assumption 1

Theorem

Suppose that:

1. ~~Taste does not affect the marginal utility of income:~~

$$\cancel{u(c) = c, c \in \mathbb{R};}$$

2. Welfare weights depend only on the ability type:

$$\lambda(t, a) = \bar{\lambda}(a), \forall(t, a);$$

3. Ability and taste types are statistically independent:

$$F(t, a) = F_t(t)F_a(a), \forall(t, a).$$

...

Assumption 1 fails (subsistence constraint has bite):

$$\mu_H \left(1 - \frac{1}{a_H} \right) < G + \underline{c} + k.$$

Relaxing Assumption 1

Theorem

Suppose $\frac{1-F(t)}{f(t)}$ is non-increasing and $\frac{F(t)}{f(t)}$ is non-decreasing.

There exists an optimal mechanism in which:

- Income is taxed at a rate $1 - 1/a_H$ (high-ability agents work);*

Relaxing Assumption 1

Theorem

Suppose $\frac{1-F(t)}{f(t)}$ is non-increasing and $\frac{F(t)}{f(t)}$ is non-decreasing.

There exists an optimal mechanism in which:

- Income is taxed at a rate $1 - 1/a_H$ (high-ability agents work);*
- Anyone can purchase either*

Relaxing Assumption 1

Theorem

Suppose $\frac{1-F(t)}{f(t)}$ is non-increasing and $\frac{F(t)}{f(t)}$ is non-decreasing.

There exists an optimal mechanism in which:

- Income is taxed at a rate $1 - 1/a_H$ (high-ability agents work);*
- Anyone can purchase either*
 - quality $\bar{x}_L$ of the good at a per-unit price $p_L < k$; or*

Relaxing Assumption 1

Theorem

Suppose $\frac{1-F(t)}{f(t)}$ is non-increasing and $\frac{F(t)}{f(t)}$ is non-decreasing.

There exists an optimal mechanism in which:

- Income is taxed at a rate $1 - 1/a_H$ (high-ability agents work);*
- Anyone can purchase either*
 - quality $\bar{x}_L$ of the good at a per-unit price $p_L < k$; or*
 - quality $\bar{x}_H$ of the good at a price $p_H(1 - \bar{x}_L/\bar{x}_H) + p_L(\bar{x}_L/\bar{x}_H)$,
where p_H is between k and the optimal monopoly price;*

Relaxing Assumption 1

Theorem

Suppose $\frac{1-F(t)}{f(t)}$ is non-increasing and $\frac{F(t)}{f(t)}$ is non-decreasing.

There exists an optimal mechanism in which:

- Income is taxed at a rate $1 - 1/a_H$ (high-ability agents work);*
- Anyone can purchase either*
 - quality $\bar{x}_L$ of the good at a per-unit price $p_L < k$; or*
 - quality $\bar{x}_H$ of the good at a price $p_H(1 - \bar{x}_L/\bar{x}_H) + p_L(\bar{x}_L/\bar{x}_H)$,
where p_H is between k and the optimal monopoly price;*
- Everyone receives a lump-sum transfer equal to $\underline{c} + p_L\bar{x}_L$.*

Relaxing Assumption 1

Discussion:

- Income tax extracts all surplus but is not distortionary.
⇒ “best-case scenario” for income taxation

Relaxing Assumption 1

Discussion:

- Income tax extracts all surplus but is not distortionary.
⇒ “best-case scenario” for income taxation
- **Non-linear pricing scheme** leads to **self-selection** in the market:

Relaxing Assumption 1

Discussion:

- Income tax extracts all surplus but is not distortionary.
⇒ “best-case scenario” for income taxation
- **Non-linear pricing scheme** leads to **self-selection** in the market:
 - low-ability agents with sufficiently high taste choose the **subsidized low-quality option**;

Relaxing Assumption 1

Discussion:

- Income tax extracts all surplus but is not distortionary.
⇒ “best-case scenario” for income taxation
- **Non-linear pricing scheme** leads to **self-selection** in the market:
 - low-ability agents with sufficiently high taste choose the **subsidized low-quality option**;
 - high-ability agents with sufficiently high taste choose the **taxed high-quality option**.

Relaxing Assumption 1

Discussion:

- Income tax extracts all surplus but is not distortionary.
⇒ “best-case scenario” for income taxation
- **Non-linear pricing scheme** leads to **self-selection** in the market:
 - low-ability agents with sufficiently high taste choose the **subsidized low-quality option**;
 - high-ability agents with sufficiently high taste choose the **taxed high-quality option**.
- High-quality option with tax ⇒ more surplus can be extracted from (some) high-ability agents.

Relaxing Assumption 1

Intuition for inclusion of a **subsidized option**:

- Suppose the good is priced at k . Low-ability agents are constrained and buy $x < 1$ despite having high taste t .

Relaxing Assumption 1

Intuition for inclusion of a **subsidized option**:

- Suppose the good is priced at k . Low-ability agents are constrained and buy $x < 1$ despite having high taste t .
- Consider perturbing the price to $p = k - \epsilon$, for small $\epsilon > 0$.

Relaxing Assumption 1

Intuition for inclusion of a **subsidized option**:

- Suppose the good is priced at k . Low-ability agents are constrained and buy $x < 1$ despite having high taste t .
- Consider perturbing the price to $p = k - \epsilon$, for small $\epsilon > 0$.
- There is a **second-order loss** in efficiency (marginal effect)

Relaxing Assumption 1

Intuition for inclusion of a **subsidized option**:

- Suppose the good is priced at k . Low-ability agents are constrained and buy $x < 1$ despite having high taste t .
- Consider perturbing the price to $p = k - \epsilon$, for small $\epsilon > 0$.
- There is a **second-order loss** in efficiency (marginal effect)
- ... but there is a **first-order gain** since all types $t > k$ can buy a good with ϵ higher quality (inframarginal effect).

Relaxing Assumption 1

Intuition under concave u : ► Extension

- Purchasing the good implies **higher marginal utility of income** $u'(c)$ relative to agents who do not buy.

Relaxing Assumption 1

Intuition under concave u : ► Extension

- Purchasing the good implies **higher marginal utility of income** $u'(c)$ relative to agents who do not buy.
- The planner endogenously values redistribution towards agents who buy the good (higher type t).

Relaxing Assumption 1

Intuition under concave u : ► Extension

- Purchasing the good implies **higher marginal utility of income** $u'(c)$ relative to agents who do not buy.
- The planner endogenously values redistribution towards agents who buy the good (higher type t).
- Welfare improvement can be accomplished by subsidizing the purchase of qualities consumed by low-ability agents.

Relaxing Assumption 1

Intuition under concave u : ► Extension

- Purchasing the good implies **higher marginal utility of income** $u'(c)$ relative to agents who do not buy.
- The planner endogenously values redistribution towards agents who buy the good (higher type t).
- Welfare improvement can be accomplished by subsidizing the purchase of qualities consumed by low-ability agents.
- Think of healthcare services as an example.

Relaxing Assumption 1

Contrast to original Atkinson-Stiglitz logic (example: **healthcare**)

- Richer (more able) agents consume more healthcare.

Relaxing Assumption 1

Contrast to original Atkinson-Stiglitz logic (example: **healthcare**)

- Richer (more able) agents consume more healthcare.
- Atkinson-Stiglitz setting: This is **only** because they have more income; hence, it is more effective to redistribute income directly (without distorting the allocation of healthcare)

Relaxing Assumption 1

Contrast to original Atkinson-Stiglitz logic (example: **healthcare**)

- Richer (more able) agents consume more healthcare.
- Atkinson-Stiglitz setting: This is **only** because they have more income; hence, it is more effective to redistribute income directly (without distorting the allocation of healthcare)
- Our setting: High demand for healthcare could stem from **both** income (ability) and need (taste).

Relaxing Assumption 1

Contrast to original Atkinson-Stiglitz logic (example: **healthcare**)

- Richer (more able) agents consume more healthcare.
- Atkinson-Stiglitz setting: This is **only** because they have more income; hence, it is more effective to redistribute income directly (without distorting the allocation of healthcare)
- Our setting: High demand for healthcare could stem from **both** income (ability) and need (taste).
- Subsidies for low-quality healthcare services improve redistribution relative to income tax alone because they **endogenously target low-ability agents who have a particularly high marginal utility of additional income.**

Relaxing Assumption 2 & 3

Relaxing Assumption 2 & 3

Relaxing Assumption 2 & 3

Theorem

Suppose that:

1. *Taste does not affect the marginal utility of income:*
 $u(c) = c, c \in \mathbb{R};$
2. *Welfare weights depend only on the ability type:*
 $\lambda(t, a) = \bar{\lambda}(a), \forall(t, a);$
3. *Ability and taste types are statistically independent:*
 $F(t, a) = F_t(t)F_a(a), \forall(t, a).$

...

Relaxing Assumption 2 & 3

Theorem

Suppose that:

1. *Taste does not affect the marginal utility of income:*
 $u(c) = c, c \in \mathbb{R};$
2. ~~**Welfare weights depend only on the ability type:**~~
 $\lambda(t, a) = \bar{\lambda}(a), \forall(t, a);$
3. ~~**Ability and taste types are statistically independent:**~~
 $F(t, a) = F_t(t)F_a(a), \forall(t, a).$

...

Relaxing Assumption 2 & 3

Theorem

Suppose that:

1. *Taste does not affect the marginal utility of income:*
 $u(c) = c, c \in \mathbb{R};$
2. ~~**Welfare weights depend only on the ability type:**~~
 $\lambda(t, a) = \bar{\lambda}(a), \forall(t, a);$
3. ~~**Ability and taste types are statistically independent:**~~
 $F(t, a) = F_t(t)F_a(a), \forall(t, a).$

...

Let $\Lambda(p)$ be the average welfare weight on agents with $t \geq p$

Relaxing Assumption 2 & 3

Theorem

Suppose that:

1. *Taste does not affect the marginal utility of income:*
 $u(c) = c, c \in \mathbb{R};$
2. ~~**Welfare weights depend only on the ability type:**~~
 $\lambda(t, a) = \bar{\lambda}(a), \forall(t, a);$
3. ~~**Ability and taste types are statistically independent:**~~
 $F(t, a) = F_t(t)F_a(a), \forall(t, a).$

...

Let $\Lambda(p)$ be the average welfare weight on agents with $t \geq p$
(generally $\neq 1$ when either Assumption 2 or 3 is relaxed).

Relaxing Assumption 2 & 3

Theorem

Under regularity conditions, there are two candidate optimal mechanisms:

Relaxing Assumption 2 & 3

Theorem

Under regularity conditions, there are two candidate optimal mechanisms:

1. *Income is taxed at a rate $1 - 1/a_H$ (high-ability agents work); the good is sold at a **single price** p^* satisfying*

$$p^* = k + (1 - \Lambda(p^*))h(p^*),$$

where h is the inverse hazard rate of the taste distribution.

Relaxing Assumption 2 & 3

Theorem

Under regularity conditions, there are two candidate optimal mechanisms:

1. *Income is taxed at a rate $1 - 1/a_H$ (high-ability agents work); the good is sold at a **single price** p^* satisfying*

$$p^* = k + (1 - \Lambda(p^*))h(p^*),$$

where h is the inverse hazard rate of the taste distribution.

2. *Income is taxed at a rate $\tau < 1 - 1/a_H$ (high-ability agents work); the good is sold at a **lower price to low-ability agents** than to high-ability agents.*

Relaxing Assumption 2 & 3

- In the first case, there is:

Relaxing Assumption 2 & 3

- In the first case, there is:
 - **Subsidy** for the good if average welfare weight on agents buying it, $\Lambda(p^*)$, exceeds the average welfare weight.

Relaxing Assumption 2 & 3

- In the first case, there is:
 - **Subsidy** for the good if average welfare weight on agents buying it, $\Lambda(p^*)$, exceeds the average welfare weight.
 - **Tax** on the good if average welfare weight on agents buying it, $\Lambda(p^*)$, is below the average welfare weight.

Relaxing Assumption 2 & 3

- In the first case, there is:
 - **Subsidy** for the good if average welfare weight on agents buying it, $\Lambda(p^*)$, exceeds the average welfare weight.
 - **Tax** on the good if average welfare weight on agents buying it, $\Lambda(p^*)$, is below the average welfare weight.
- In the second case:

Relaxing Assumption 2 & 3

- In the first case, there is:
 - **Subsidy** for the good if average welfare weight on agents buying it, $\Lambda(p^*)$, exceeds the average welfare weight.
 - **Tax** on the good if average welfare weight on agents buying it, $\Lambda(p^*)$, is below the average welfare weight.
- In the second case:
 - The reduced price of the good is available only to agents who do not work—the instruments are **bundled** together.

Relaxing Assumption 2 & 3

- In the first case, there is:
 - **Subsidy** for the good if average welfare weight on agents buying it, $\Lambda(p^*)$, exceeds the average welfare weight.
 - **Tax** on the good if average welfare weight on agents buying it, $\Lambda(p^*)$, is below the average welfare weight.
- In the second case:
 - The reduced price of the good is available only to agents who do not work—the instruments are **bundled** together.
 - Incentive compatibility requires decreasing tax on labor: high-ability agents get surplus from working.

Relaxing Assumption 2 & 3

- In the first case, there is:
 - **Subsidy** for the good if average welfare weight on agents buying it, $\Lambda(p^*)$, exceeds the average welfare weight.
 - **Tax** on the good if average welfare weight on agents buying it, $\Lambda(p^*)$, is below the average welfare weight.
- In the second case:
 - The reduced price of the good is available only to agents who do not work—the instruments are **bundled** together.
 - Incentive compatibility requires decreasing tax on labor: high-ability agents get surplus from working.
 - The optimal mechanism can be implemented as a **means-tested subsidy for the good**.

Relaxing Assumption 2 & 3

What drives the trade-off between mechanism 1 and mechanism 2?

- Because income reveals ability, using **income for** (third-degree) **price discrimination** in the goods market is (generally) useful.

Relaxing Assumption 2 & 3

What drives the trade-off between mechanism 1 and mechanism 2?

- Because income reveals ability, using **income for** (third-degree) **price discrimination** in the goods market is (generally) useful.
- But: income-dependent prices interact with incentives to work.

Relaxing Assumption 2 & 3

What drives the trade-off between mechanism 1 and mechanism 2?

- Because income reveals ability, using **income for** (third-degree) **price discrimination** in the goods market is (generally) useful.
- But: income-dependent prices interact with incentives to work.
- Mechanism 1 makes the IC constraint **bind** in the labor market, so income cannot be used for price discrimination.

Relaxing Assumption 2 & 3

What drives the trade-off between mechanism 1 and mechanism 2?

- Because income reveals ability, using **income for** (third-degree) **price discrimination** in the goods market is (generally) useful.
- But: income-dependent prices interact with incentives to work.
- Mechanism 1 makes the IC constraint **bind** in the labor market, so income cannot be used for price discrimination.
- Mechanism 2 makes the income tax **less progressive**, and uses the resulting **slack** to implement income-dependent prices.

Relaxing Assumption 2 & 3

What drives the trade-off between mechanism 1 and mechanism 2?

- Because income reveals ability, using **income for** (third-degree) **price discrimination** in the goods market is (generally) useful.
- But: income-dependent prices interact with incentives to work.
- Mechanism 1 makes the IC constraint **bind** in the labor market, so income cannot be used for price discrimination.
- Mechanism 2 makes the income tax **less progressive**, and uses the resulting **slack** to implement income-dependent prices.
- Mechanism 1 is optimal iff the motive to redistribute across ability types is sufficiently strong.

Derivation of the optimal mechanism

Derivation of the optimal mechanism

Derivation of the optimal mechanism

Source of tractability: **linearity** + **2-by-continuum** of types
(see also Fiat et al., 2016, Li, 2021)

Derivation of the optimal mechanism

Source of tractability: **linearity** + **2-by-continuum** of types
(see also Fiat et al., 2016, Li, 2021)

Incentive constraints:

- No misreport of t conditional on true report of $a = a_L$
 $\implies x_L(t)$ is non-decreasing.

Derivation of the optimal mechanism

Source of tractability: **linearity** + **2-by-continuum** of types
(see also Fiat et al., 2016, Li, 2021)

Incentive constraints:

- No misreport of t conditional on true report of $a = a_L$
 $\implies x_L(t)$ is non-decreasing.
- No misreport of t conditional on true report of $a = a_H$
 $\implies x_H(t)$ is non-decreasing.

Derivation of the optimal mechanism

Source of tractability: **linearity** + **2-by-continuum** of types
(see also Fiat et al., 2016, Li, 2021)

Incentive constraints:

- No misreport of t conditional on true report of $a = a_L$
 $\implies x_L(t)$ is non-decreasing.
- No misreport of t conditional on true report of $a = a_H$
 $\implies x_H(t)$ is non-decreasing.
- No misreport of t and of $a = a_L$
 $\iff$ ruled out by the assumption that $a_L \approx 0$.

Derivation of the optimal mechanism

Source of tractability: **linearity** + **2-by-continuum** of types
(see also Fiat et al., 2016, Li, 2021)

Incentive constraints:

- No misreport of t conditional on true report of $a = a_L$
 $\implies x_L(t)$ is non-decreasing.
- No misreport of t conditional on true report of $a = a_H$
 $\implies x_H(t)$ is non-decreasing.
- No misreport of t and of $a = a_L$
 $\iff$ ruled out by the assumption that $a_L \approx 0$.
- No misreport of t and of $a = a_H$
 $\iff$ an **outside option constraint**.

Derivation of the optimal mechanism

- We have an outside-option constraint: $\forall t \in [0, \bar{t}]$,

$$\Delta + \int_0^t x_H(\tau) d\tau \equiv U_H(t) \geq U_L(t)$$

Derivation of the optimal mechanism

- We have an outside-option constraint: $\forall t \in [0, \bar{t}]$,

$$\Delta + \int_0^t x_H(\tau) d\tau \equiv U_H(t) \geq U_L(t) \equiv \int_0^t x_L(\tau) d\tau.$$

Derivation of the optimal mechanism

- We have an outside-option constraint: $\forall t \in [0, \bar{t}]$,

$$\Delta + \int_0^t x_H(\tau) d\tau \equiv U_H(t) \geq U_L(t) \equiv \int_0^t x_L(\tau) d\tau.$$

- Fixing x_L , this is a linear mechanism design problem with a **type-dependent outside-option constraint** (Jullien, 2000)

Derivation of the optimal mechanism

- We have an outside-option constraint: $\forall t \in [0, \bar{t}]$,

$$\Delta + \int_0^t x_H(\tau) d\tau \equiv U_H(t) \geq U_L(t) \equiv \int_0^t x_L(\tau) d\tau.$$

- Fixing x_L , this is a linear mechanism design problem with a **type-dependent outside-option constraint** (Jullien, 2000)
- Dworczak and Muir (2024): This problem can be solved by adapting the **ironing technique** (Myerson, 1981).

Derivation of the optimal mechanism

- We have an outside-option constraint: $\forall t \in [0, \bar{t}]$,

$$\Delta + \int_0^t x_H(\tau) d\tau \equiv U_H(t) \geq U_L(t) \equiv \int_0^t x_L(\tau) d\tau.$$

- Fixing x_L , this is a linear mechanism design problem with a **type-dependent outside-option constraint** (Jullien, 2000)
- Dworczak and Muir (2024): This problem can be solved by adapting the **ironing technique** (Myerson, 1981).
 $\implies$ The optimal (Δ, x_H) depend **linearly** on x_L .

Derivation of the optimal mechanism

- We have an outside-option constraint: $\forall t \in [0, \bar{t}]$,

$$\Delta + \int_0^t x_H(\tau) d\tau \equiv U_H(t) \geq U_L(t) \equiv \int_0^t x_L(\tau) d\tau.$$

- Fixing x_L , this is a linear mechanism design problem with a **type-dependent outside-option constraint** (Jullien, 2000)
- Dworczak and Muir (2024): This problem can be solved by adapting the **ironing technique** (Myerson, 1981).
 $\implies$ The optimal (Δ, x_H) depend **linearly** on x_L .
- The problem of optimizing over x_L is linear with no constraints:

Derivation of the optimal mechanism

- We have an outside-option constraint: $\forall t \in [0, \bar{t}]$,

$$\Delta + \int_0^t x_H(\tau) d\tau \equiv U_H(t) \geq U_L(t) \equiv \int_0^t x_L(\tau) d\tau.$$

- Fixing x_L , this is a linear mechanism design problem with a **type-dependent outside-option constraint** (Jullien, 2000)
- Dworczak and Muir (2024): This problem can be solved by adapting the **ironing technique** (Myerson, 1981).
 $\implies$ The optimal (Δ, x_H) depend **linearly** on x_L .
- The problem of optimizing over x_L is linear with no constraints:
 $\implies$ Optimal x_L takes the form $x_L(t) = \bar{x}_L \cdot \mathbf{1}_{\{t \geq t_L^*\}}$.

Derivation of the optimal mechanism

- We have an outside-option constraint: $\forall t \in [0, \bar{t}]$,

$$\Delta + \int_0^t x_H(\tau) d\tau \equiv U_H(t) \geq U_L(t) \equiv \int_0^t x_L(\tau) d\tau.$$

- Fixing x_L , this is a linear mechanism design problem with a **type-dependent outside-option constraint** (Jullien, 2000)
- Dworczak and Muir (2024): This problem can be solved by adapting the **ironing technique** (Myerson, 1981).
 - $\implies$ The optimal (Δ, x_H) depend **linearly** on x_L .
- The problem of optimizing over x_L is linear with no constraints:
 - $\implies$ Optimal x_L takes the form $x_L(t) = \bar{x}_L \cdot \mathbf{1}_{\{t \geq t_L^*\}}$.
 - $\implies$ Optimal x_H has at most three steps.

Concluding Remarks

Concluding Remarks

Concluding Remarks

- We investigated the optimal **joint design** of an **income tax** and a **market** for goods.

Concluding Remarks

- We investigated the optimal **joint design** of an **income tax** and a **market** for goods.
- Income taxation is used as the only redistributive instrument under narrow assumptions...

Concluding Remarks

- We investigated the optimal **joint design** of an **income tax** and a **market** for goods.
- Income taxation is used as the only redistributive instrument under narrow assumptions... but in most cases, **inequality-aware market design** plays a role in resolving the equity-efficiency trade-off:

Concluding Remarks

- We investigated the optimal **joint design** of an **income tax** and a **market** for goods.
- Income taxation is used as the only redistributive instrument under narrow assumptions... but in most cases, **inequality-aware market design** plays a role in resolving the equity-efficiency trade-off:
 - **Non-linear pricing** (subsidizing low quality and taxing high quality) complements a redistributive income tax when high need for the good induces significant income effects;

Concluding Remarks

- We investigated the optimal **joint design** of an **income tax** and a **market** for goods.
- Income taxation is used as the only redistributive instrument under narrow assumptions... but in most cases, **inequality-aware market design** plays a role in resolving the equity-efficiency trade-off:
 - **Non-linear pricing** (subsidizing low quality and taxing high quality) complements a redistributive income tax when high need for the good induces significant income effects;
 - **Taxing or subsidizing** the good may be optimal when consuming it is informative about redistribution-worthiness;

Concluding Remarks

- We investigated the optimal **joint design** of an **income tax** and a **market** for goods.
- Income taxation is used as the only redistributive instrument under narrow assumptions... but in most cases, **inequality-aware market design** plays a role in resolving the equity-efficiency trade-off:
 - **Non-linear pricing** (subsidizing low quality and taxing high quality) complements a redistributive income tax when high need for the good induces significant income effects;
 - **Taxing or subsidizing** the good may be optimal when consuming it is informative about redistribution-worthiness;
 - **Means-tested subsidies** may sometimes be more effective than a strongly redistributive income tax.

Concluding Remarks

- This is **not** to say that all market interventions motivated by redistributive motives are justified...

Concluding Remarks

- This is **not** to say that all market interventions motivated by redistributive motives are justified...
- ... but this does mean that **traditional theoretical foundations opposing goods market distortions are insufficient.**

Concluding Remarks

- This is **not** to say that all market interventions motivated by redistributive motives are justified...
- ... but this does mean that **traditional theoretical foundations opposing goods market distortions are insufficient.**
- More research is needed on this topic!

Concluding Remarks

- This is **not** to say that all market interventions motivated by redistributive motives are justified...
- ... but this does mean that **traditional theoretical foundations opposing goods market distortions are insufficient.**
- More research is needed on this topic!

Questions?

Appendix

Appendix

Relaxing Assumption 2

- Suppose that Assumptions 1 and 3 from the AS result hold:

Relaxing Assumption 2

- Suppose that Assumptions 1 and 3 from the AS result hold:
 - $F_L(t) = F_H(t) = F(t)$, for all t ;

Relaxing Assumption 2

- Suppose that Assumptions 1 and 3 from the AS result hold:
 - $F_L(t) = F_H(t) = F(t)$, for all t ;
 - $u(c) = c$, $c \in \mathbb{R}$.

Relaxing Assumption 2

- Suppose that Assumptions 1 and 3 from the AS result hold:
 - $F_L(t) = F_H(t) = F(t)$, for all t ;
 - $u(c) = c$, $c \in \mathbb{R}$.
- Suppose that welfare weights can depend on the taste type.

Relaxing Assumption 2

- Suppose that Assumptions 1 and 3 from the AS result hold:
 - $F_L(t) = F_H(t) = F(t)$, for all t ;
 - $u(c) = c$, $c \in \mathbb{R}$.
- Suppose that welfare weights can depend on the taste type.
 - Let $\Lambda_i(t)$ be the average welfare weight on all types higher than t , conditional on $i \in \{L, H\}$.

Relaxing Assumption 2

- Suppose that Assumptions 1 and 3 from the AS result hold:
 - $F_L(t) = F_H(t) = F(t)$, for all t ;
 - $u(c) = c$, $c \in \mathbb{R}$.
- Suppose that welfare weights can depend on the taste type.
 - Let $\Lambda_i(t)$ be the average welfare weight on all types higher than t , conditional on $i \in \{L, H\}$.
 - Let $\Lambda(t) = \mu_L \Lambda_L(t) + \mu_H \Lambda_H(t)$.

Relaxing Assumption 2

- Suppose that Assumptions 1 and 3 from the AS result hold:
 - $F_L(t) = F_H(t) = F(t)$, for all t ;
 - $u(c) = c$, $c \in \mathbb{R}$.
- Suppose that welfare weights can depend on the taste type.
 - Let $\Lambda_i(t)$ be the average welfare weight on all types higher than t , conditional on $i \in \{L, H\}$.
 - Let $\Lambda(t) = \mu_L \Lambda_L(t) + \mu_H \Lambda_H(t)$.
 - Let $h(t)$ be the inverse hazard rate, and $J(t) = t - h(t)$ be the virtual surplus function associated with distribution F .

Relaxing Assumption 2

- Suppose that Assumptions 1 and 3 from the AS result hold:
 - $F_L(t) = F_H(t) = F(t)$, for all t ;
 - $u(c) = c$, $c \in \mathbb{R}$.
- Suppose that welfare weights can depend on the taste type.
 - Let $\Lambda_i(t)$ be the average welfare weight on all types higher than t , conditional on $i \in \{L, H\}$.
 - Let $\Lambda(t) = \mu_L \Lambda_L(t) + \mu_H \Lambda_H(t)$.
 - Let $h(t)$ be the inverse hazard rate, and $J(t) = t - h(t)$ be the virtual surplus function associated with distribution F .
- Assume that

Relaxing Assumption 2

- Suppose that Assumptions 1 and 3 from the AS result hold:
 - $F_L(t) = F_H(t) = F(t)$, for all t ;
 - $u(c) = c$, $c \in \mathbb{R}$.
- Suppose that welfare weights can depend on the taste type.
 - Let $\Lambda_i(t)$ be the average welfare weight on all types higher than t , conditional on $i \in \{L, H\}$.
 - Let $\Lambda(t) = \mu_L \Lambda_L(t) + \mu_H \Lambda_H(t)$.
 - Let $h(t)$ be the inverse hazard rate, and $J(t) = t - h(t)$ be the virtual surplus function associated with distribution F .
- Assume that
 - $(\Lambda_i(t)h(t) + J(t) - k)f(t)$ is non-decreasing whenever it is negative, for $i \in \{L, H\}$.

Relaxing Assumption 2

- Suppose that Assumptions 1 and 3 from the AS result hold:
 - $F_L(t) = F_H(t) = F(t)$, for all t ;
 - $u(c) = c$, $c \in \mathbb{R}$.
- Suppose that welfare weights can depend on the taste type.
 - Let $\Lambda_i(t)$ be the average welfare weight on all types higher than t , conditional on $i \in \{L, H\}$.
 - Let $\Lambda(t) = \mu_L \Lambda_L(t) + \mu_H \Lambda_H(t)$.
 - Let $h(t)$ be the inverse hazard rate, and $J(t) = t - h(t)$ be the virtual surplus function associated with distribution F .
- Assume that
 - $(\Lambda_i(t)h(t) + J(t) - k)f(t)$ is non-decreasing whenever it is negative, for $i \in \{L, H\}$.
 - $\Lambda_L(t) \geq \Lambda_H(t)$, for all t (weaker than $\lambda_L(t) \geq \lambda_H(t)$ for all t).

Relaxing Assumption 2

Parametrize the welfare weights by $\lambda_a(t) \equiv \bar{\lambda}_a \omega_a(t)$, where $\omega_a(t)$ is a function with mean 1.

Relaxing Assumption 2

Parametrize the welfare weights by $\lambda_a(t) \equiv \bar{\lambda}_a \omega_a(t)$, where $\omega_a(t)$ is a function with mean 1.

Proposition

Under regularity conditions, and fixing all parameters other than $\bar{\lambda}_L$ (while adjusting $\bar{\lambda}_H$ to keep the mean welfare weight constant at 1), there exists a cutoff value $\bar{\lambda}_L^0 \geq 1$ such that mechanism 1 is optimal if $\bar{\lambda}_L > \bar{\lambda}_L^0$ and mechanism 2 is optimal if $\bar{\lambda}_L < \bar{\lambda}_L^0$.

Relaxing Assumption 2

Parametrize the welfare weights by $\lambda_a(t) \equiv \bar{\lambda}_a \omega_a(t)$, where $\omega_a(t)$ is a function with mean 1.

Proposition

Under regularity conditions, and fixing all parameters other than $\bar{\lambda}_L$ (while adjusting $\bar{\lambda}_H$ to keep the mean welfare weight constant at 1), there exists a cutoff value $\bar{\lambda}_L^0 \geq 1$ such that mechanism 1 is optimal if $\bar{\lambda}_L > \bar{\lambda}_L^0$ and mechanism 2 is optimal if $\bar{\lambda}_L < \bar{\lambda}_L^0$.

If the designer is primarily concerned with **redistribution across ability** types, it is optimal to redistribute as much as possible through the income tax.

Relaxing Assumption 2

Parametrize the welfare weights by $\lambda_a(t) \equiv \bar{\lambda}_a \omega_a(t)$, where $\omega_a(t)$ is a function with mean 1.

Proposition

Under regularity conditions, and fixing all parameters other than $\bar{\lambda}_L$ (while adjusting $\bar{\lambda}_H$ to keep the mean welfare weight constant at 1), there exists a cutoff value $\bar{\lambda}_L^0 \geq 1$ such that mechanism 1 is optimal if $\bar{\lambda}_L > \bar{\lambda}_L^0$ and mechanism 2 is optimal if $\bar{\lambda}_L < \bar{\lambda}_L^0$.

If the designer is primarily concerned with **redistribution across ability** types, it is optimal to redistribute as much as possible through the income tax.

Otherwise, it may be optimal to limit the role of income taxes to make income useful for **price discrimination** in market design:

⇒ income tax and market design are to some extent **substitutes**.

Relaxing Assumption 3

- Suppose that Assumptions 1 and 2 from the AS result hold:

Relaxing Assumption 3

- Suppose that Assumptions 1 and 2 from the AS result hold:
 - $\lambda_i(t) = \bar{\lambda}_i$, for all t ;

Relaxing Assumption 3

- Suppose that Assumptions 1 and 2 from the AS result hold:
 - $\lambda_i(t) = \bar{\lambda}_i$, for all t ;
 - $u(c) = c$, $c \in \mathbb{R}$.

Relaxing Assumption 3

- Suppose that Assumptions 1 and 2 from the AS result hold:
 - $\lambda_i(t) = \bar{\lambda}_i$, for all t ;
 - $u(c) = c$, $c \in \mathbb{R}$.
- Suppose that $F_L(t) \neq F_H(t)$.

Relaxing Assumption 3

- Suppose that Assumptions 1 and 2 from the AS result hold:
 - $\lambda_i(t) = \bar{\lambda}_i$, for all t ;
 - $u(c) = c$, $c \in \mathbb{R}$.
- Suppose that $F_L(t) \neq F_H(t)$.
- Let $\Lambda(t)$ be the average welfare weight on all types higher than t :

$$\Lambda(p) = \frac{\bar{\lambda}_L \mu_L(1 - F_L(p)) + \bar{\lambda}_H \mu_H(1 - F_H(p))}{\mu_L(1 - F_L(p)) + \mu_H(1 - F_H(p))}.$$

Relaxing Assumption 3

- Suppose that Assumptions 1 and 2 from the AS result hold:
 - $\lambda_i(t) = \bar{\lambda}_i$, for all t ;
 - $u(c) = c$, $c \in \mathbb{R}$.
- Suppose that $F_L(t) \neq F_H(t)$.
- Let $\Lambda(t)$ be the average welfare weight on all types higher than t :

$$\Lambda(p) = \frac{\bar{\lambda}_L \mu_L(1 - F_L(p)) + \bar{\lambda}_H \mu_H(1 - F_H(p))}{\mu_L(1 - F_L(p)) + \mu_H(1 - F_H(p))}.$$

- Assume that

Relaxing Assumption 3

- Suppose that Assumptions 1 and 2 from the AS result hold:
 - $\lambda_i(t) = \bar{\lambda}_i$, for all t ;
 - $u(c) = c$, $c \in \mathbb{R}$.
- Suppose that $F_L(t) \neq F_H(t)$.
- Let $\Lambda(t)$ be the average welfare weight on all types higher than t :

$$\Lambda(p) = \frac{\bar{\lambda}_L \mu_L(1 - F_L(p)) + \bar{\lambda}_H \mu_H(1 - F_H(p))}{\mu_L(1 - F_L(p)) + \mu_H(1 - F_H(p))}.$$

- Assume that
 - $(t - (1 - \bar{\lambda}_H)h_H(t) - k) f_H(t)$ is increasing when negative;

Relaxing Assumption 3

- Suppose that Assumptions 1 and 2 from the AS result hold:
 - $\lambda_i(t) = \bar{\lambda}_i$, for all t ;
 - $u(c) = c$, $c \in \mathbb{R}$.
- Suppose that $F_L(t) \neq F_H(t)$.
- Let $\Lambda(t)$ be the average welfare weight on all types higher than t :

$$\Lambda(p) = \frac{\bar{\lambda}_L \mu_L(1 - F_L(p)) + \bar{\lambda}_H \mu_H(1 - F_H(p))}{\mu_L(1 - F_L(p)) + \mu_H(1 - F_H(p))}.$$

- Assume that
 - $(t - (1 - \bar{\lambda}_H)h_H(t) - k) f_H(t)$ is increasing when negative;
 - $(t - (1 - \bar{\lambda}_L)h_L(t) - k) f_L(t)$ crosses zero once from below;

Relaxing Assumption 3

- Suppose that Assumptions 1 and 2 from the AS result hold:
 - $\lambda_i(t) = \bar{\lambda}_i$, for all t ;
 - $u(c) = c$, $c \in \mathbb{R}$.
- Suppose that $F_L(t) \neq F_H(t)$.
- Let $\Lambda(t)$ be the average welfare weight on all types higher than t :

$$\Lambda(p) = \frac{\bar{\lambda}_L \mu_L(1 - F_L(p)) + \bar{\lambda}_H \mu_H(1 - F_H(p))}{\mu_L(1 - F_L(p)) + \mu_H(1 - F_H(p))}.$$

- Assume that
 - $(t - (1 - \bar{\lambda}_H)h_H(t) - k) f_H(t)$ is increasing when negative;
 - $(t - (1 - \bar{\lambda}_L)h_L(t) - k) f_L(t)$ crosses zero once from below;
 - $(\bar{\lambda}_L - 1)h_L(r) \geq (\bar{\lambda}_H - 1)h_H(r)$, for all t .

Relaxing Assumption 1

Robustness of results to concavity in the utility function u :

- Suppose that u is a strictly increasing, strictly concave, continuously differentiable function.

Relaxing Assumption 1

Robustness of results to concavity in the utility function u :

- Suppose that u is a strictly increasing, strictly concave, continuously differentiable function.
- We prove that:

Relaxing Assumption 1

Robustness of results to concavity in the utility function u :

- Suppose that u is a strictly increasing, strictly concave, continuously differentiable function.
- We prove that:
 - Any optimal mechanism distorts the goods allocation away from efficiency;

Relaxing Assumption 1

Robustness of results to concavity in the utility function u :

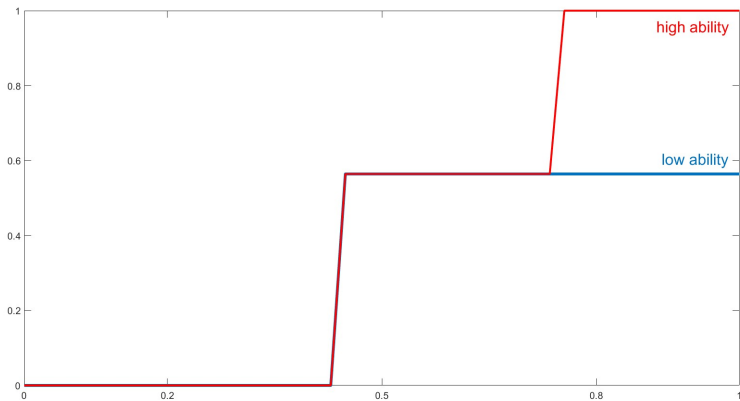
- Suppose that u is a strictly increasing, strictly concave, continuously differentiable function.
- We prove that:
 - Any optimal mechanism distorts the goods allocation away from efficiency;
 - The consumption of low-ability agents is distorted upwards;

Relaxing Assumption 1

Robustness of results to concavity in the utility function u :

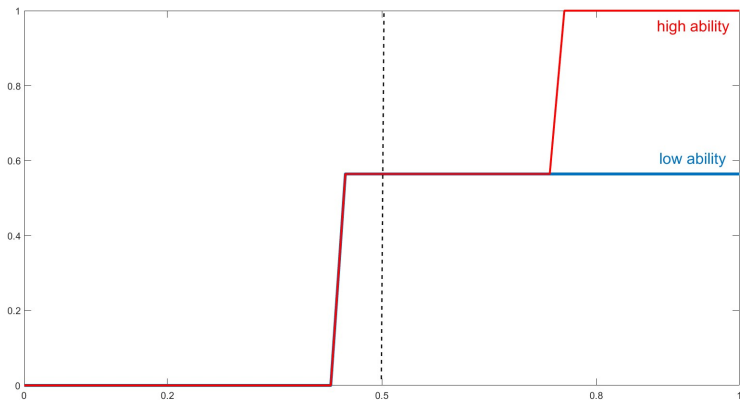
- Suppose that u is a strictly increasing, strictly concave, continuously differentiable function.
- We prove that:
 - Any optimal mechanism distorts the goods allocation away from efficiency;
 - The consumption of low-ability agents is distorted upwards;
 - The consumption of high-ability agents with high enough taste type is distorted downwards.

Relaxing Assumption 1



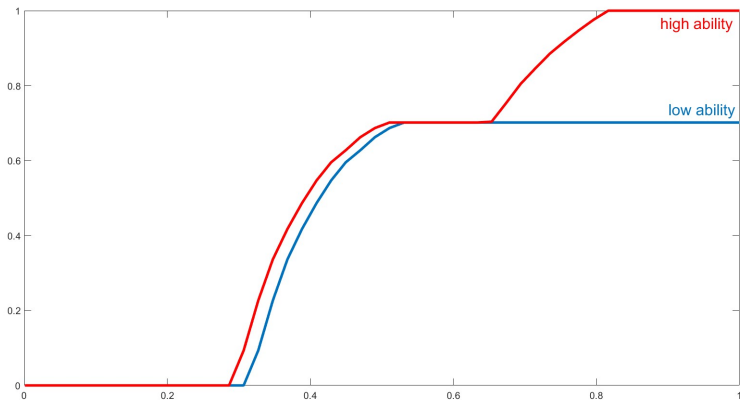
Goods allocation in optimal mechanism

Relaxing Assumption 1



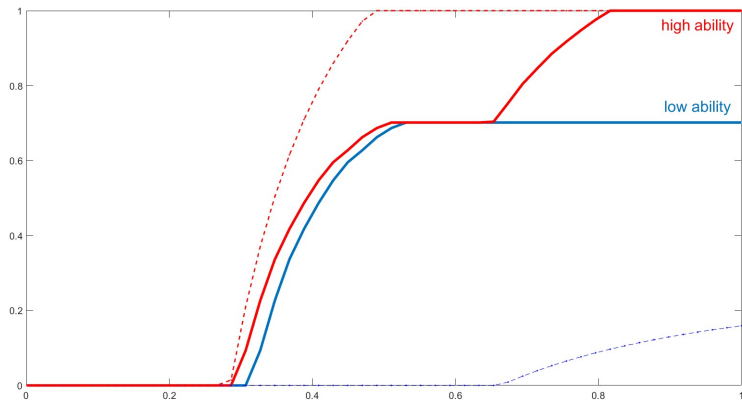
Goods allocation in optimal mechanism

Relaxing Assumption 1



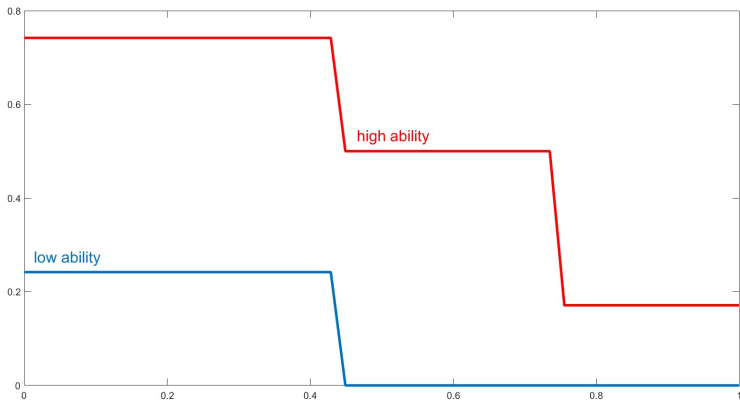
Goods allocation in optimal mechanism ($u(c) = \sqrt{c}$)

Relaxing Assumption 1



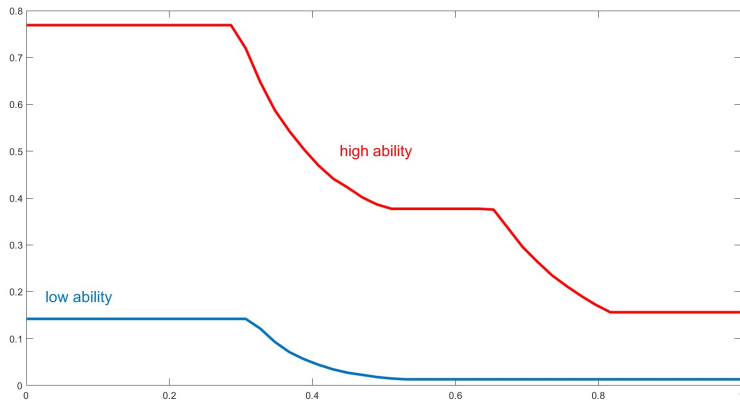
Goods allocation in optimal mechanism ($u(c) = \sqrt{c}$)

Relaxing Assumption 1



Numeraire consumption in optimal mechanism

Relaxing Assumption 1



Numeraire consumption in optimal mechanism ($u(c) = \sqrt{c}$)