

A mechanism-design approach to property rights

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- Example: How to optimally design **spectrum licenses**?

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- Key modeling idea: Designer **doesn't control the allocation** directly but she **can design an agent's outside option**.
- This perspective allows us to characterize the **optimal property right** using a mechanism-design approach.
- Result: The optimal property right is simple but more flexible than a conventional property right, often featuring an **option to own**.

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- Property rights affect **investment incentives**:
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- Attractive properties of options to own were recognized.
- We provide an **optimality foundation** (in a “refreshed” framework, allowing for market-design applications).

Other related papers

- **Allocation mechanisms versus efficient investment:** Rogerson (1992), Bergemann and Välimäki (2002), Hatfield, Kojima, and Kominers (2019), Gershkov, Moldovanu, Strack, and Zhang (2021), Akbarpour, Kominers, Li, Li, and Milgrom (2023), ...
- **Related techniques:** Jullien (2000), Kleiner, Moldovanu, and Strack (2021), Loertscher and Muir (2022), Kang (2023), Akbarpour [®] Dworczak [®] Kominers (2024), Valenzuela-Stookey (2025), Augias and Uhe (2025), ...
- **Optimal design of contractual environment:** Hermalin and Katz (1993), Bhaskar, McClellan and Sadler (2023), Hitzig and Niswonger (2023), Thereze and Vaidaya (2025), Lüelfesmann (2025), ...
- **Spectrum license design:** Posner and Weyl (2017), Milgrom, Weyl and Zhang (2017), Weyl and Zhang (2017), ...

Outline

- ➊ Simple model (without investment)
- ➋ Main result and proof outline
- ➌ Full model (with investment)
- ➍ Applications

Model

Simple model (without investment)

Model

Standard contracting problem between principal and agent:

- **Agent's** private type $\theta \in \Theta := [\underline{\theta}, \bar{\theta}] \subset \mathbb{R}$ is drawn from a distribution F with continuous positive density f .
- **Principal** chooses a trading mechanism $\langle x, t \rangle$ subject to IC and IR, where $x : [\underline{\theta}, \bar{\theta}] \rightarrow [0, 1]$ denotes an allocation rule, and $t : [\underline{\theta}, \bar{\theta}] \rightarrow \mathbb{R}$ denotes a transfer rule.
- Agent's utility is $\theta \cdot x - t$.
- Principal maximizes $V(\theta) \cdot x + \alpha \cdot t$, where $\alpha > 0$.

Model

Running example

- Principal is a **seller**, agent is a **buyer** of an indivisible good.
- Type θ is the buyer's **value**.
- Seller has opportunity **cost** c and maximizes **revenue** ($V(\theta) = -c$ and $\alpha = 1$).

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$$M = \{(x_i, t_i)\}_{i \in I},$$

where $x_i \in [0, 1]$, $t_i \in \mathbb{R}$, and set I is arbitrary.

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- Agent can “execute” any one of these rights in lieu of participating in mechanism.
- Designer's payoff is given by $V^*(\theta) \cdot x + \alpha^* \cdot t$, where $\alpha^* \geq 0$.

Model

Running example

- Designer maximizes **weighted sum of seller's and buyer's payoffs**.
- $V^*(\theta) = \lambda\theta - c$ and $\alpha^* = 1 - \lambda$, where $\lambda \in [0, 1]$ is the weight on the buyer.

Comments about the model

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- Renewable lease/option to own: $M = \{(1, p)\}$
- Partial property right: $M = \{(y, 0)\}$, where $y \in (0, 1)$
- Flexible property right: $M = \{(s, p(s))\}_{s \in [0, 1]}$

Comments about the model

- **Two key frictions:** Non-commitment and private information



- **Non-commitment friction** is fundamental challenge that property rights address.
- Private information avoids the **Coasian irrelevance**.

Comments about the model



Running example

- $\lambda = 0$: Designer optimally offers no rights to buyer (non-commitment friction turned off)
- $\lambda = 1$: Designer commits seller to efficient mechanism by assigning to the buyer an option to own with price $p = c$ (private information friction effectively turned off)
- $\lambda \in (0, 1)$: Both frictions are present and interact (optimal property right non-trivial)

Main result

Main result and proof outline

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Running example

The optimal property right is an **option to buy the good from the seller** at price p (chosen by the designer).

Proof outline

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Step 1: Outside option constraint

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Step 1: Outside option constraint

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A choice of menu M by the designer is equivalent to choosing an outside option function $R(\theta)$ for the agent in the mechanism, where $R(\theta)$ is non-negative, non-decreasing, convex, and has slope bounded above by 1.

- This lemma follows immediately from the observation that, for any M , we can set

$$R(\theta) = \max\{0, \max_{i \in I} \{x_i \theta - t_i\}\}.$$

- Implication: The designer **maximizes over set of increasing, convex type-dependent outside options** for the agent.

Step 2: Principal's problem

The principal solves

$$\max_{x: \Theta \rightarrow [0,1], \underline{u} \geq 0} \int_{\underline{\theta}}^{\bar{\theta}} W(\theta) x(\theta) d\theta - \alpha \underline{u}$$

s.t. x is non-decreasing;

$$U(\theta) := \underline{u} + \int_{\underline{\theta}}^{\theta} x(\tau) d\tau \geq R(\theta), \quad \forall \theta \in \Theta,$$

where

$$W(\theta) := \left[V(\theta) + \alpha \left(\theta - \frac{1 - F(\theta)}{f(\theta)} \right) \right] f(\theta).$$

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where

$$W(\theta) := \left[V(\theta) + \alpha \left(\theta - \frac{1 - F(\theta)}{f(\theta)} \right) \right] f(\theta).$$

This is an instance of the problem of Jullien (2000).

Step 3: Ironing Jullien (2000)

$$W(\theta)$$

$$\downarrow$$

$$\mathcal{W}(\theta) := \int_{\theta}^{\bar{\theta}} W(\tau) dF(\tau)$$

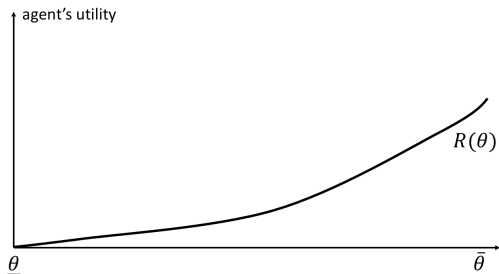
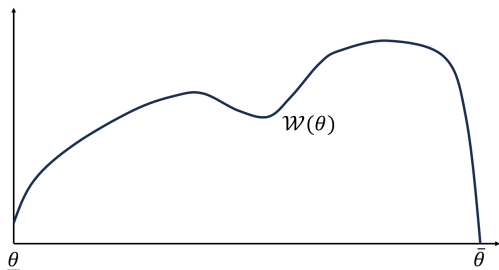
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$$\overline{\mathcal{W}} := \text{co}(\mathcal{W})$$

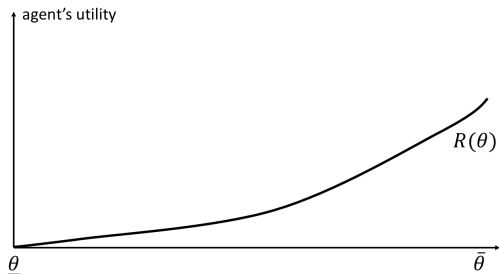
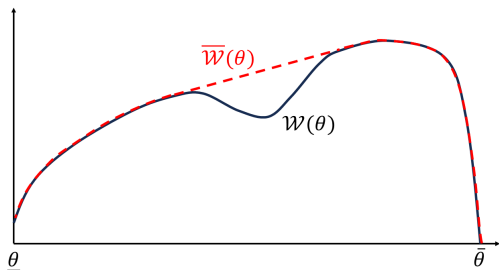
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$$\overline{W}(\theta) := -\frac{d}{d\theta} [\text{co}(\mathcal{W})(\theta)]$$

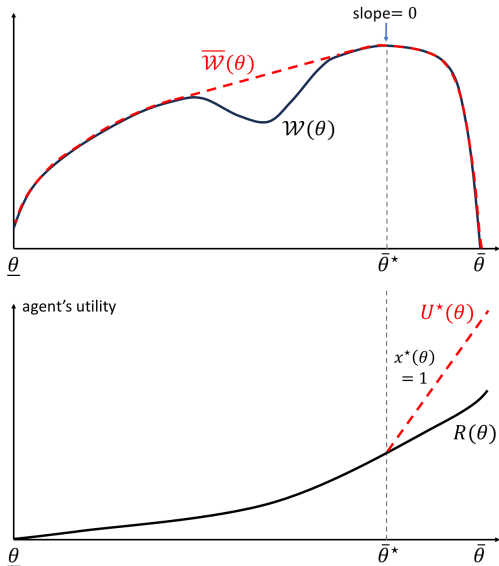
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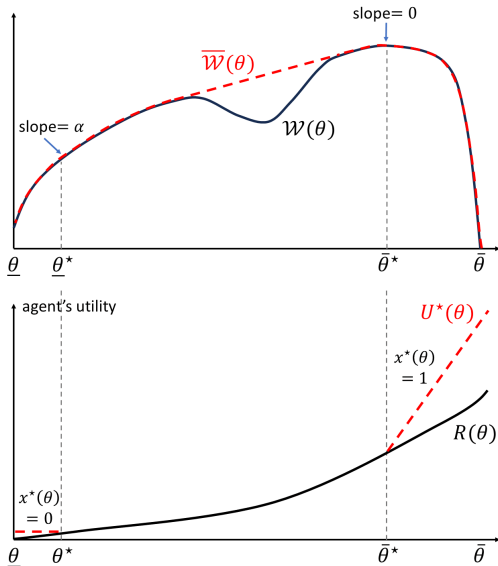
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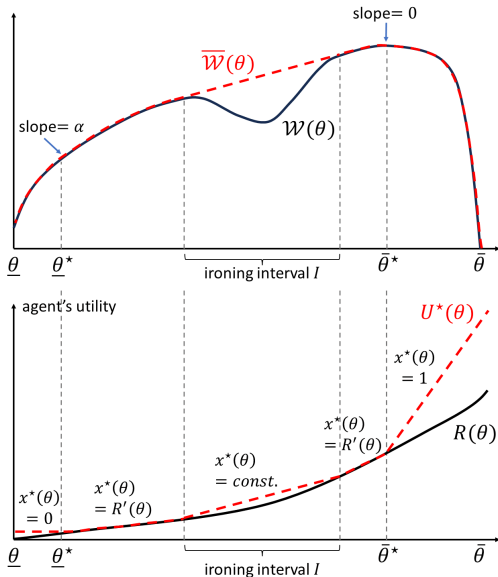
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- Why does ironing work?
- The outside option constraint is:

$$U(\theta) \geq R(\theta) \iff \underline{u} + \int_{\underline{\theta}}^{\theta} x(\tau) d\tau \geq \underline{u}_0 + \int_{\underline{\theta}}^{\theta} x_0(\tau) d\tau.$$

- This is (almost) a **second-order stochastic dominance** constraint if x and x_0 are treated as CDFs.
- Ironing corresponds to taking a mean-preserving spread of the distribution—it preserves the outside-option constraint.
- Related work: Kleiner, Moldovanu, and Strack (2021), Loertscher and Muir (2022), Akbarpour[®] Dworczak[®] Kominers (2024).

Step 4: Linearity of the designer's problem

We now go back to the designer's problem...

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- **Intuition:** Ironing procedure identifies a set of types such that:
 - the outside-option constraint holds with equality on that set;
 - all other outside-option constraints can be relaxed;
 - the set does not depend on R !
- The designer's objective is **linear** in x^* .

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- Extreme points of the feasible set of outside-option functions are precisely **options to own**.

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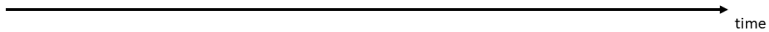
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- A linear maximization problem on a convex set admits a solution that is an **extreme point** of the feasible set.
- Extreme points of the feasible set of outside-option functions are precisely **options to own**.
- This gives us the form of the optimal menu. □

Model

Model with investment

Timeline



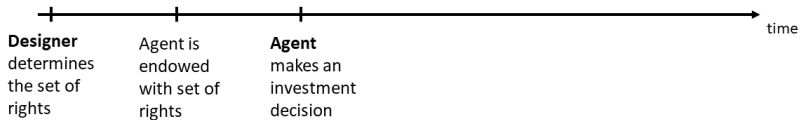
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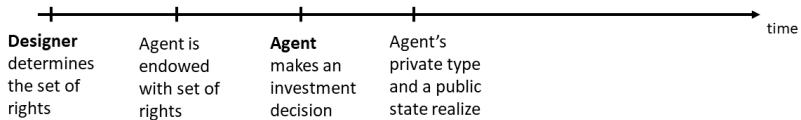
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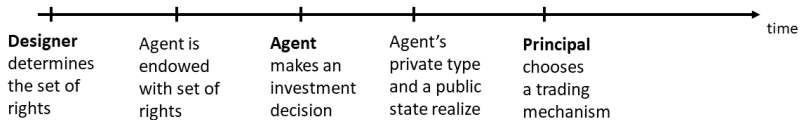
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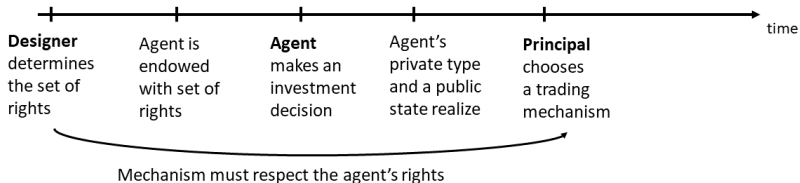
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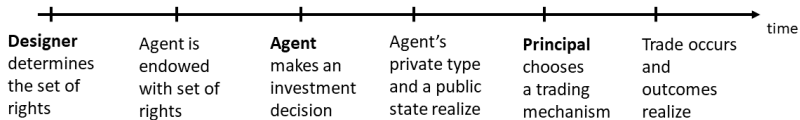
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Key model changes

- Agent makes binary **investment decision**.
 - Investing is associated with (sunk) cost $c > 0$.
 - Investment decision determines joint distribution of agent's type θ and public state ω .
 - If agent invests, public state is drawn from distribution G ; otherwise this distribution is $\underline{G}$.
 - Agent's type is drawn from conditional distribution F_ω .
 - Investment increases agent's type in FOSD sense.

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- Ex-ante uncertainty about the state:
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- Ex-ante uncertainty about the state:
 - A **public state** $\omega \in \Omega$ (finite) is realized before principal selects trading mechanism.
 - Designer and principal's payoffs depend on ω .
- Property rights cannot condition on realization state ω (**contractual incompleteness**).
- Property rights *can* condition on a partially informative **signal**.
- The contractible state $s \in \mathcal{S}$ is given by $s = S(\omega)$.
 - $S : \Omega \rightarrow \mathcal{S}$ denotes a deterministic function that pins down realized s as function of ω .

Key model changes

- We assume that the designer wants to induce investment (and that it is feasible to do so).
- Whether property rights are contingent on investment depends on contractible state:
 - If $S(\text{supp}(G)) \cap S(\text{supp}(\underline{G})) = \emptyset$, then **investment is contractible**.
 - If $S(\omega)$ has the same distribution under G and $\underline{G}$, then **investment is non-contractible**.

Main result

Theorem

There exists an optimal menu that takes the form

$M^ = \{(1, p_s), (y_s, p'_s)\}$ for some $p_s, p'_s \in \mathbb{R}$ and $y_s \in [0, 1)$.*

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There exists an optimal menu that takes the form
 $M^* = \{(1, p_s), (y_s, p'_s)\}$ *for some* $p_s, p'_s \in \mathbb{R}$ *and* $y_s \in [0, 1)$.

Comments:

- The optimal menu now consists of at most **two types of rights**.
- One right is an **option to own**.

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Comments:

- The optimal menu now consists of at most **two types of rights**.
- One right is an **option to own**.
- Proof: Same argument as before but designer now faces additional constraint that is **linear** in outside-option function R .

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 - Satisfying FOC at the target investment level e^* is sufficient for obedience.
 - FOC is linear in $R(\theta) \implies$ we need two options in the optimal menu.

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If investment is **non-contractible**, there exists an optimal menu in which

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with $y_s > 0$, for each $s \in S$.

If investment is **contractible**, there exists an optimal menu in which

$$M_s^* = \begin{cases} \{(1, p_s), (0, -T)\} & s \in S(\text{supp}(G)), \\ \emptyset & s \in S(\text{supp}(\underline{G})), \end{cases}$$

for some $T > 0$.

Prices under regularity

We can say more about optimal prices under additional assumptions.

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Assume that:

- Buyer and seller virtual surpluses are **monotone**:

$$B_{\omega}(\theta) \equiv \theta - \frac{1 - F_{\omega}(\theta)}{f_{\omega}(\theta)}$$

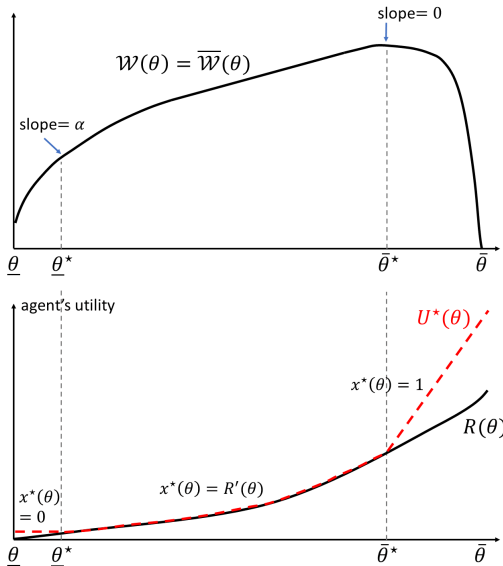
and

$$S_{\omega}(\theta) \equiv \theta + \frac{F_{\omega}(\theta)}{f_{\omega}(\theta)}$$

are non-decreasing in θ ;

- Both the **designer's and the principal's objective** functions $V_{\omega}^*(\theta)$ and $V_{\omega}(\theta)$ are **non-decreasing** in the agent's type θ .

Prices under regularity



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Lemma (The monotone case)

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- For $\theta \leq \underline{\theta}_\omega^*$, the principal prefers to “**buy out**” the rights using a monetary payment, so the constraint is again slack.

Prices under regularity

If investment-obedience constraint is slack and $\alpha^* = 0$ (dropped dependence on s to simplify exposition):

$$\mathbb{E}_{\omega \sim G} \left[V_{\omega}^*(p) f_{\omega}(p) \mid p \in [\underline{\theta}_{\omega}^*, \bar{\theta}_{\omega}^*] \right] = 0.$$

Prices under regularity

If investment-obedience constraint is slack and $\alpha^* > 0$: (dropped dependence on s to simplify exposition):

$$\begin{aligned} \mathbb{E}_{\omega \sim G} \left[(V_{\omega}^*(p) + \alpha^* B_{\omega}(p)) f_{\omega}(p) \mathbf{1}_{p \in [\underline{\theta}_{\omega}^*, \bar{\theta}_{\omega}^*]} \right] \\ - \alpha^* \mathbb{P}_{\omega \sim G}(p < \underline{\theta}_{\omega}^*) = 0. \end{aligned}$$

Prices under regularity

If investment is contractible and $\alpha^* > 0$, then the price in the option-to-own offered **on top of** the cash payment satisfies:

$$\mathbb{E}_{\omega \sim G} \left[(V_{\omega}^*(p) + \alpha^* p) f_{\omega}(p) \mid p \in [\underline{\theta}_{\omega}^*, \bar{\theta}_{\omega}^*] \right] = 0.$$

Prices under regularity

If investment is non-contractible and $\alpha^* > 0$, then with γ denoting the Lagrange multiplier on the investment constraint:

$$\begin{aligned} \mathbb{E}_{\omega \sim G} \left[(V_{\omega}^*(p) + \alpha^* B_{\omega}(p)) f_{\omega}(p) \mathbf{1}_{p \in [\underline{\theta}_{\omega}^*, \bar{\theta}_{\omega}^*]} \right] - \alpha^* \mathbb{P}_{\omega \sim G}(p < \underline{\theta}_{\omega}^*) \\ + \gamma \left(\mathbb{E}_{\omega \sim G - \underline{G}} \left[(1 - F_{\omega}(p)) \mathbf{1}_{p \in [\underline{\theta}_{\omega}^*, \bar{\theta}_{\omega}^*]} \right] \right) \\ + \gamma \left(\mathbb{P}_{\omega \sim G}(p < \underline{\theta}_{\omega}^*) - \mathbb{P}_{\omega \sim \underline{G}}(p < \underline{\theta}_{\omega}^*) \right) = 0. \end{aligned}$$

Applications

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We look at several **applications**:

① Dynamic resource allocation;

► Details

② Regulating a private rental market;

► Details

③ Optimal procurement contract;

► Details

④ Patent policy;

► Details

⑤ Supply chain contracting.

► Details

Conclusions

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- Optimal **allocation of contracts** at $t = 0$ to multiple agents?

A mechanism-design approach to property rights

Questions?

Appendix

Appendix

Application #1: Dynamic resource allocation

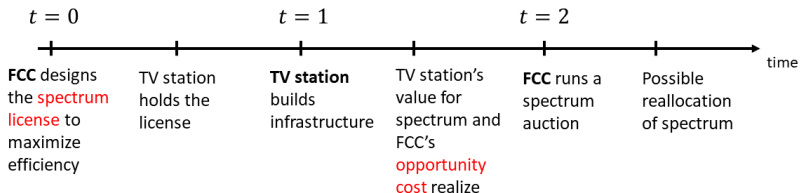
- Players:
 - Agent: Firm
 - Designer: Regulator
 - Principal: Regulator
- Agent invests in infrastructure determining value θ ;
- State ω represents the value for an alternative use;
- The designer maximizes a combination of efficiency and revenue:

$$V_{\omega}^*(\theta)x + \alpha^*t = (\theta - \omega)x + \alpha^*t.$$

- The principal might put more weight on revenue:

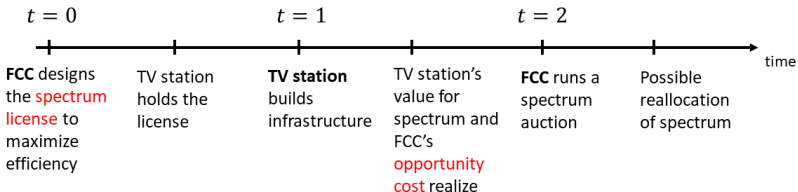
$$V_{\omega}(\theta)x + \alpha t = (\theta - \omega)x + \alpha t, \text{ where } \alpha \geq \alpha^*.$$

Application #1: Dynamic resource allocation



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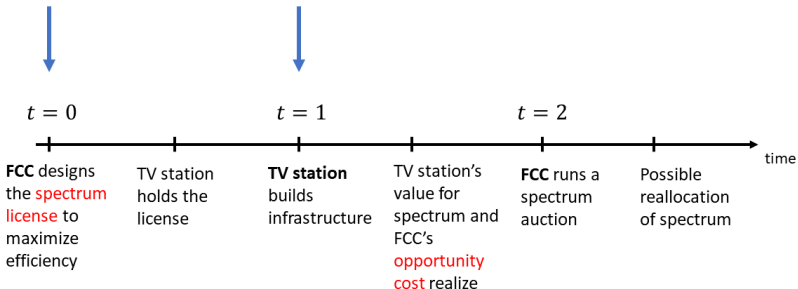
Contractual
incompleteness



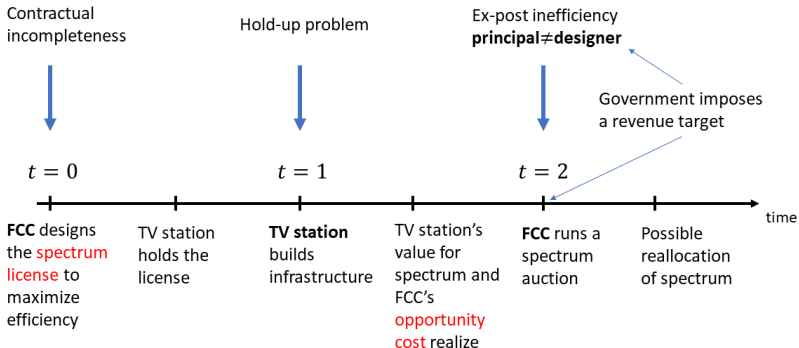
Application #1: Dynamic resource allocation

Contractual
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Hold-up problem



Application #1: Dynamic resource allocation



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(consistent with Rogerson (1992))

Application #2: Regulating a private rental market

- Players:
 - Agent: Tenant
 - Designer: Regulator
 - Principal: Rental company
- Agent invests in the property determining her value θ for staying.
- The state ω is the seller's outside option (market rental price).
- The seller wants to maximize revenue:
$$V_{\omega}(\theta)x + \alpha t = -\omega x + t.$$
- The designer wants to maximize efficiency:
$$V_{\omega}^*(\theta)x + \alpha^* t = (\theta - \omega)x.$$

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where γ is the Lagrange multiplier on the investment constraint.

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where γ is the Lagrange multiplier on the investment constraint.

- If investment constraint is slack, $p^* = \omega$, so the designer “forces” the seller to use a VCG mechanism.
- If ω is random, then (assuming interior solution)

$$p^* = \mathbb{E}[\omega \mid \underline{\theta}_\omega^* \leq p^* \leq \bar{\theta}_\omega^*] - \gamma\Delta.$$

Application #3: Vaccine development

- Players:
 - Agent: Pharmaceutical company
 - Designer: Health agency
 - Principal: Health agency
- Company develops a vaccine; the marginal cost of production conditional on investment is $k \sim \tilde{F}(\theta = -k)$;
- Allocation x is the number of units purchased by the health agency, with 1 being the mass of the patient population;
- Health agency maximizes the total value of allocation,
 $V_{\omega}^*(\theta)x = V_{\omega}(\theta)x = \omega x$,
where ω measures the severity of the health crisis.
- Assume that $1 = \alpha \geq \alpha^*$.

Application #3: Vaccine development

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- If the designer cares about revenue as much as the principal, she sets a price equal to ω , **as if** she maximized efficiency.
- For α^* close enough to 0, the optimal contract is a guaranteed sale (for all producer types).

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- If ω is stochastic, then the optimal price satisfies

$$p^* = \min \left\{ \frac{\mathbb{E} [\omega | \omega \in [\underline{\omega}_{p^*}, \bar{\omega}_{p^*}]]}{\alpha^*}, \bar{k} \right\},$$

where $[\underline{\omega}_{p^*}, \bar{\omega}_{p^*}]$ is the interval of ω 's for which the choice of p^* changes the second-period mechanism.

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- If the realized ω is high (health crisis is severe), the principal will offer a price better than p^* .
- If the realized ω is low (health crisis is mild), the principal will offer a price lower than p^* and compensate the producer with an additional cash payment (on top of the payment for investment).

Application #4: Patent policy

- Players:
 - Agent: Firm
 - Designer: Regulator
 - Principal: Patent agency
- Firm invests in a new technology; if investment is made, the firm can produce at constant marginal cost $k \sim \tilde{F}$.
- Conditional on $x = 1$, the firm provides a monopoly quantity to maximize profits; conditional on $x = 0$, the firm competes in a competitive market. Market demand is $D(p) = 1 - p$.
- The regulator and the patent agency maximize consumer surplus with weight ω (but attach a positive weight to revenue).

Application #4: Patent policy

- This setting maps into our framework with $\theta = \frac{1}{4}(1 - k)^2$ and

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- Perfect competition is more beneficial when marginal cost is low.
- Suppose that the density of θ is non-decreasing, and the weight on revenue α is small enough.
- Then, an optimal policy is to offer full patent protection to the invention ($x = 1$) with exogenous probability $y \in (0, 1]$ at no payment.

Application #5: Supply chain contracting

- Players:
 - Agent: Small supplier
 - Designer: Large producer
 - Principal: Large producer
- Producer buys some amount x of inputs from the supplier.
- The supplier must invest at time $t = 1$ in relationship-specific technology to produce the inputs at marginal cost $c \equiv -\theta$.
- Through the interaction with the supplier, the large firm can learn the supplier's costs: The state ω is a noisy signal of θ .
- Producer maximizes profits having a constant marginal value 1 for each unit of the input: $V_{\omega}(\theta) = 1$ and $\alpha = 1$.
- Producer proposes a contract: $V_{\omega}^*(\theta) = 1$ and $\alpha^* = 1$.

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- The producer will in general commit to a **two-price scheme**, committing to buy up to y units at some price p_H , and any number of units at some lower price p_L .
- If investment by the small supplier is **contractible**, assuming the cost of investment is high enough, the producer will offer an **upfront payment** for setting up production and a **guaranteed purchase price** for any number of units.