Searchable Menus

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- Optimal mechanisms may have **counterintuitive** properties (e.g., Hart and Reny, 2015).
- “Simple” mechanisms are generally **poor approximations** of fully optimal ones (e.g., Hart and Nisan, 2019).
- **Final nail in the coffin?** (Lahr and Niemeyer, 2025): The set of mechanisms uniquely optimal for some type distribution is **dense** in all IC and IR mechanisms—“anything goes.”

Multidimensional screening is important...

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- We require that the mechanism makes the agent's problem of choosing the optimal option **simple**.
- Specifically, the mechanism is an **ordered menu** where, for every type of the agent, **local maxima are global maxima**.
- Intuitively, **greedy search** leads to globally optimal behavior by the agent ($\Longrightarrow$ **searchable menu**).

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  - multi-product monopolist;
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- **Extension**: a more permissive notion of **grid-searchability**.

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- **Searchability and unrestricted optimality** need not conflict:

  Haghpanah and Hartline (2021), Bergemann, Bonatti, Haupt and Smolin (2021), Ghili (2023), Yang (2025, 2026).

Model

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- Each type θ has utility $u(x, \theta)$ over outcomes $x \in X$.
- **Outside option** $\underline{x} \in X$ with $u(\underline{x}, \theta) = 0$ for all θ.

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- **IC** and **IR** defined as usual.
- Mechanism x is **implemented** by a menu $M \subseteq X$ (always assumed to contain the outside option) if, for each θ,

$$x(\theta) \in \operatorname{argmax}_{m \in M} \{ u(m, \theta) \}.$$

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- (Feel free to think about finite menus.)

Searchable menu

- Interval $[\underline{s}, \overline{s}] \subset S$ is a **neighborhood** of $s \in S$ if either

$$\underline{s} < s < \overline{s}, \quad \underline{s} < s = \overline{s} = \max S, \quad \text{or} \quad \min S = \underline{s} = s < \overline{s}.$$

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Intuition: "greedy search" finds the agent's optimal choice (can be formalized by one-step dominance of Pycia and Troyan, 2023). Details

No bite in one-dimensional environments

- Outcome space $X \subseteq \mathbb{R}^2$ with $(q, p) \in X$, and type space $\Theta \subseteq \mathbb{R}$;
- $u((q, p), \theta)$ is strictly decreasing in p;
- **Single-crossing:** for $q < q'$, $\theta < \theta'$, $(q, p), (q', p') \in X$,

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- Then, each type's utility is **semi-strictly quasi-concave** on the menu (strictly increasing, flat, then strictly decreasing). $\square$

Results

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- **Incremental nesting condition** (INC):

$$H^{k+1} \subseteq H^k \text{ and } G^{k+1} \subseteq G^k, \forall k.$$

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- Number of **tiers** $= |M| - 1$.

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- Upgrade menus are **undominated** (Manelli and Vincent, 2007).

Multi-product monopoly **(proof sketch)**

- In a payoff type space, searchability $\iff$ WINC:

$$H^{k+1} \subseteq H^k, \forall k,$$

where $H^k = \{\theta : u(x^{k-1}, \theta) \leq u(x^k, \theta)\}$ (and $H^1 = \Theta$).

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- Define indifference regions:

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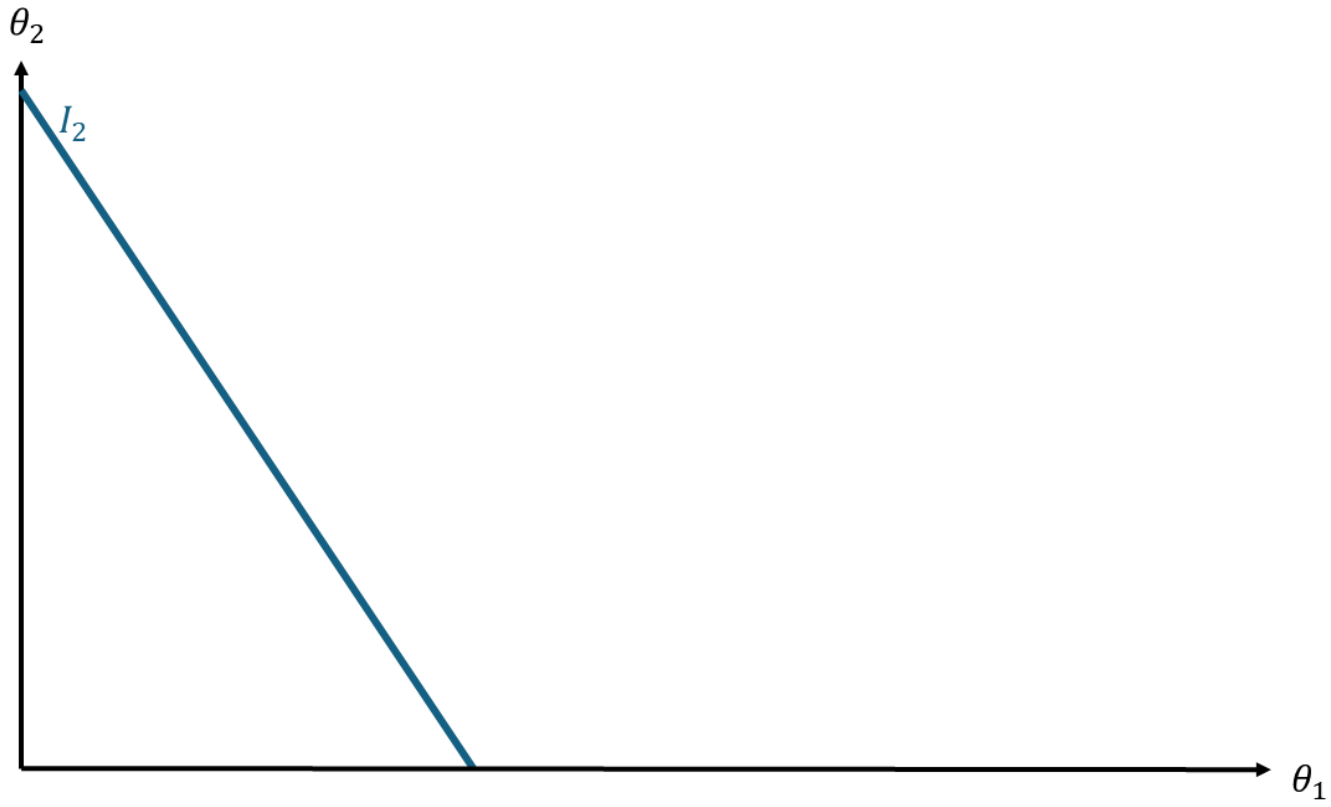

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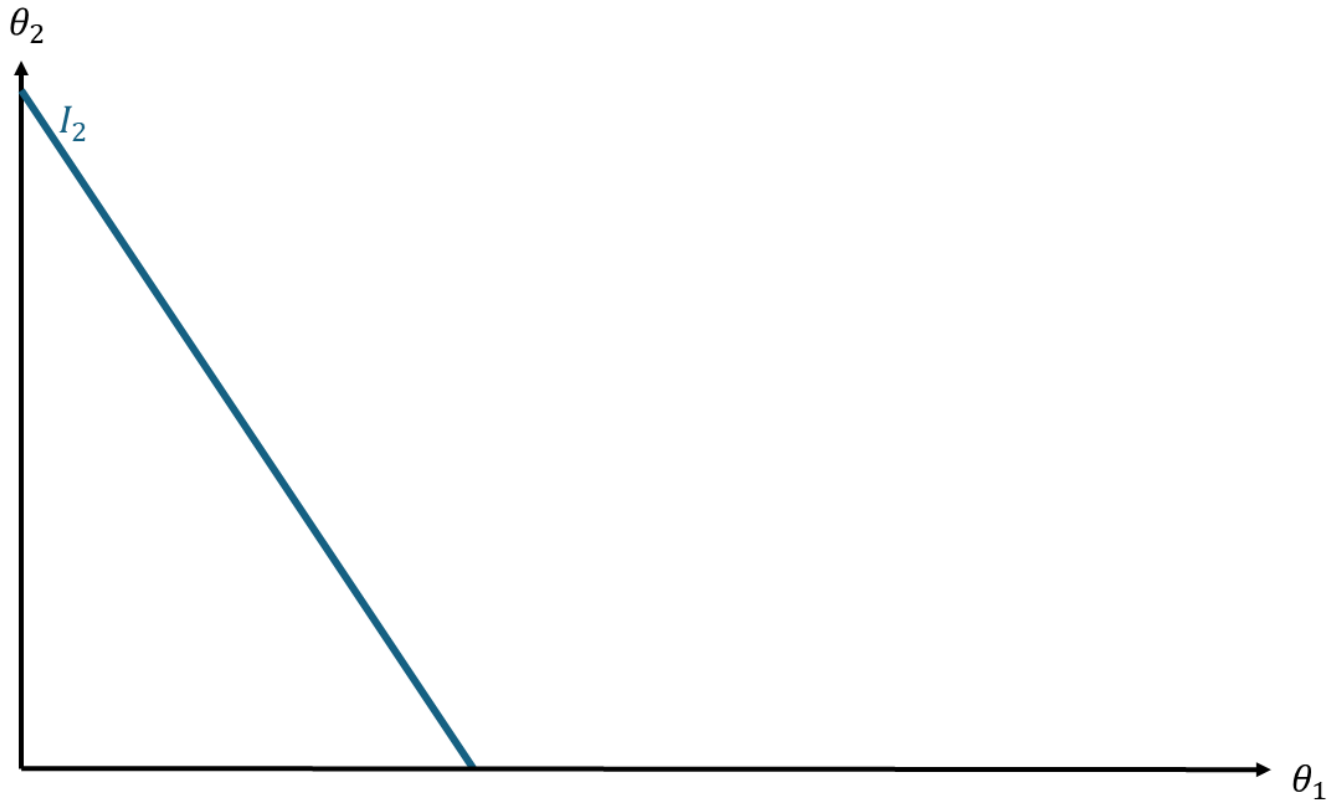

Let $\Delta q^k = q^k - q^{k-1}$ and $\Delta p^k = p^k - p^{k-1}$.

$$I^k = \{\theta : \theta \cdot \Delta q^k = \Delta p^k\}$$

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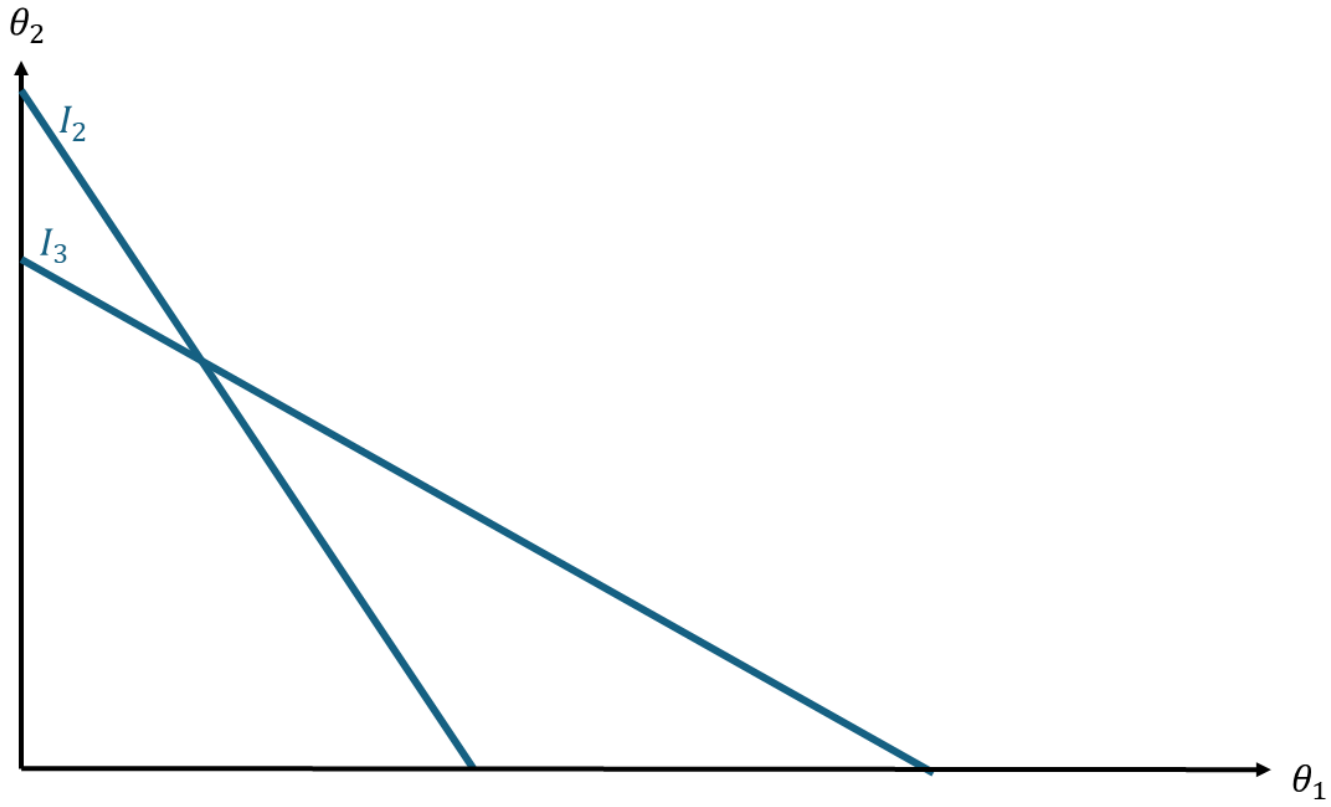

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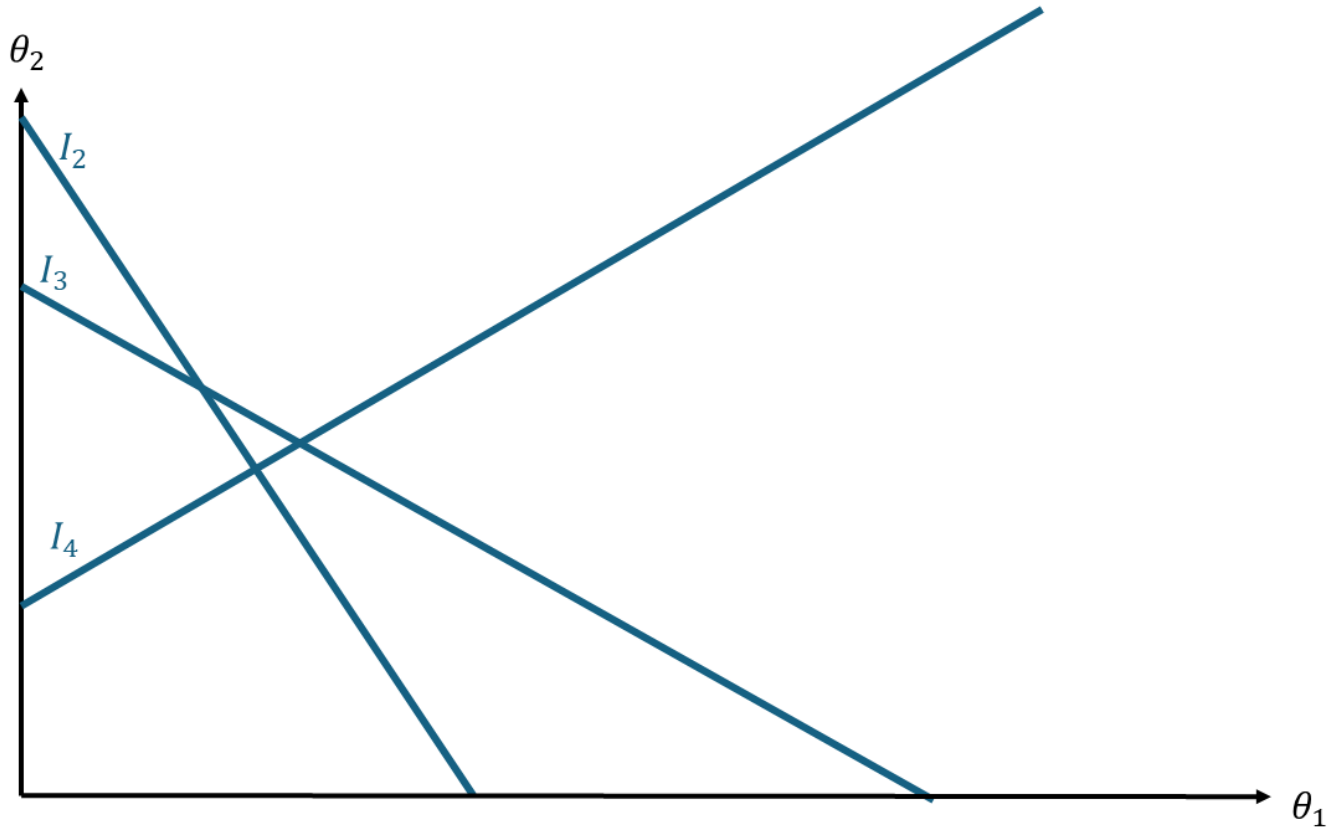

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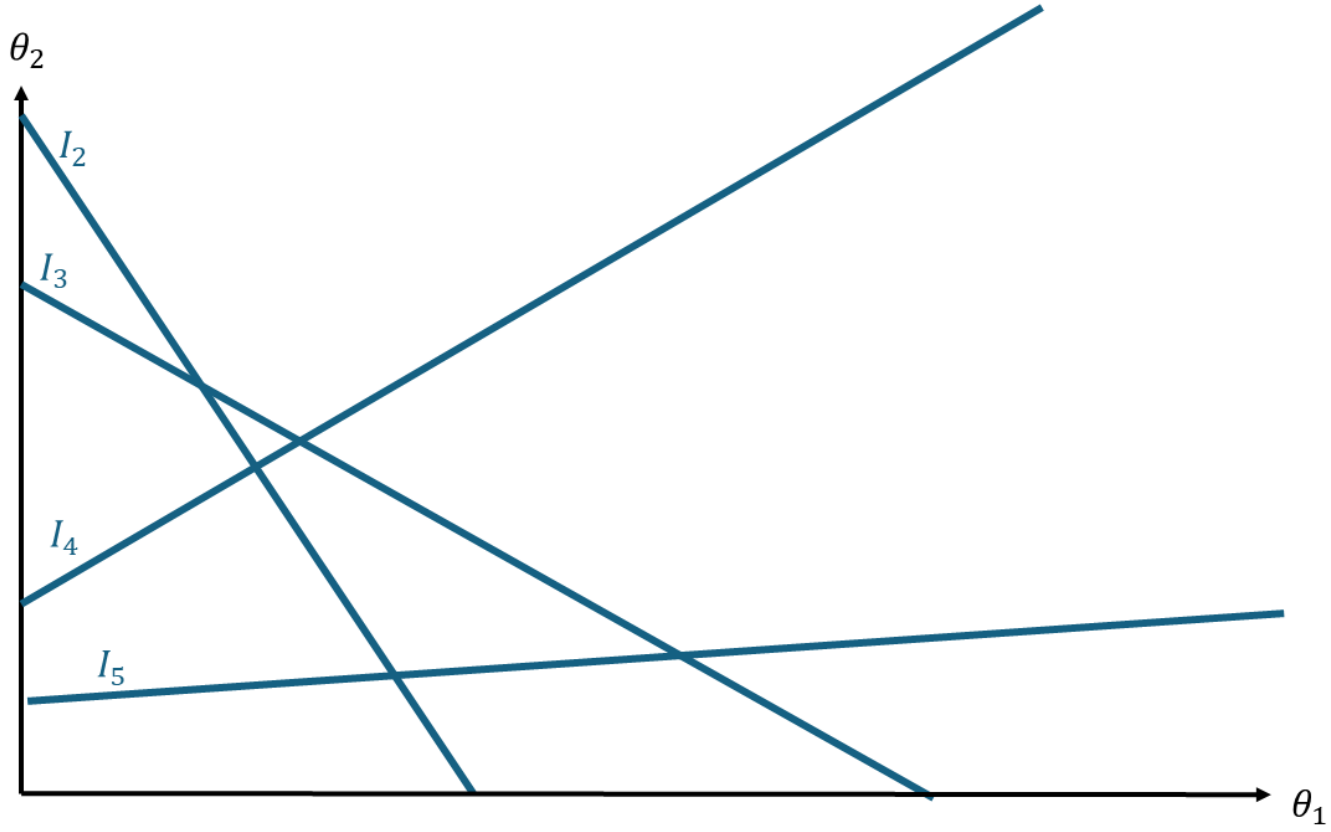

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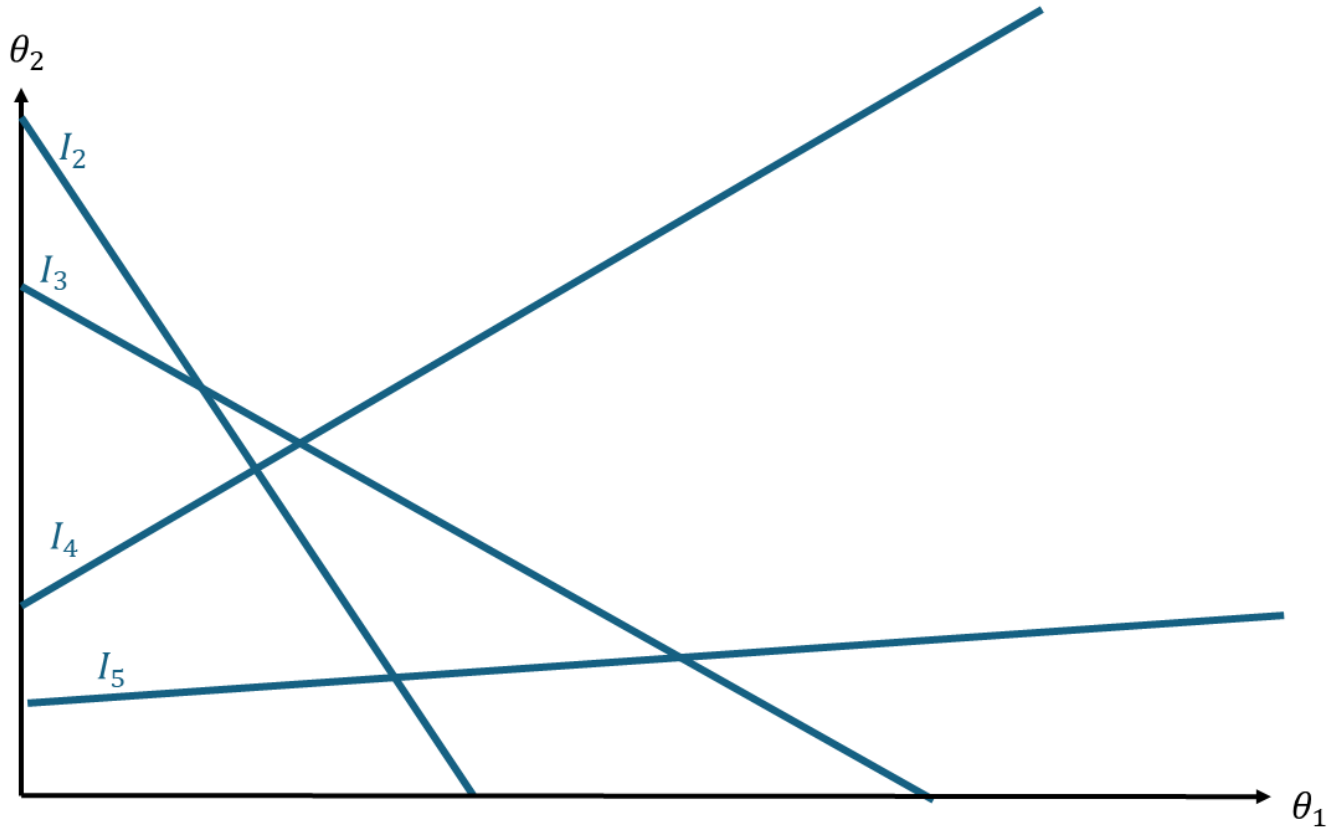

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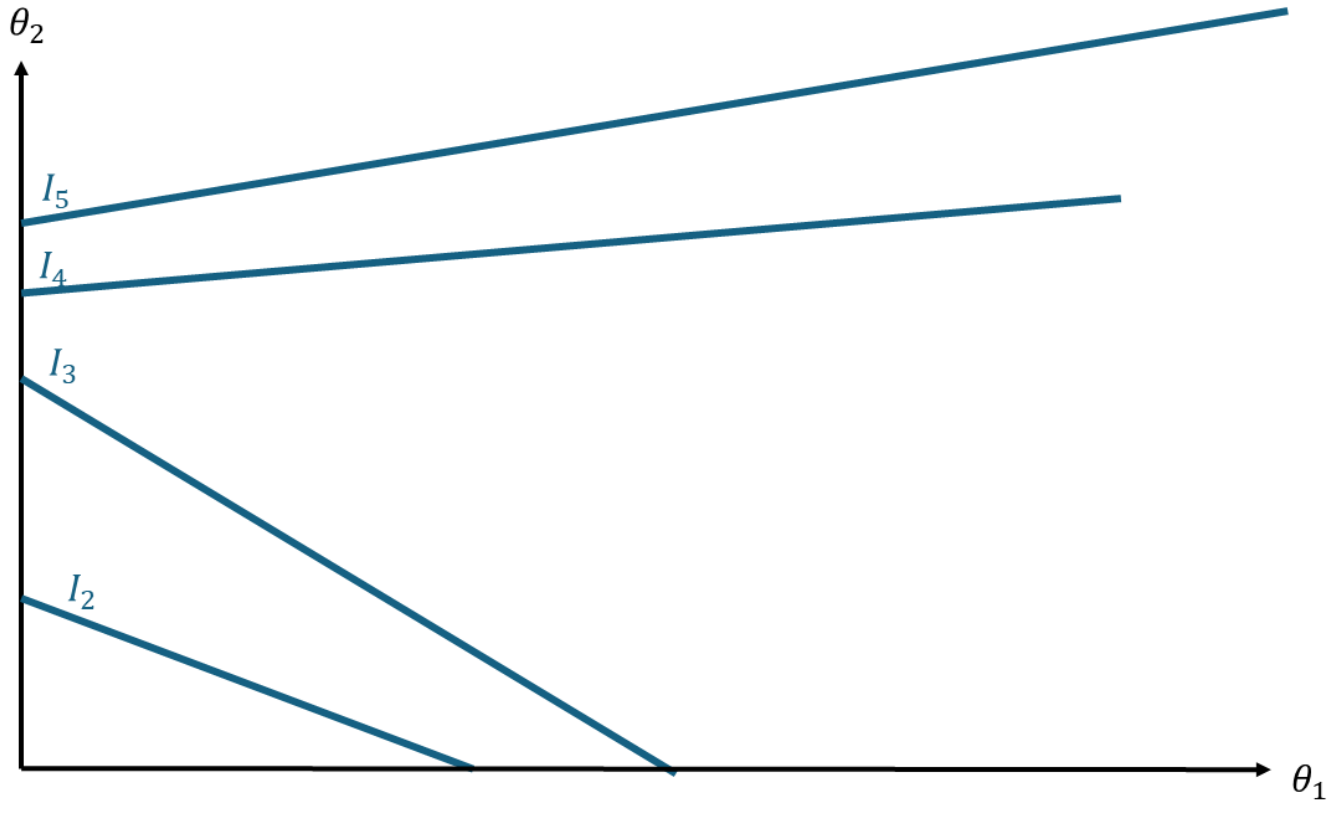

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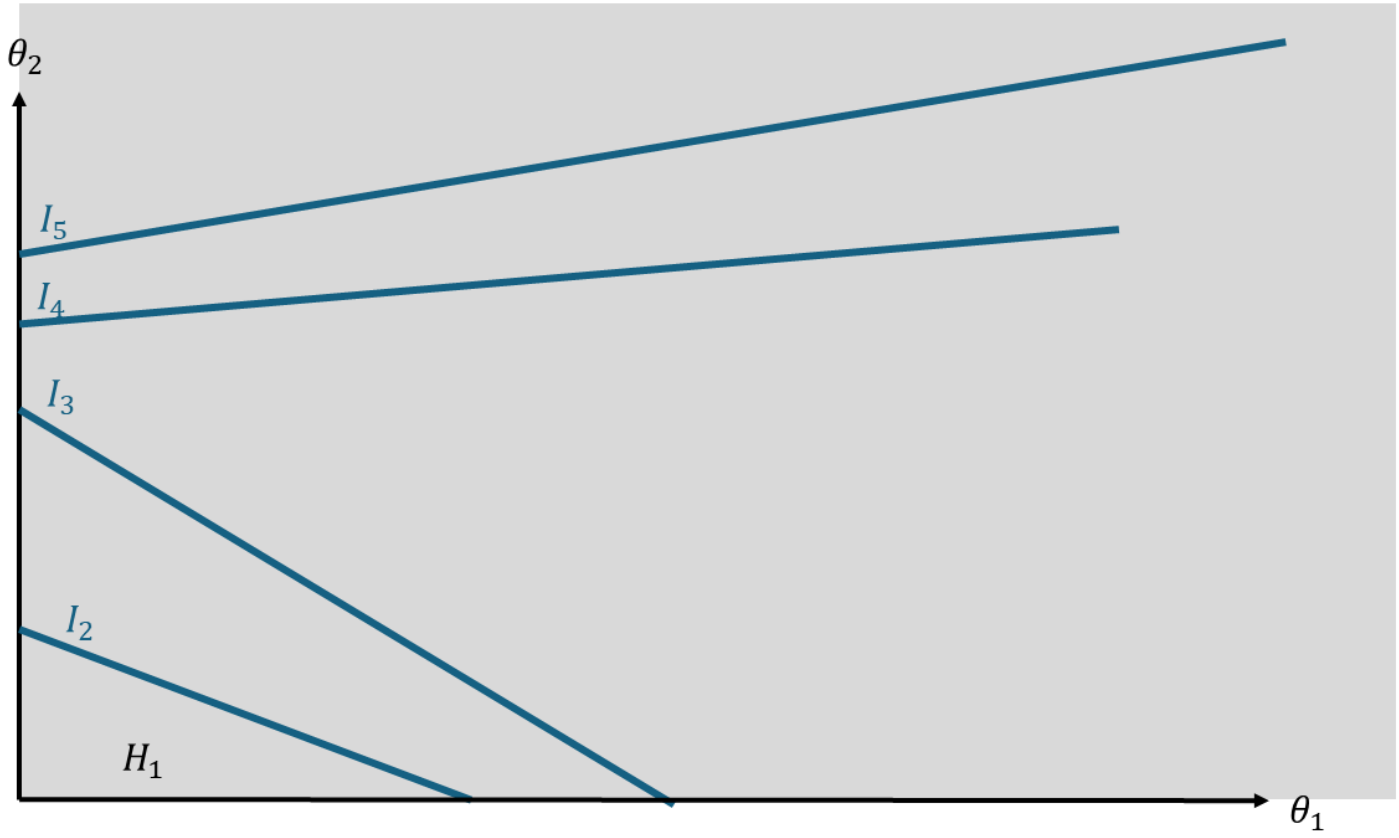

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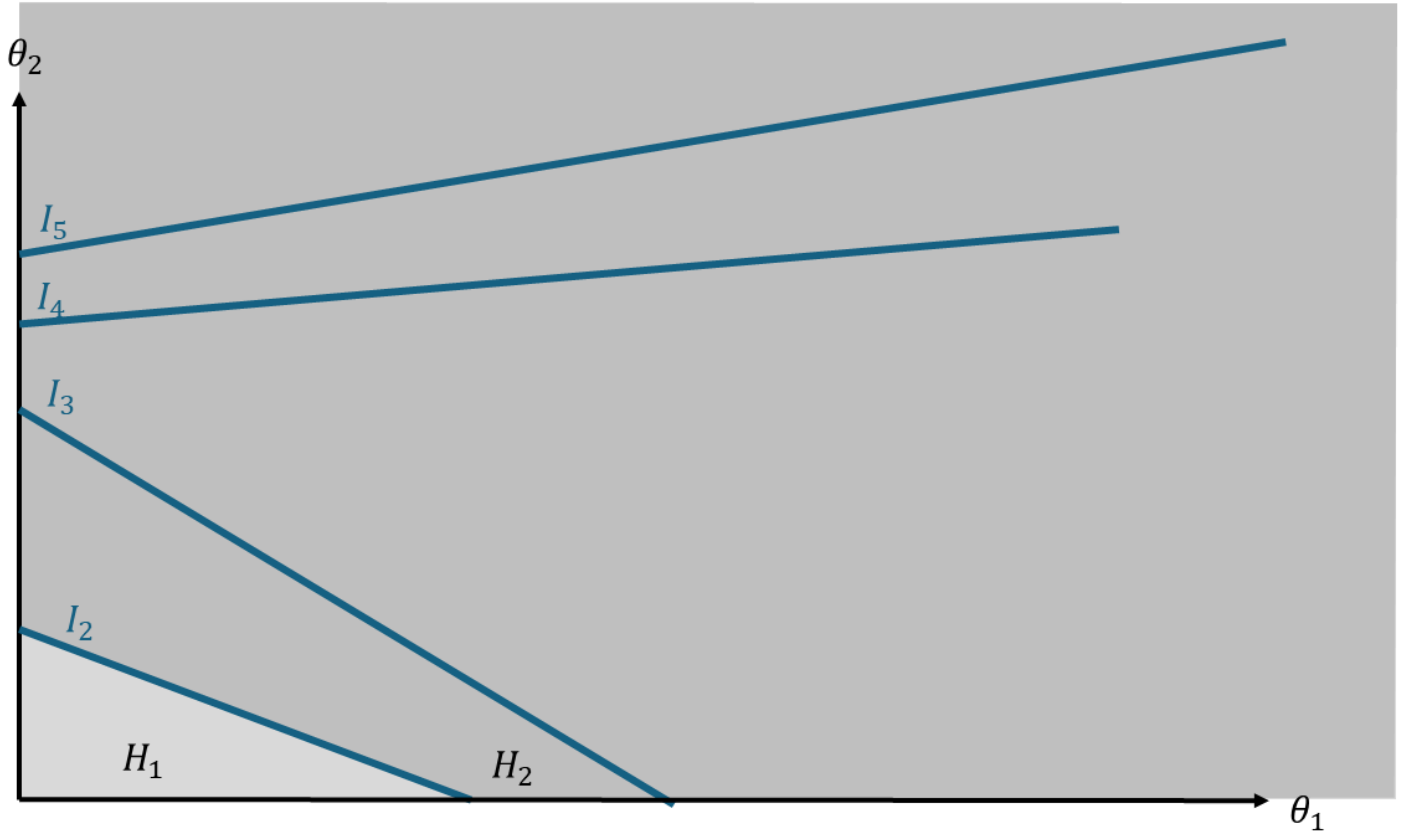

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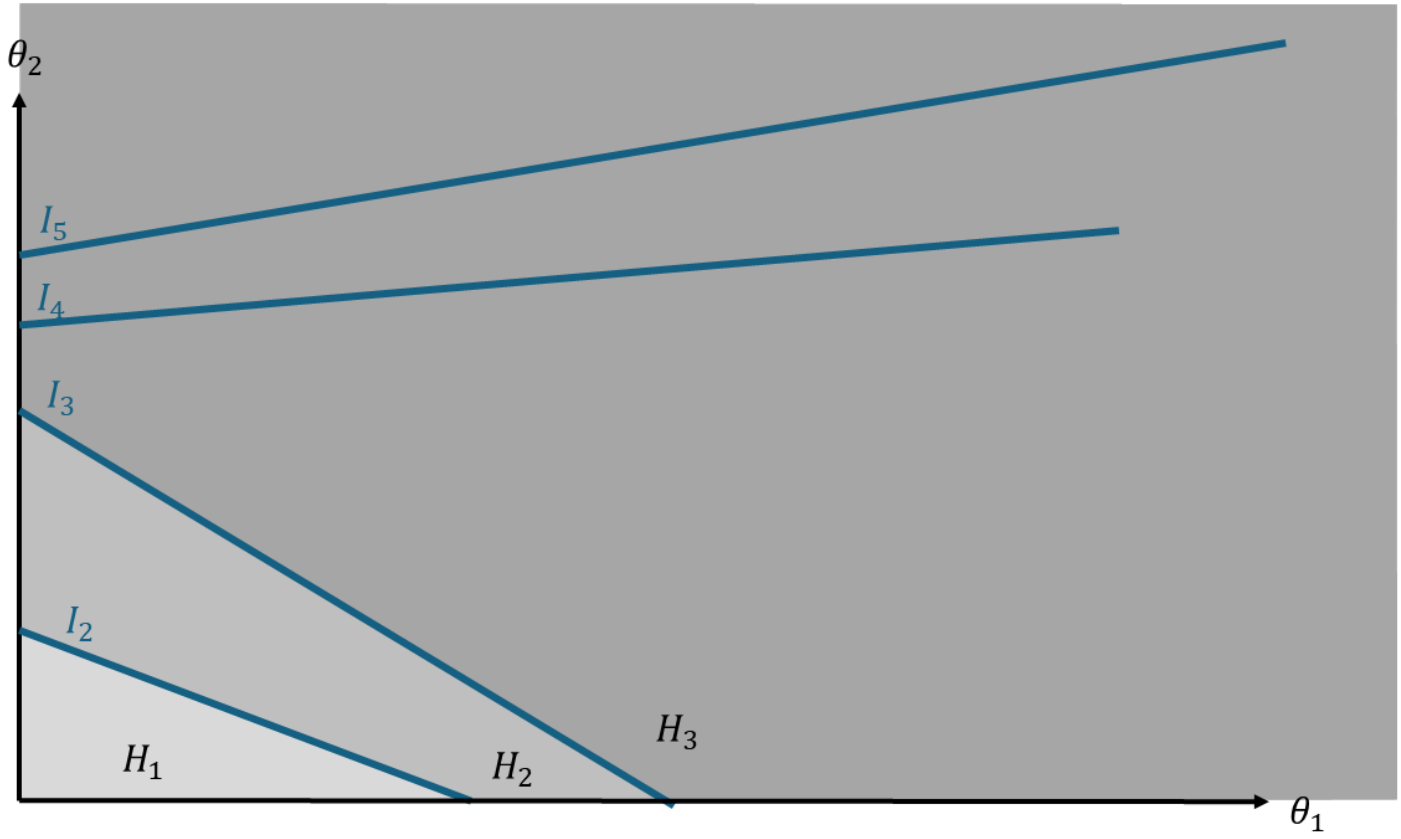

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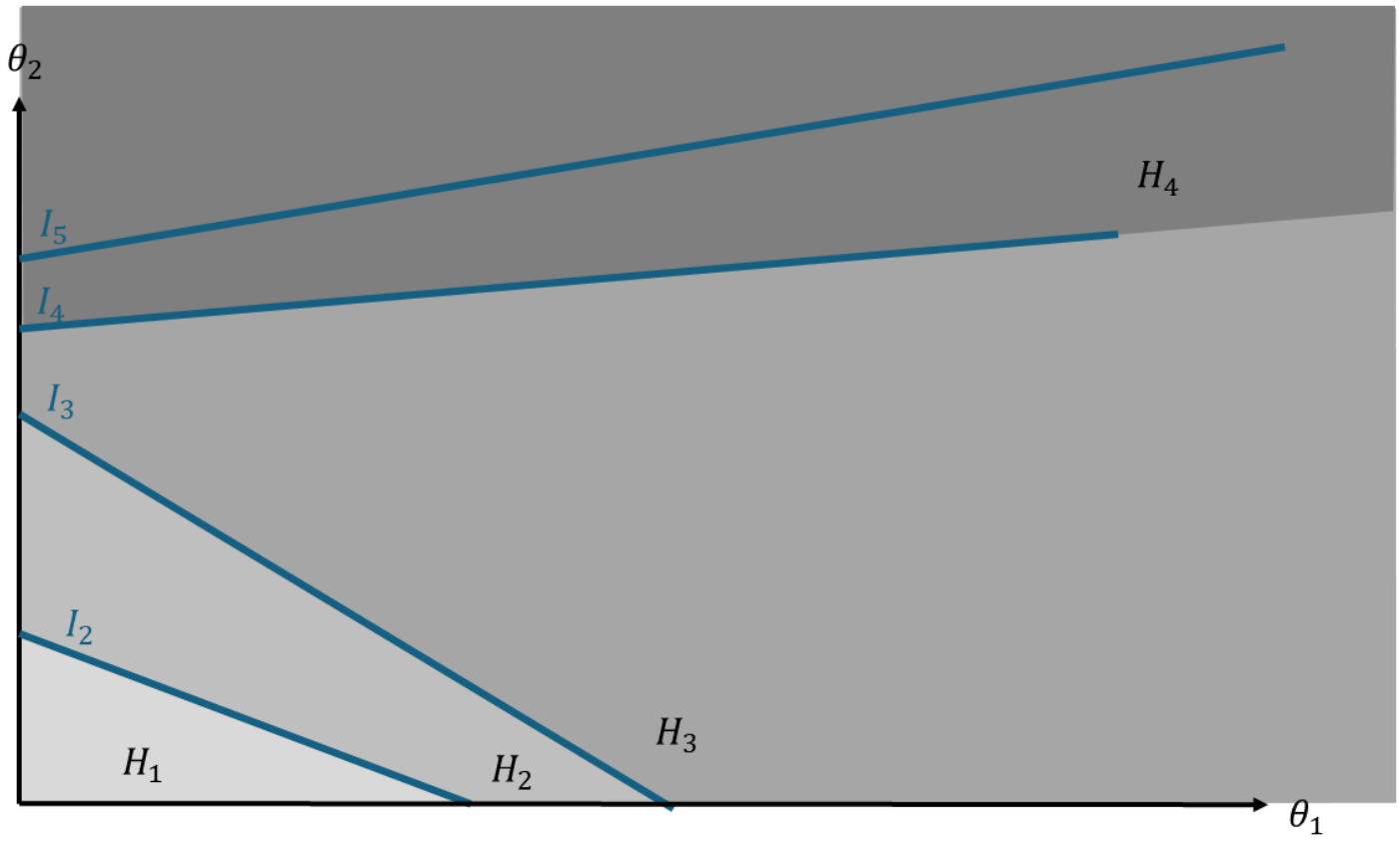

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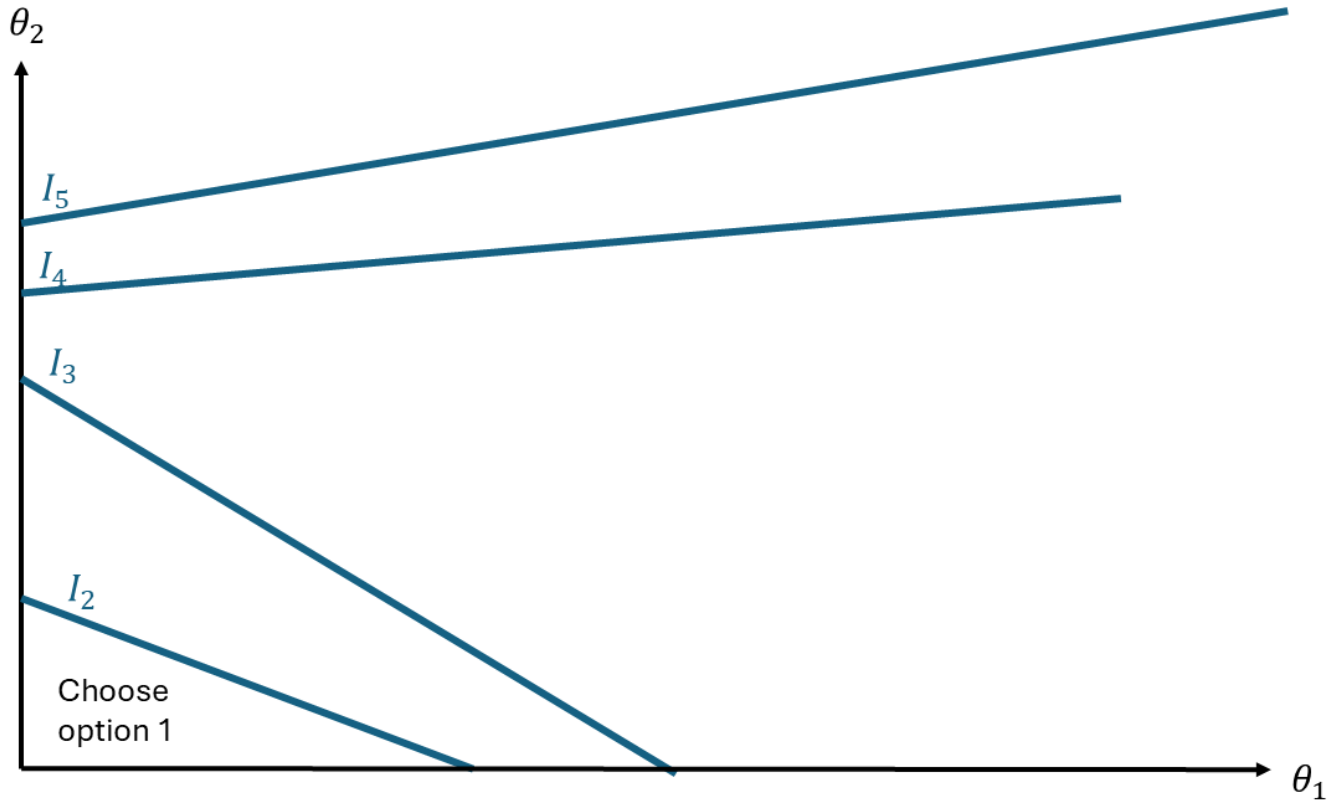

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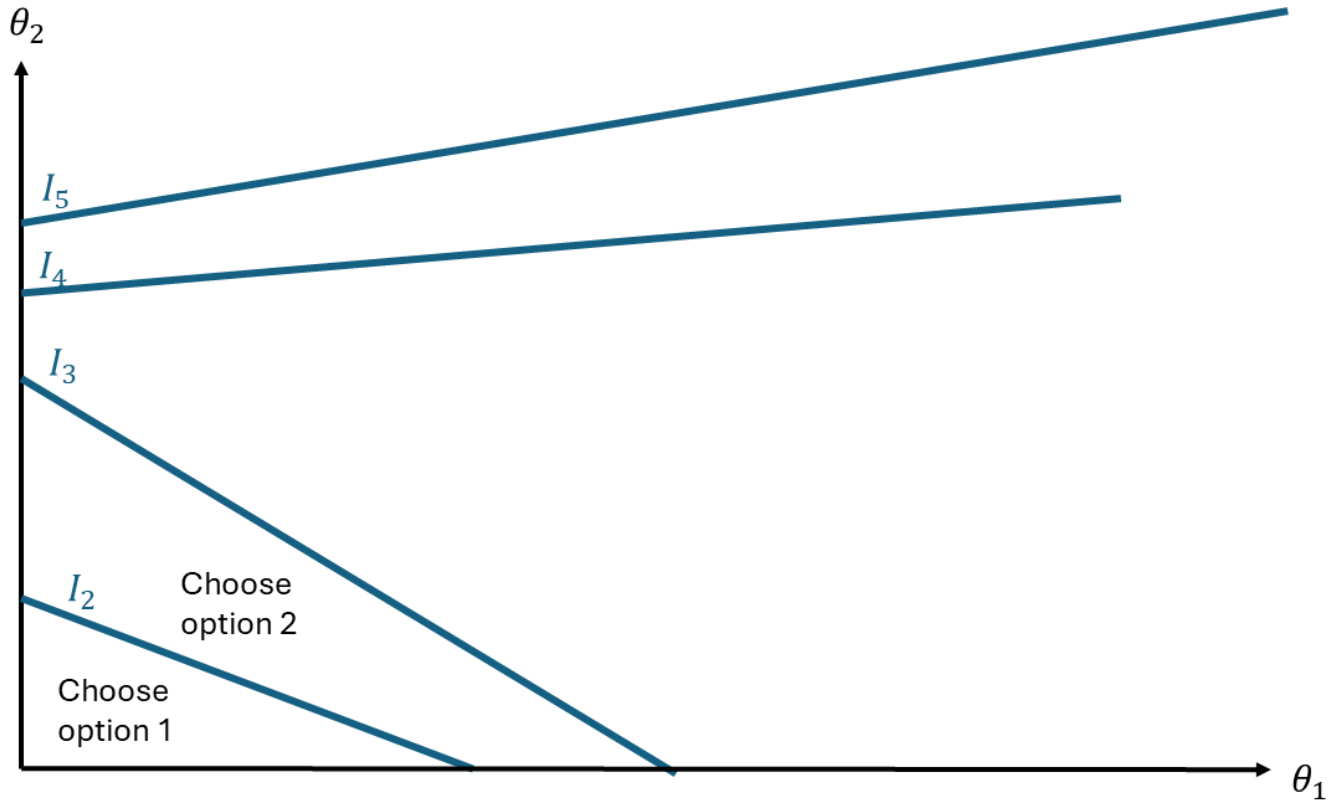

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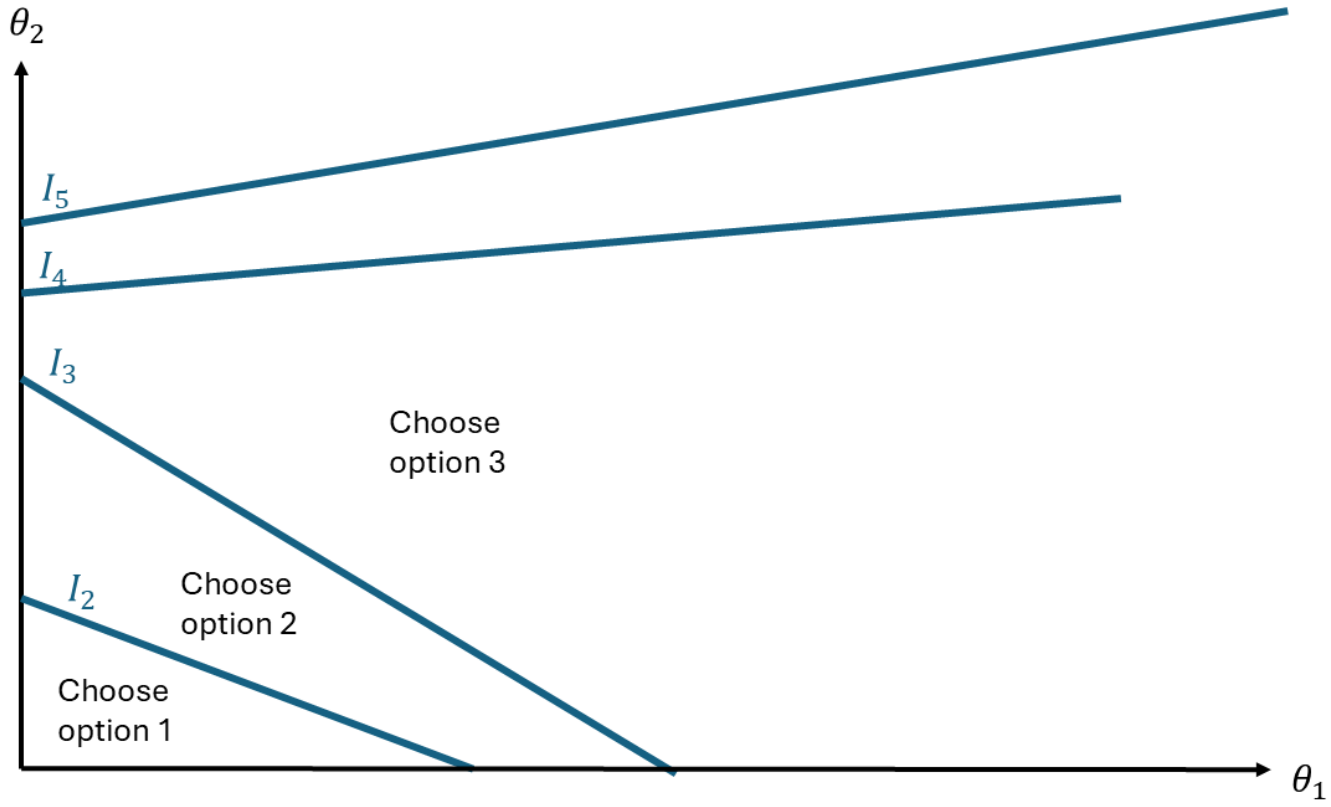

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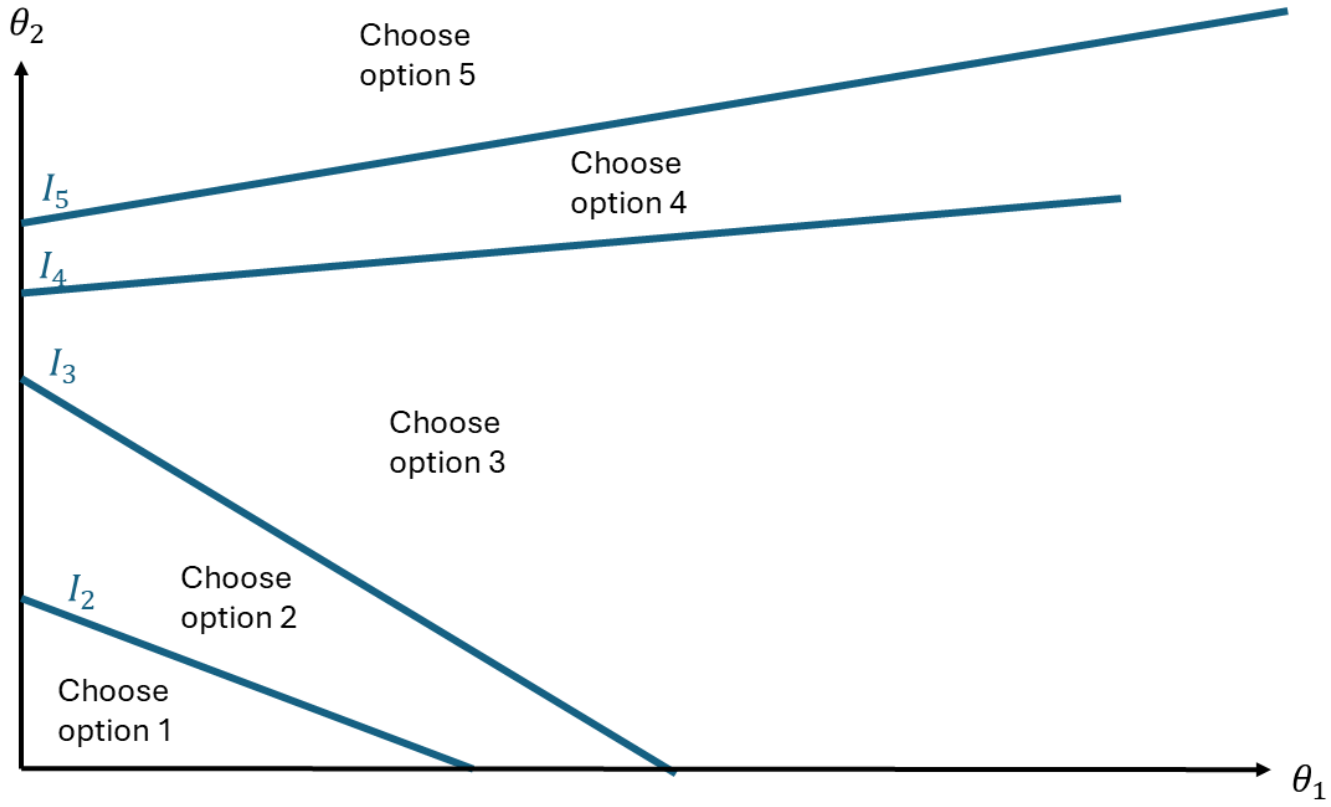

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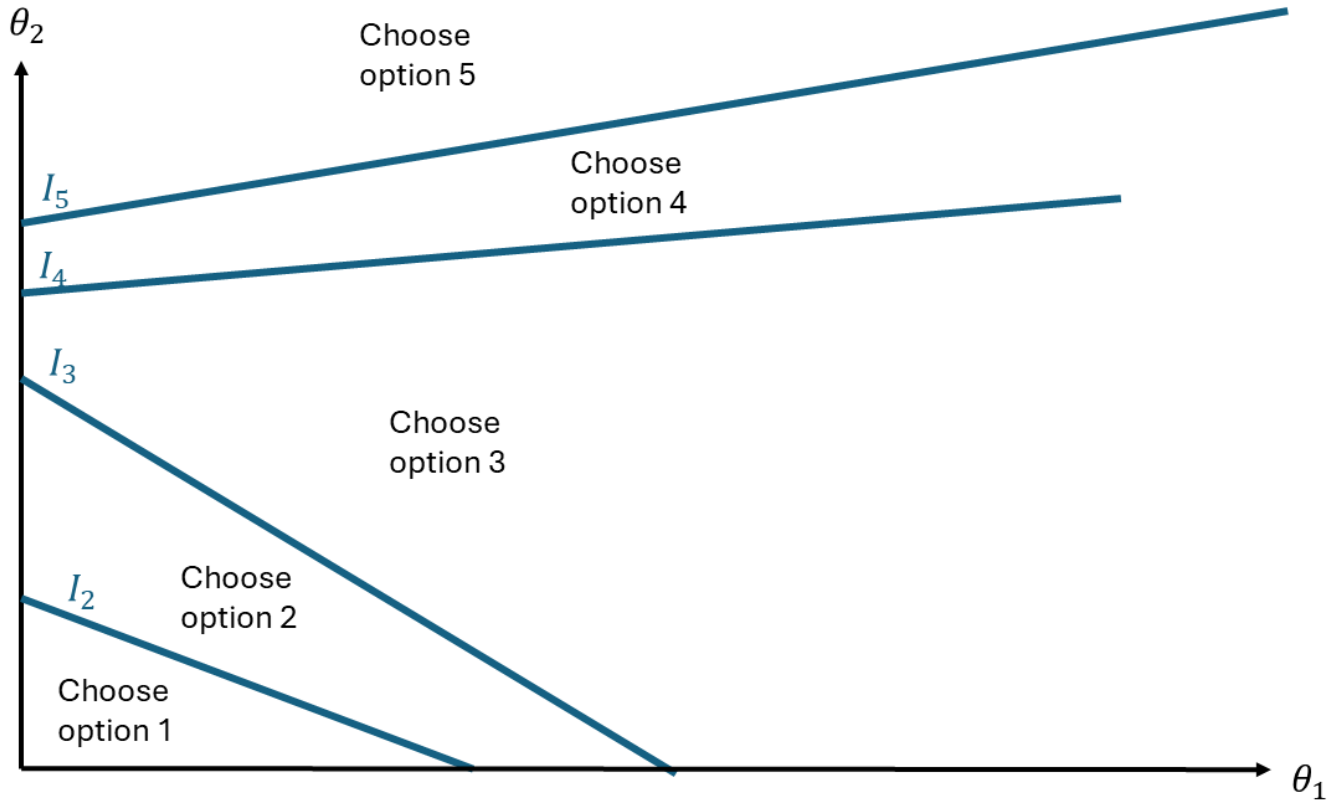
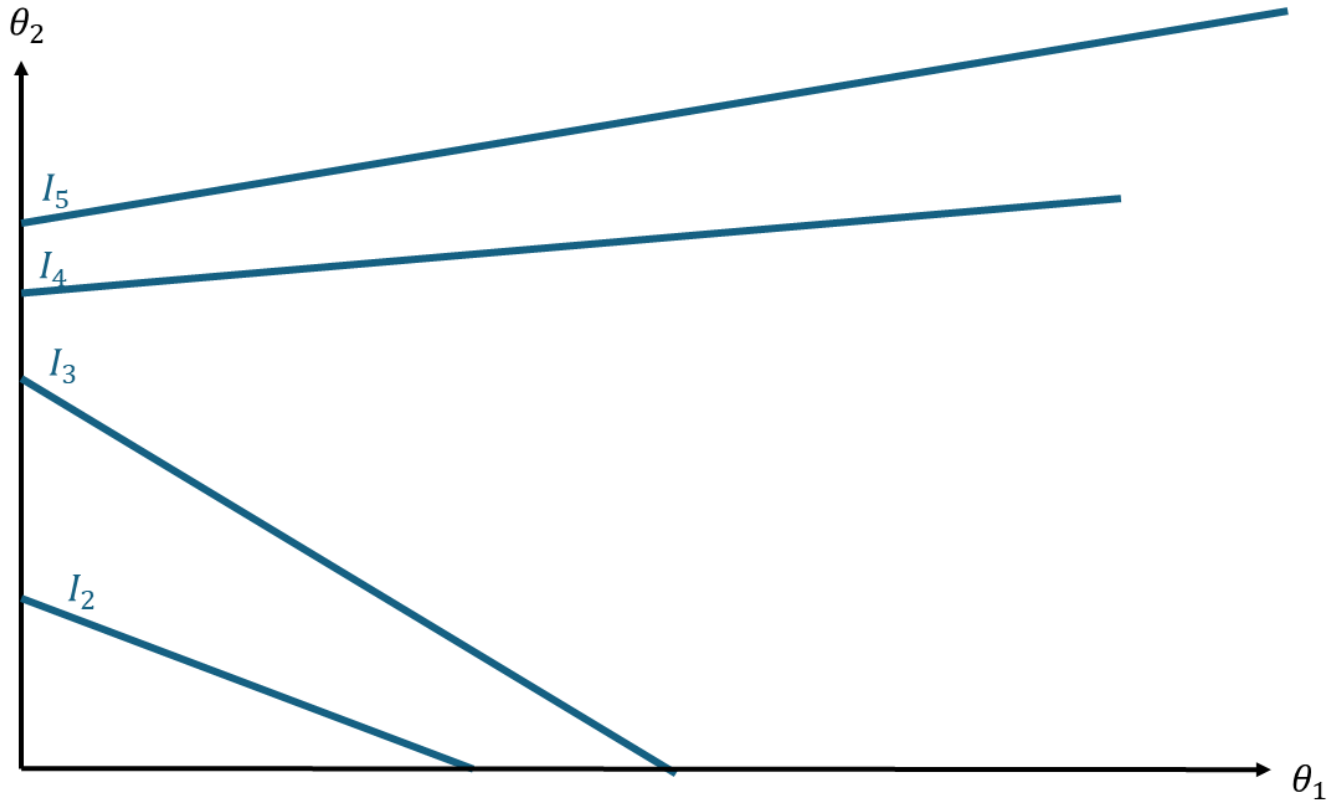


$$Revenue = \sum_{k=1}^{m} \Delta p^k \mu(H^k).$$

Multi-product monopoly **(proof sketch)**



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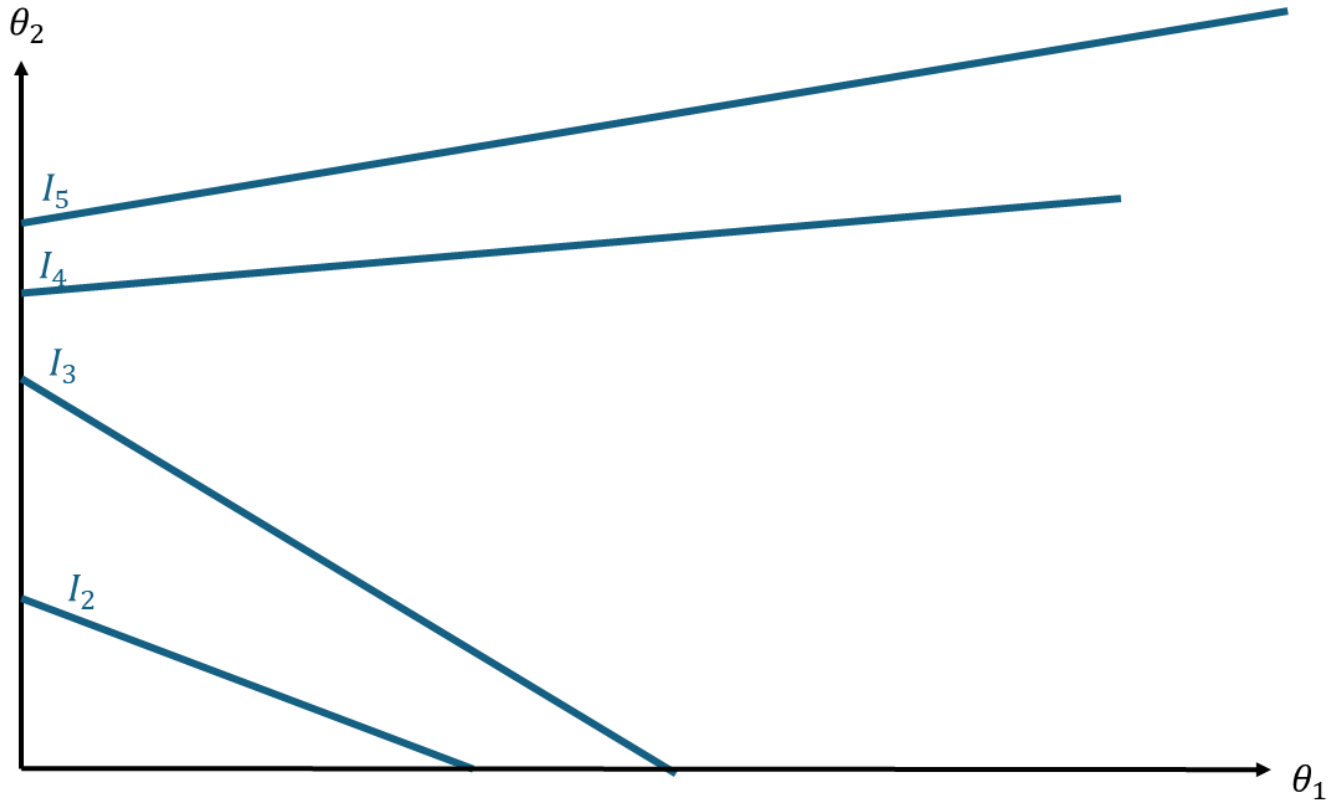

Claim: $\Delta p^k \geq 0$ for all k.

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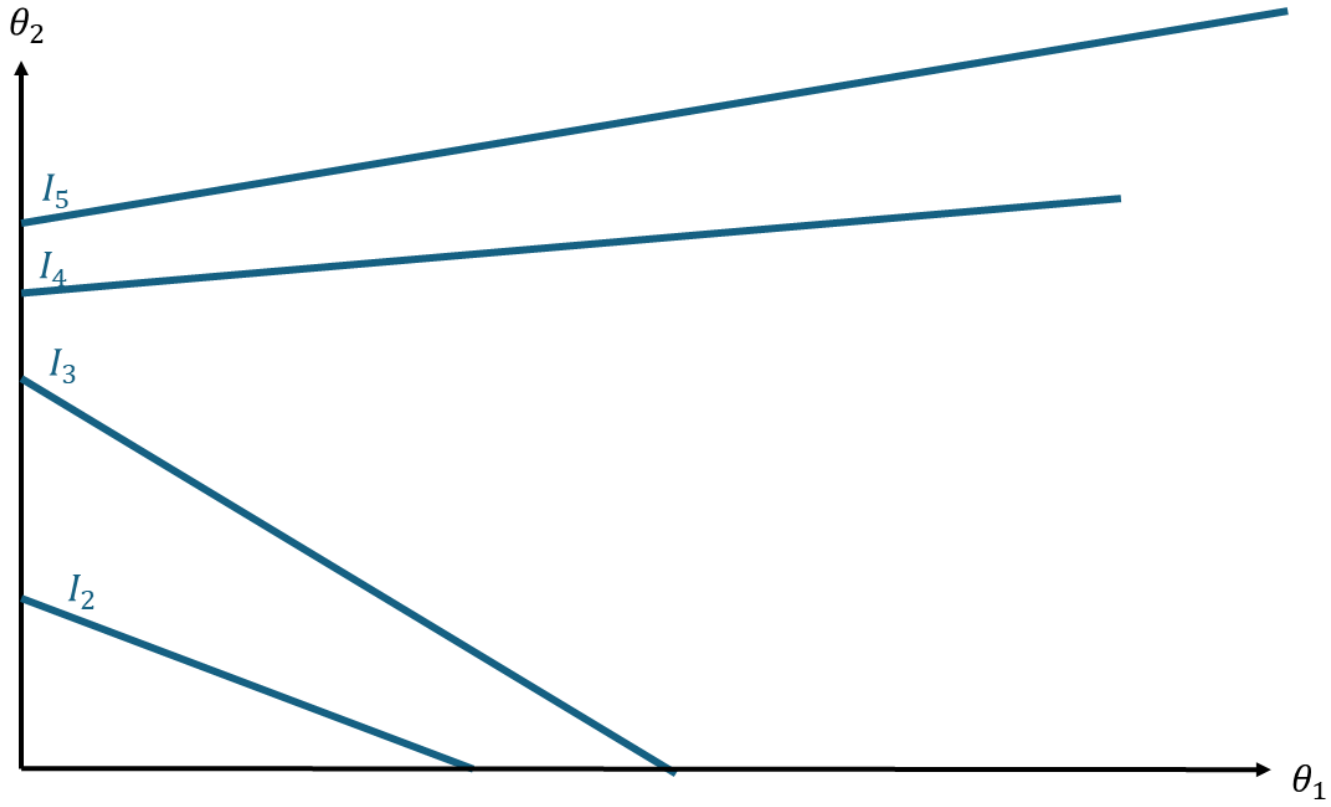

Intuitively: Searchability for type 0 $\implies$ p^k is quasi-convex.

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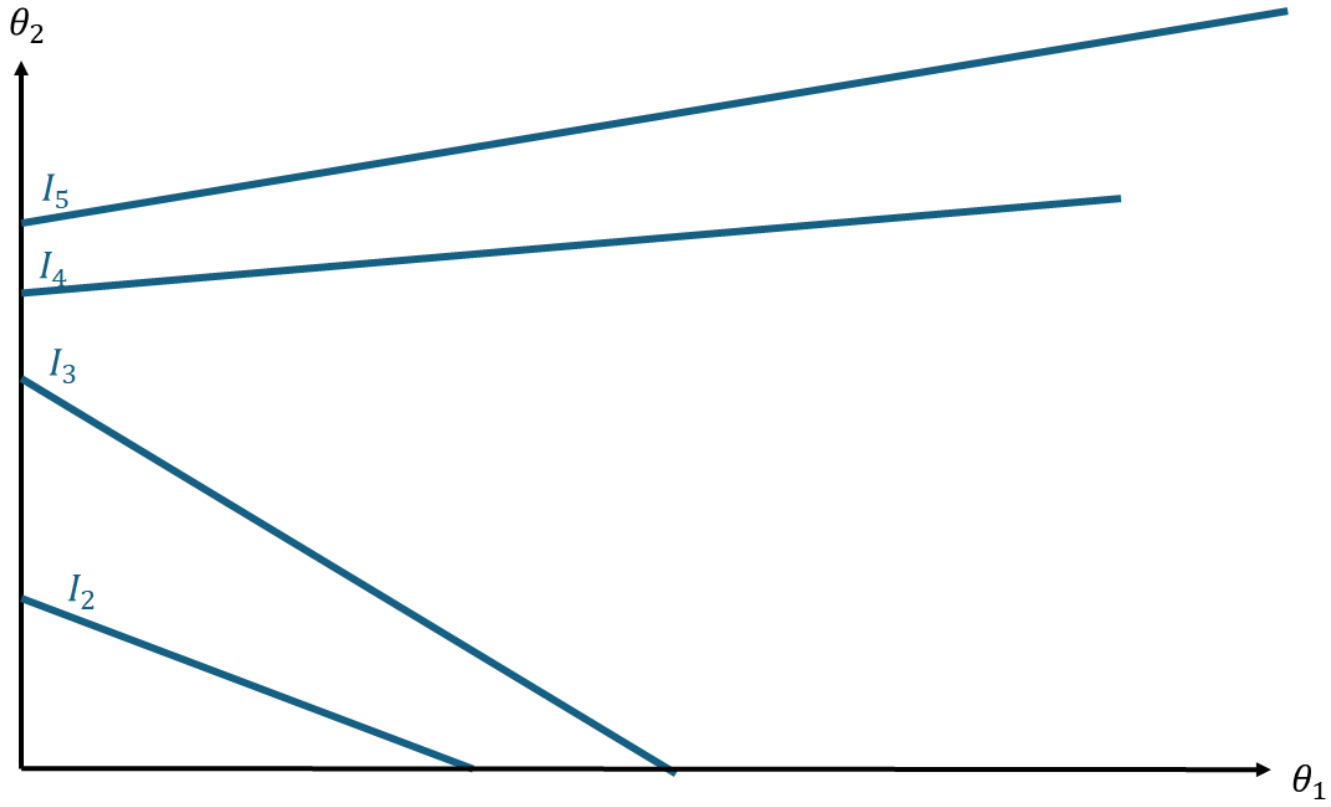

Intuitively: Searchability for type 0 $\implies$ p^k is quasi-convex.

We can "chop off" negative prices.

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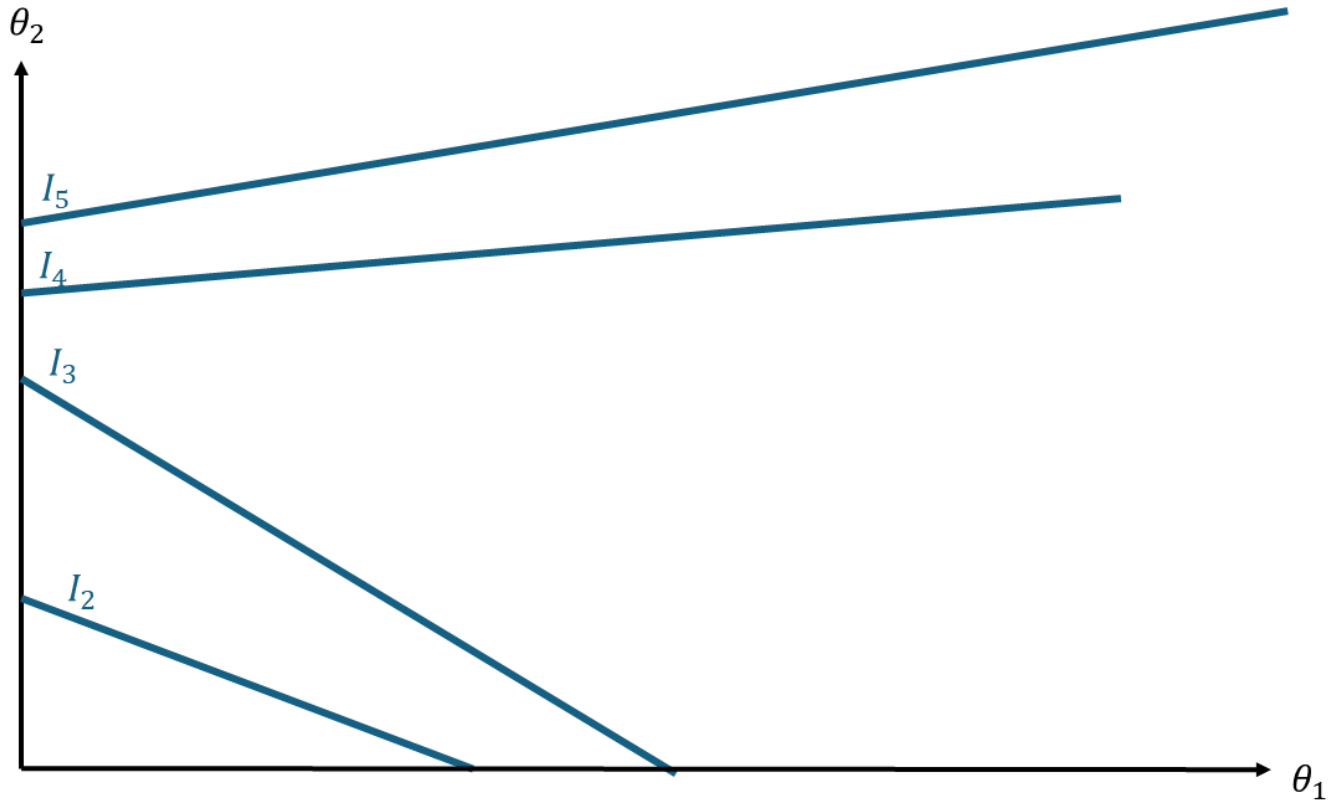

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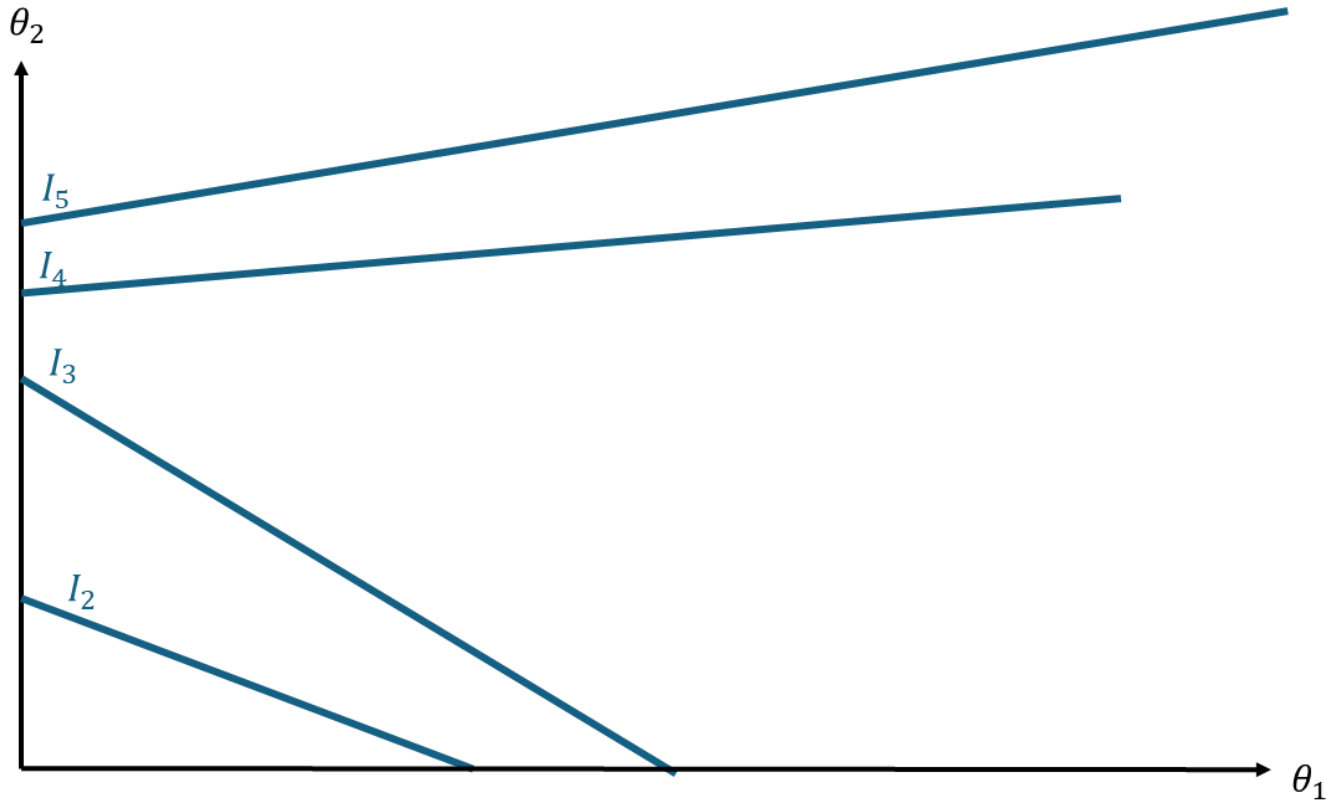

Negative slope of $I^k \iff \Delta q_1^k,\ \Delta q_2^k \geq 0$

Positive slope of $I^k \iff \Delta q_1^k \leq 0 \leq \Delta q_2^k$.

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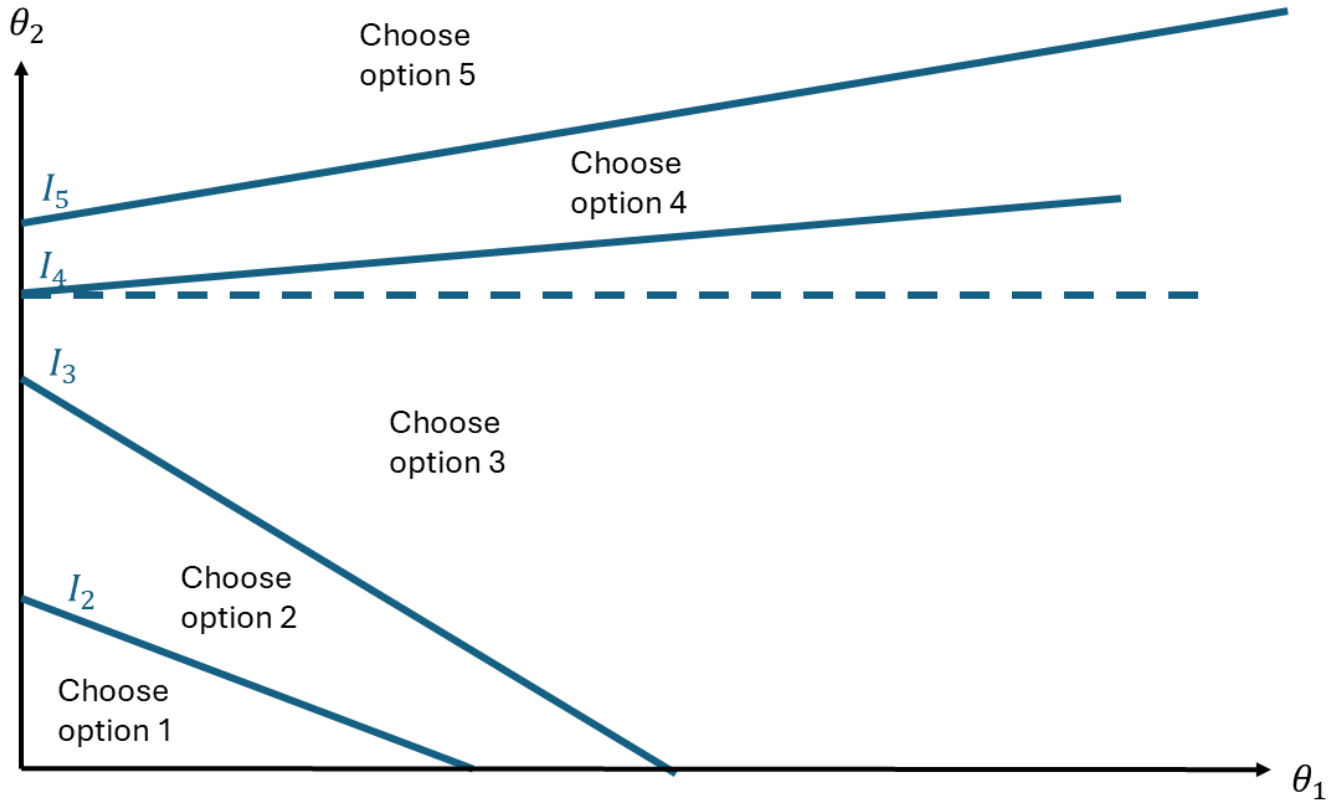

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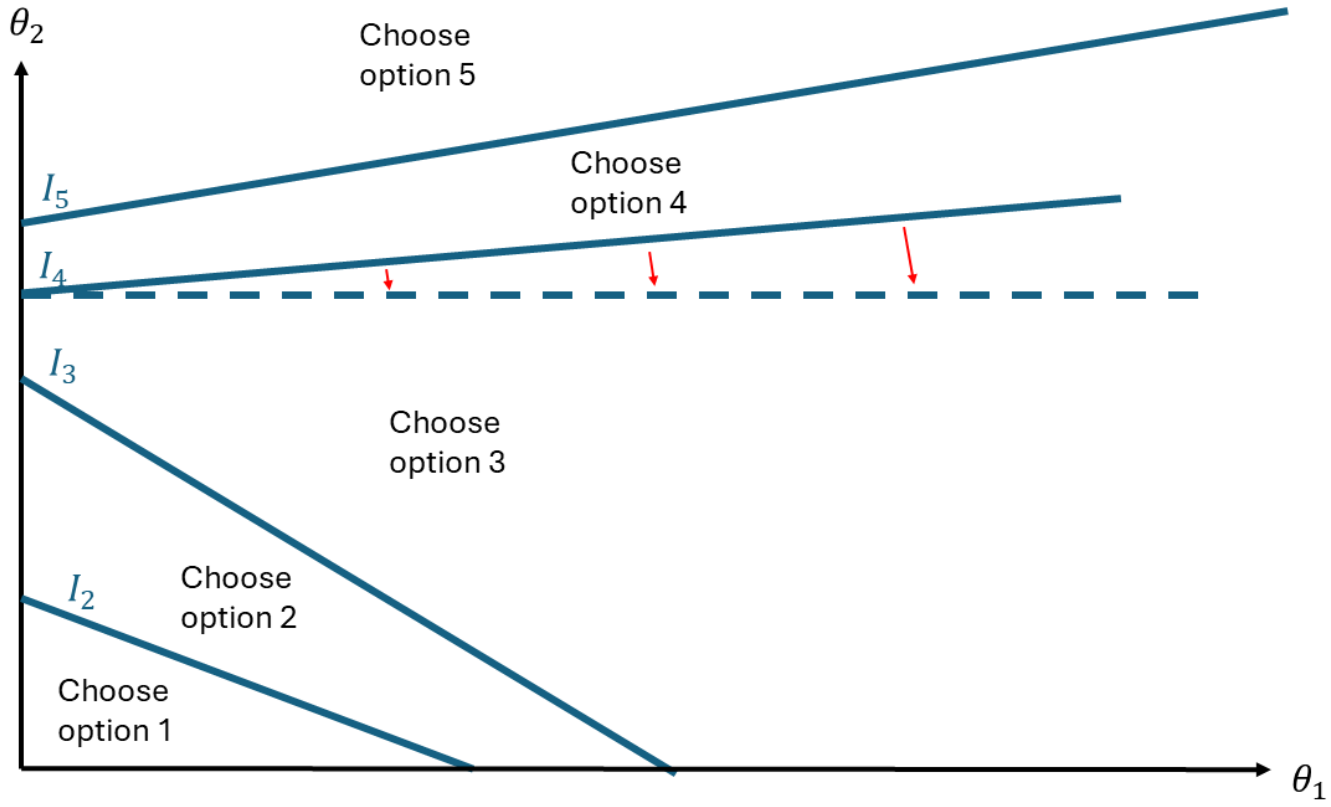

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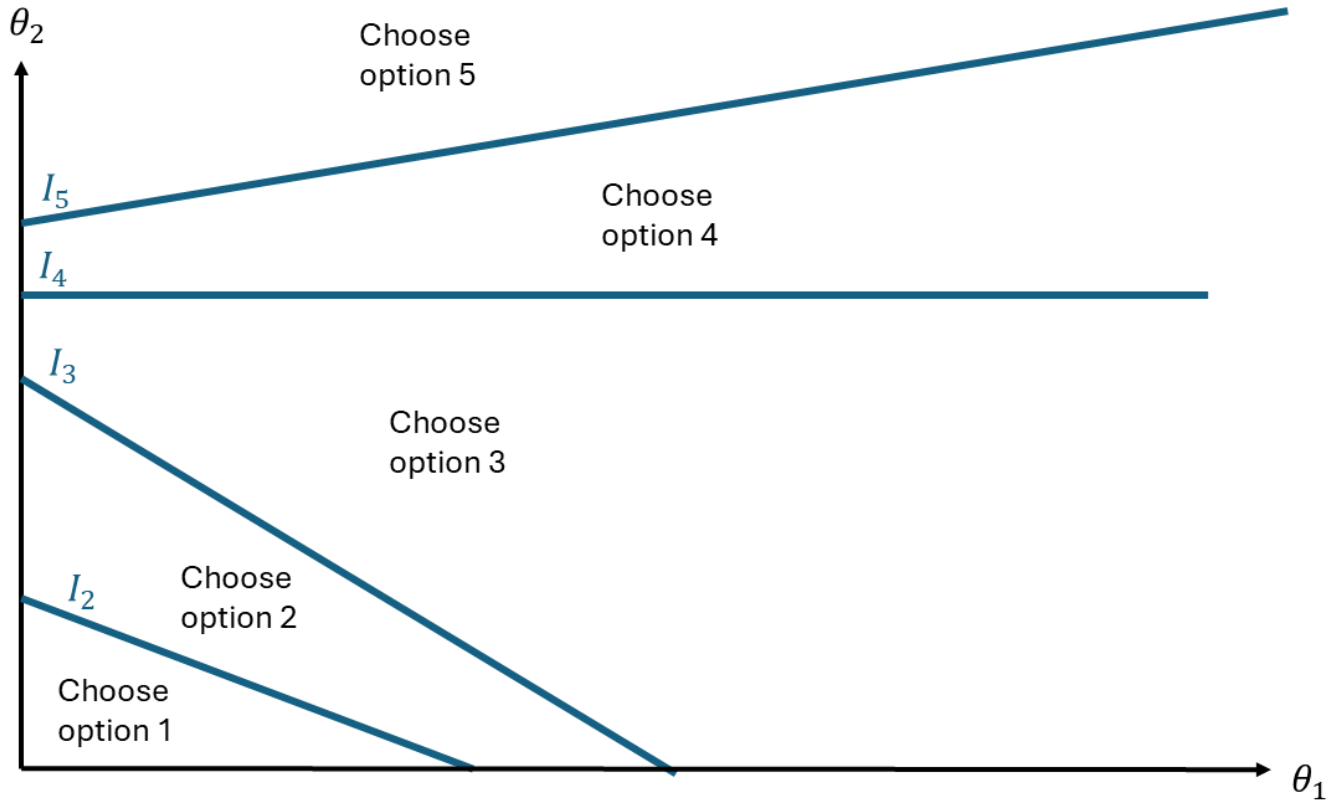

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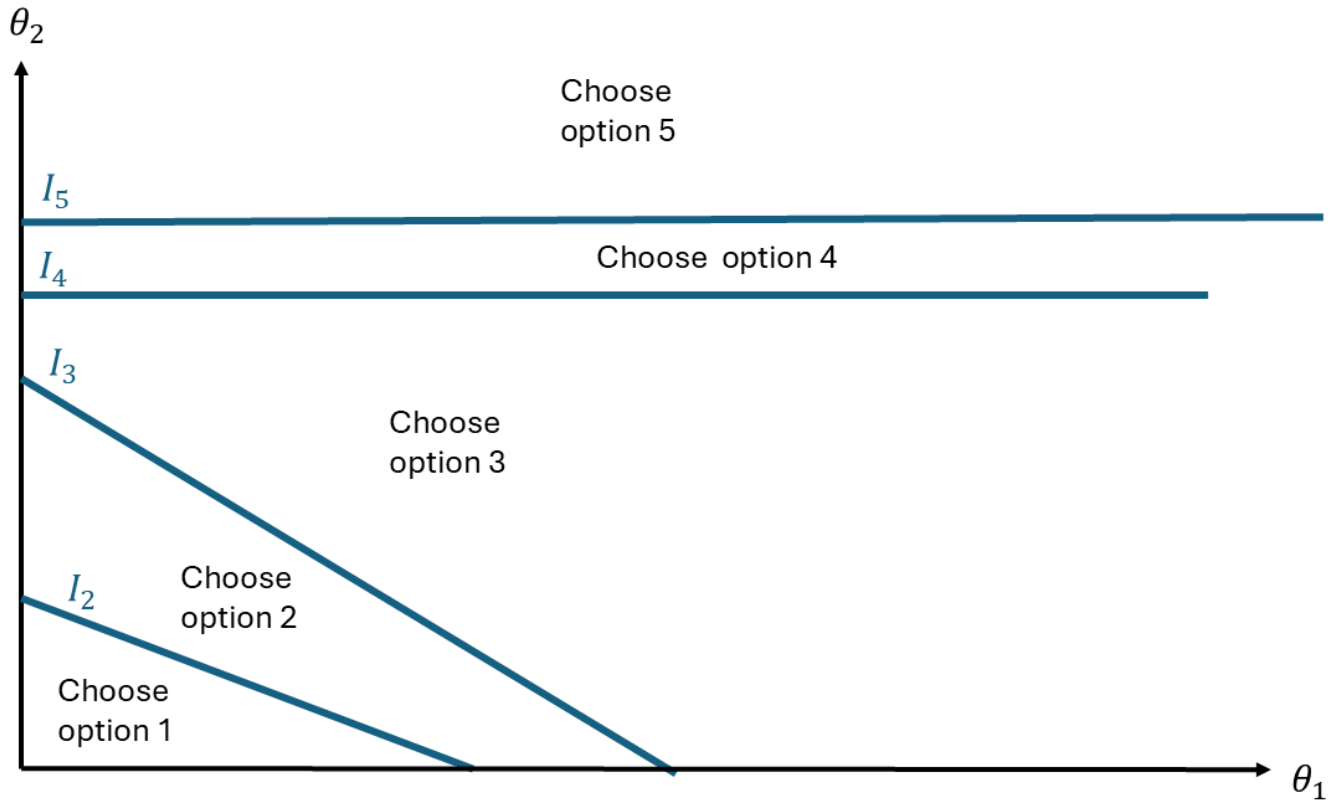

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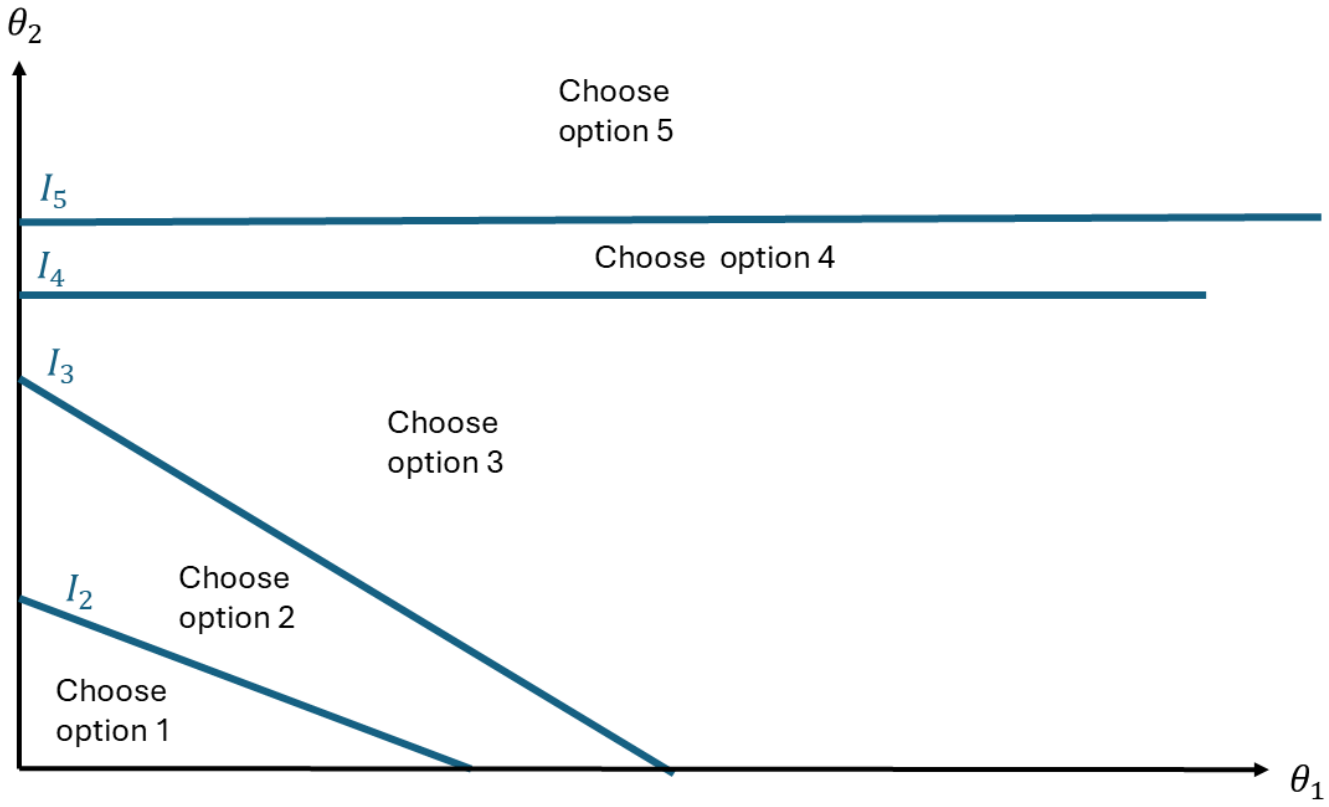

Thus, it is wlog to have $\Delta q^k \geq 0$ for all k.

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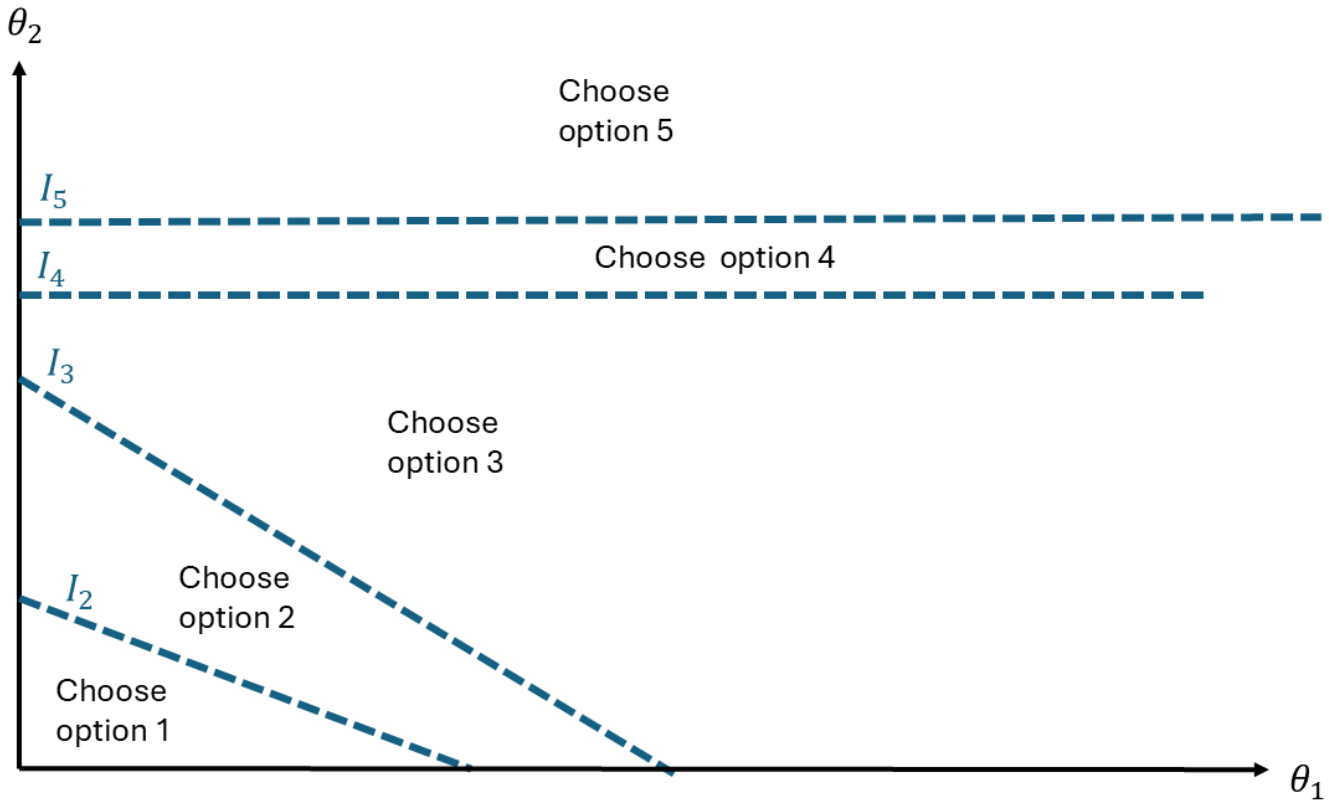

Keeping I^k fixed, we can optimize over weights $\lambda(k) \geq 0$:

$$\Delta\hat{p}^k = \lambda(k)\Delta p^k \text{ and } \Delta\hat{q}^k = \lambda(k)\Delta q^k$$

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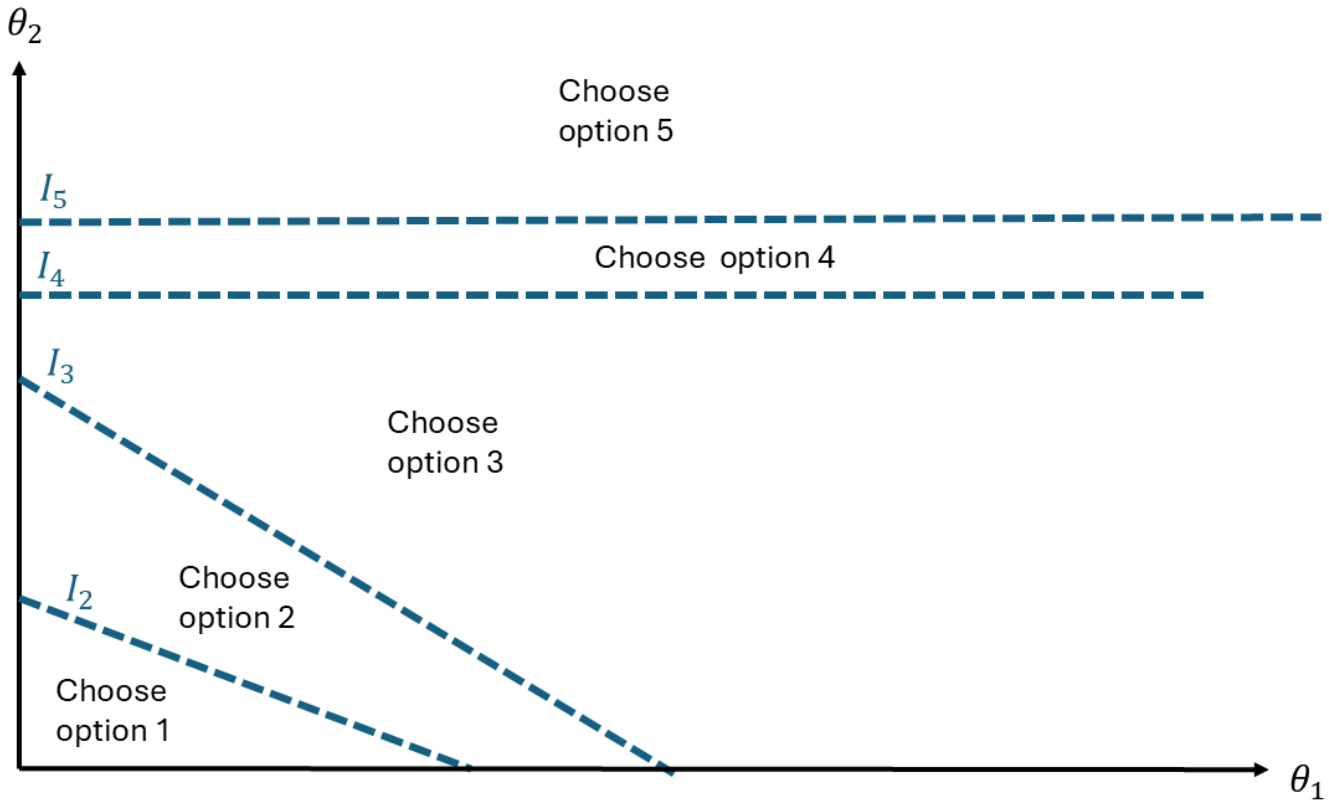

Because $\Delta q^k \geq 0$ for all k, the constraint on $\lambda(k)$'s:

$$\sum_{k=1}^{m} \lambda(k) \Delta q_i^k \leq 1, \text{ for all } i.$$

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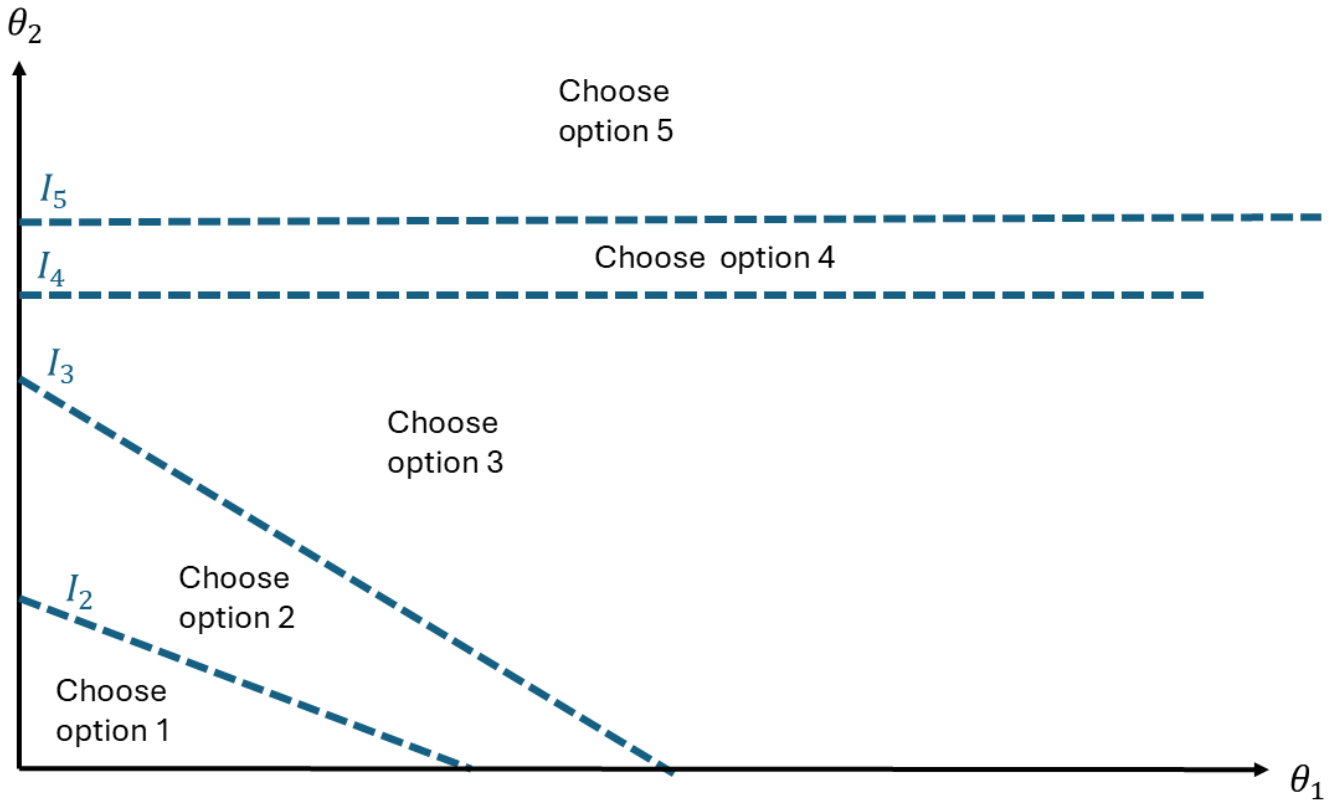

Revenue is linear in the $\lambda(k)$'s.

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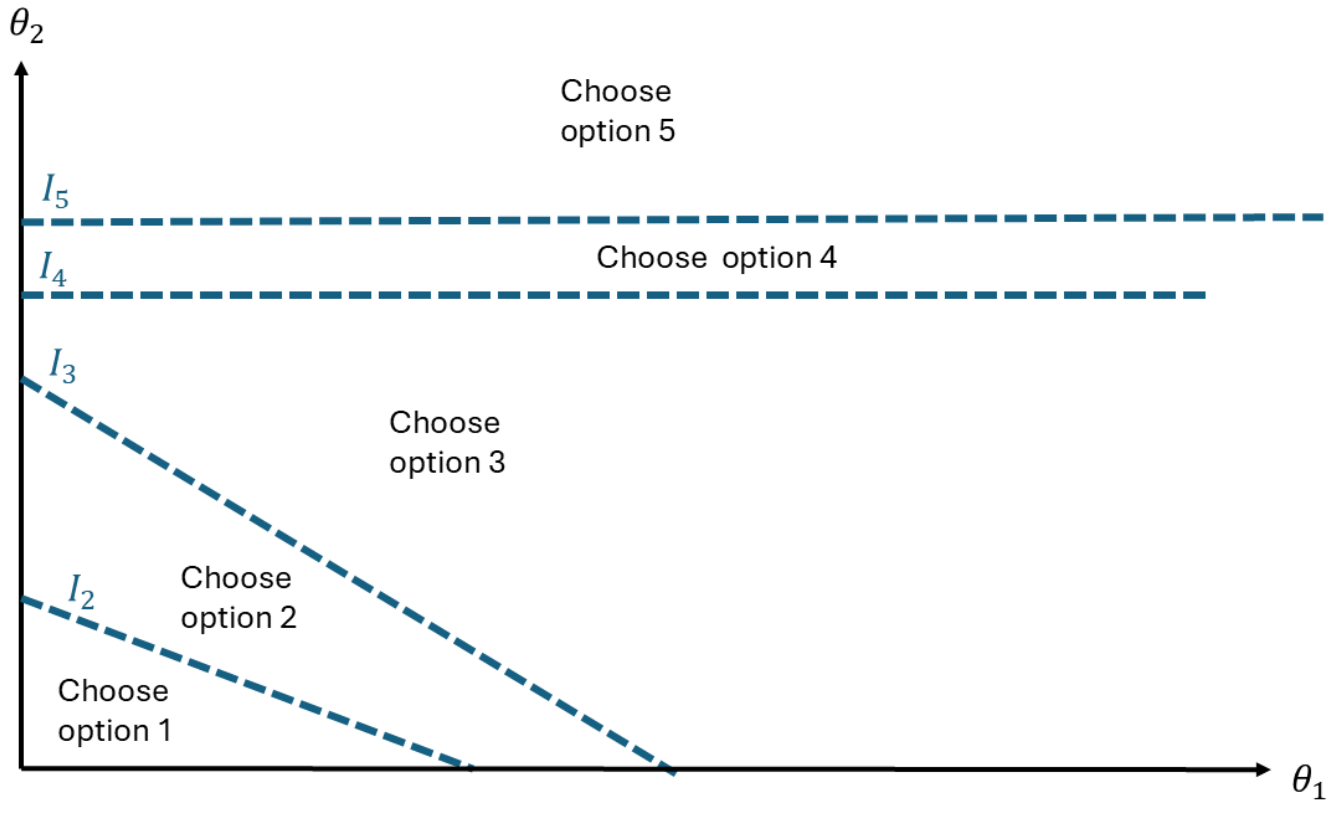

This gives us a linear program with *n* linear constraints.

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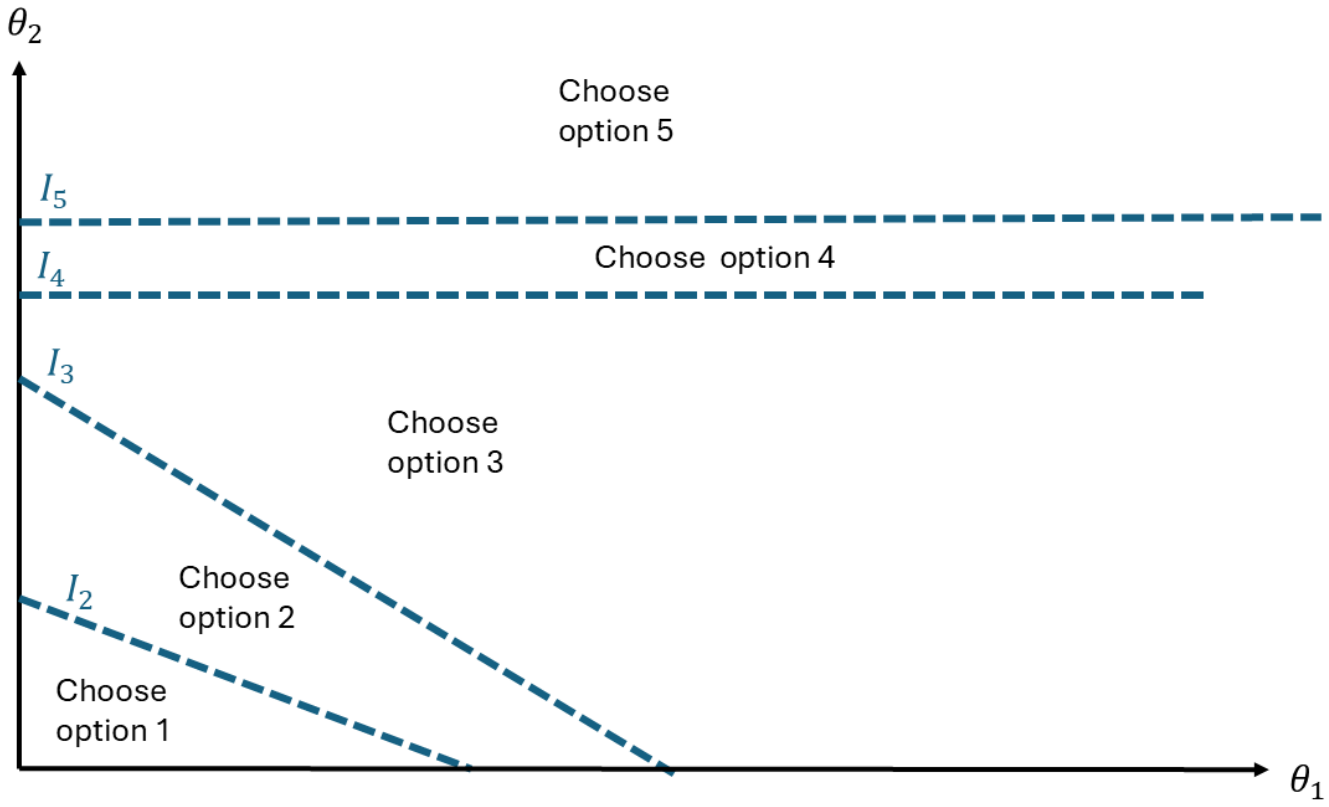

This gives us a linear program with n linear constraints.

$\implies$ only n of the $\lambda(k)$'s are non-zero.

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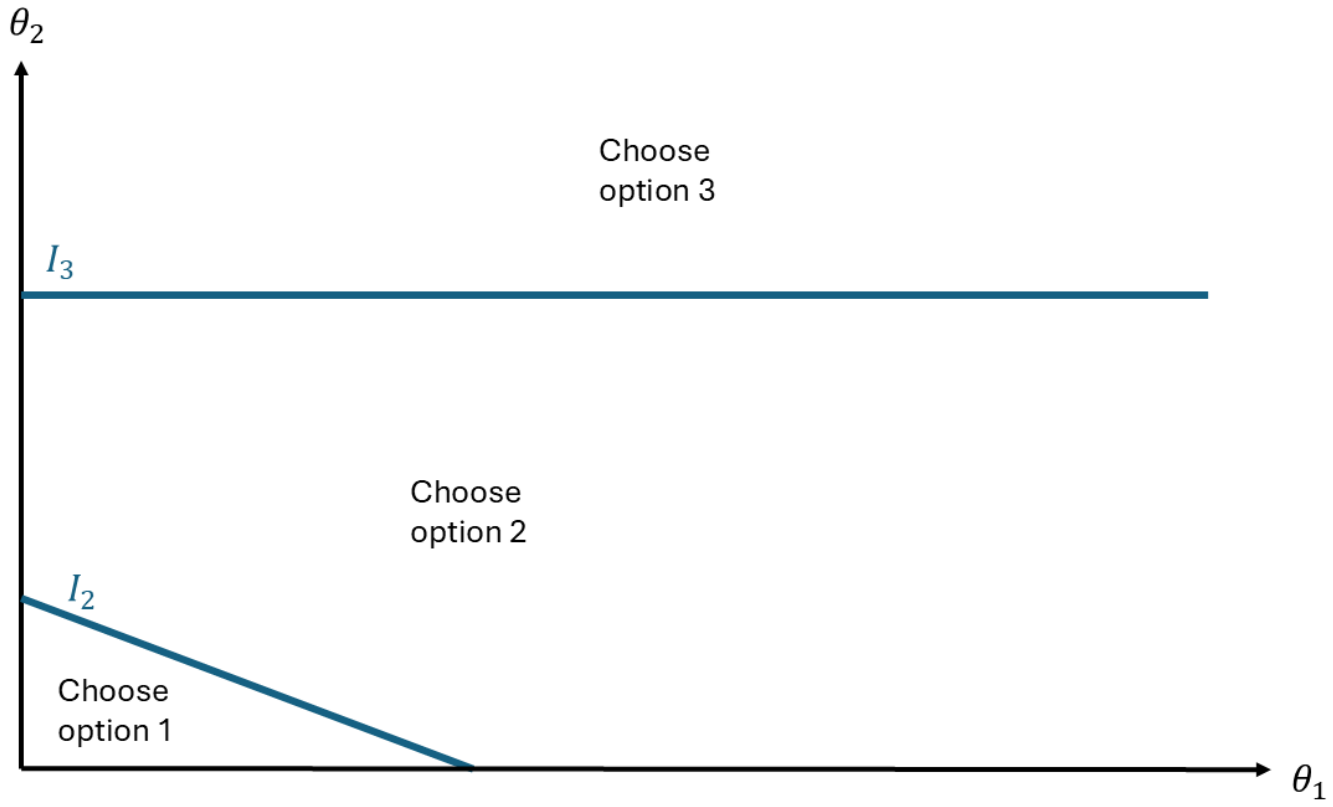

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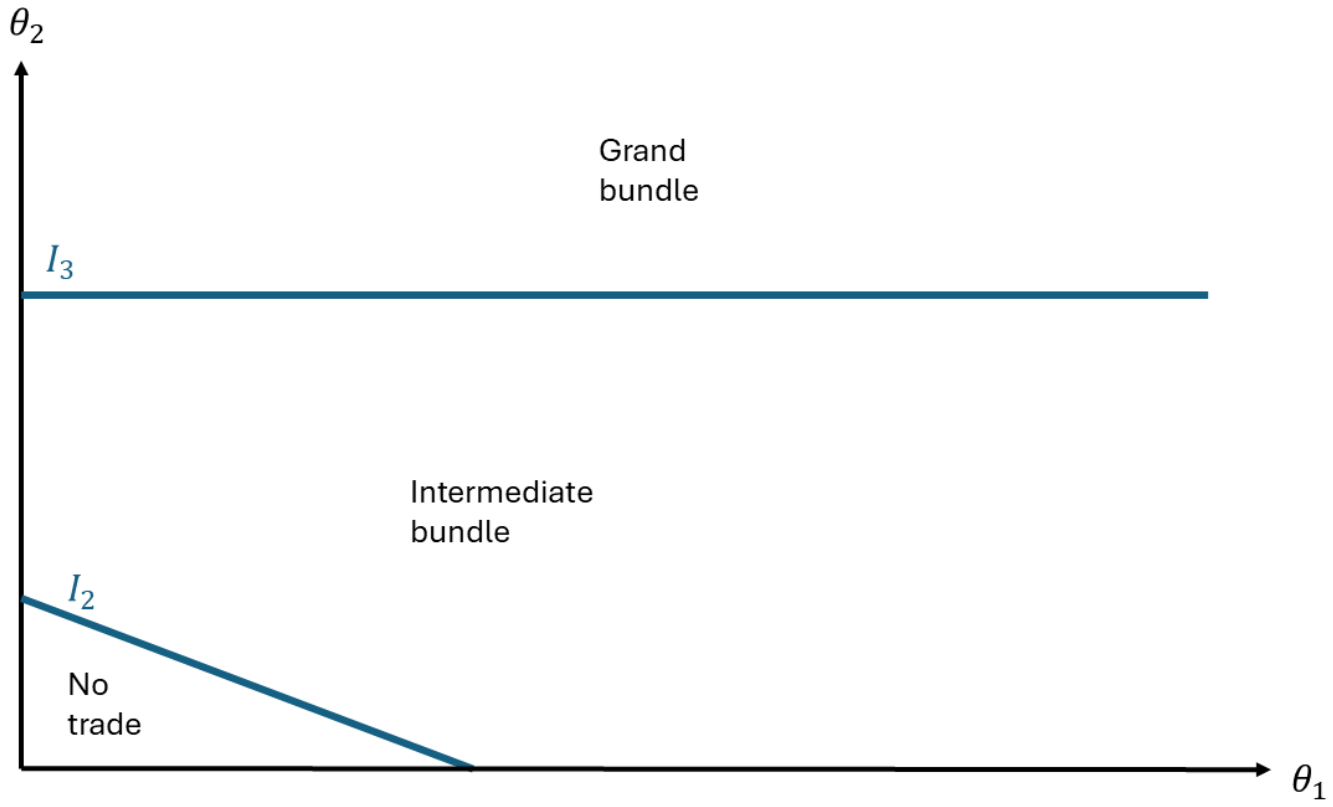

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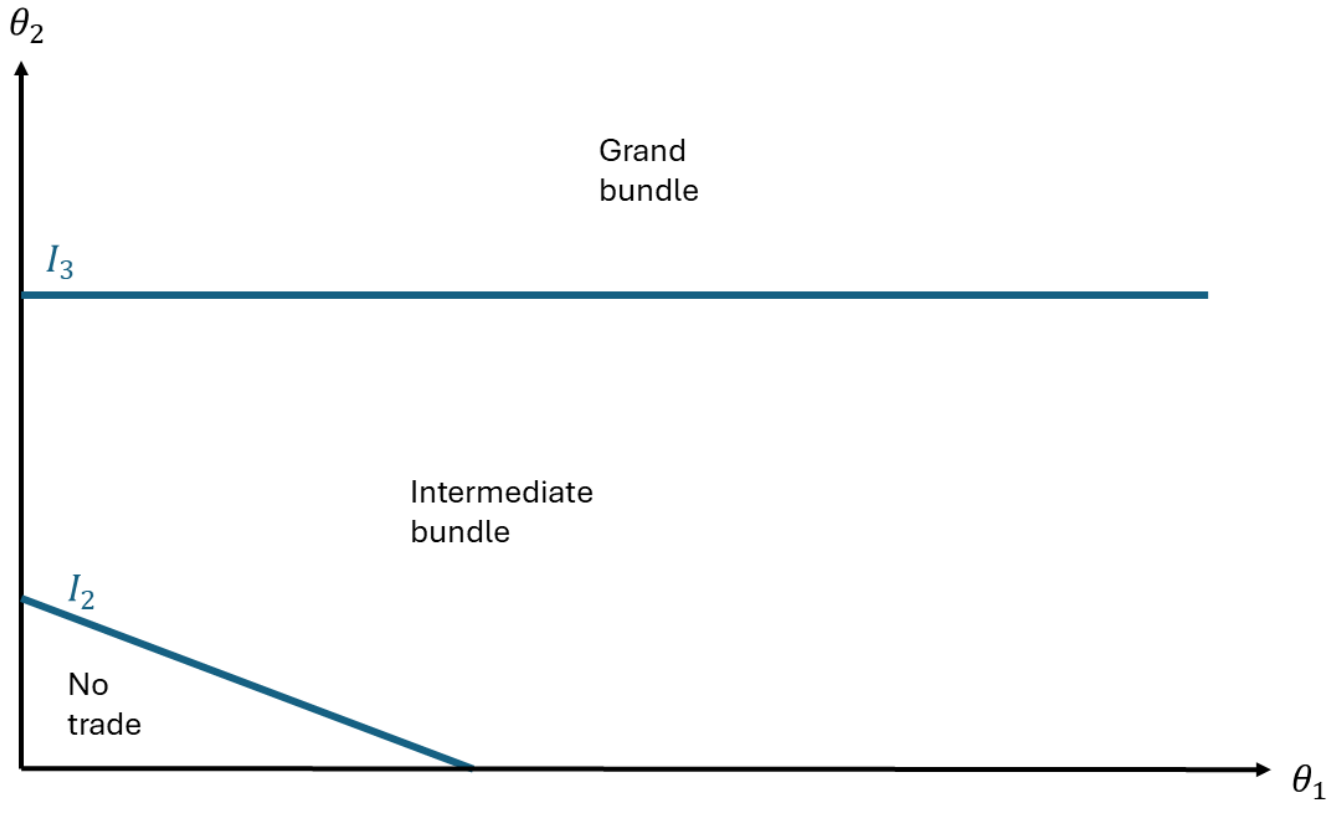

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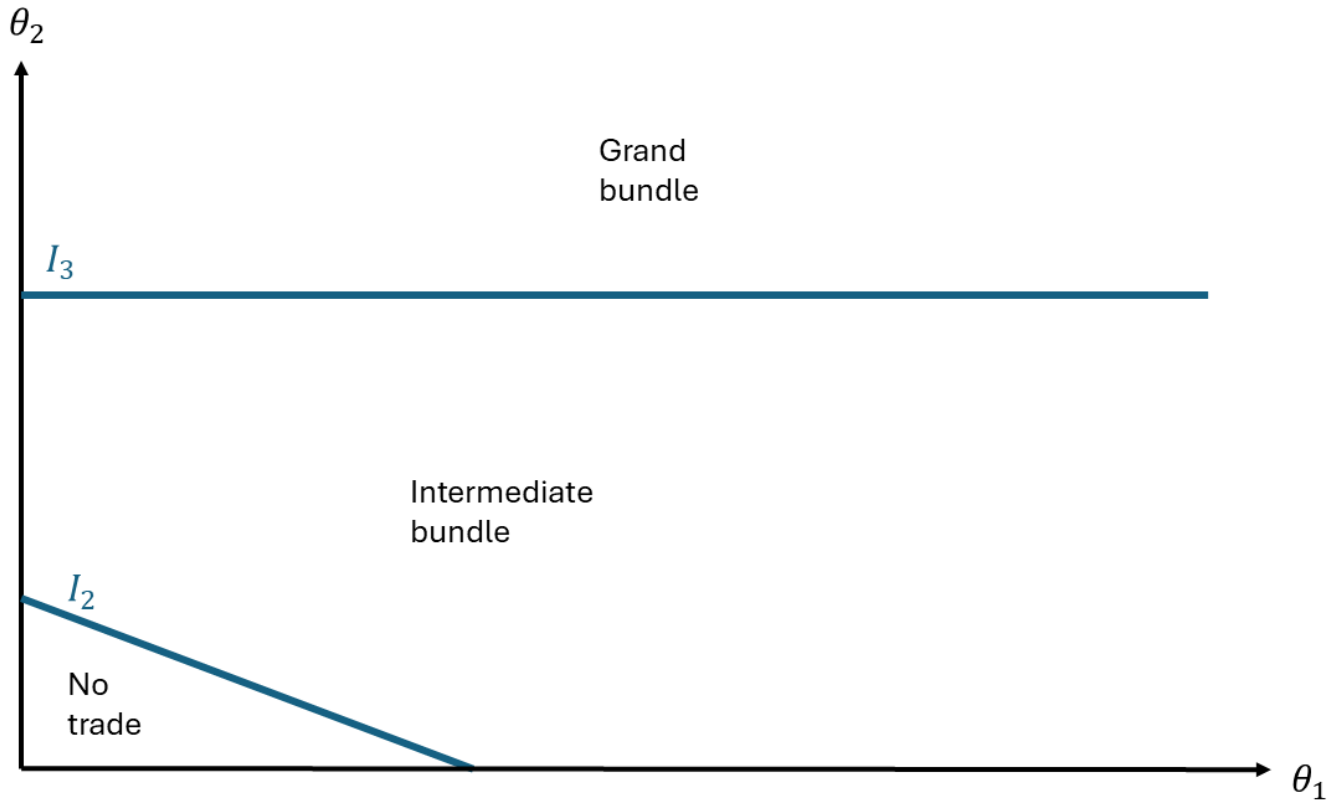

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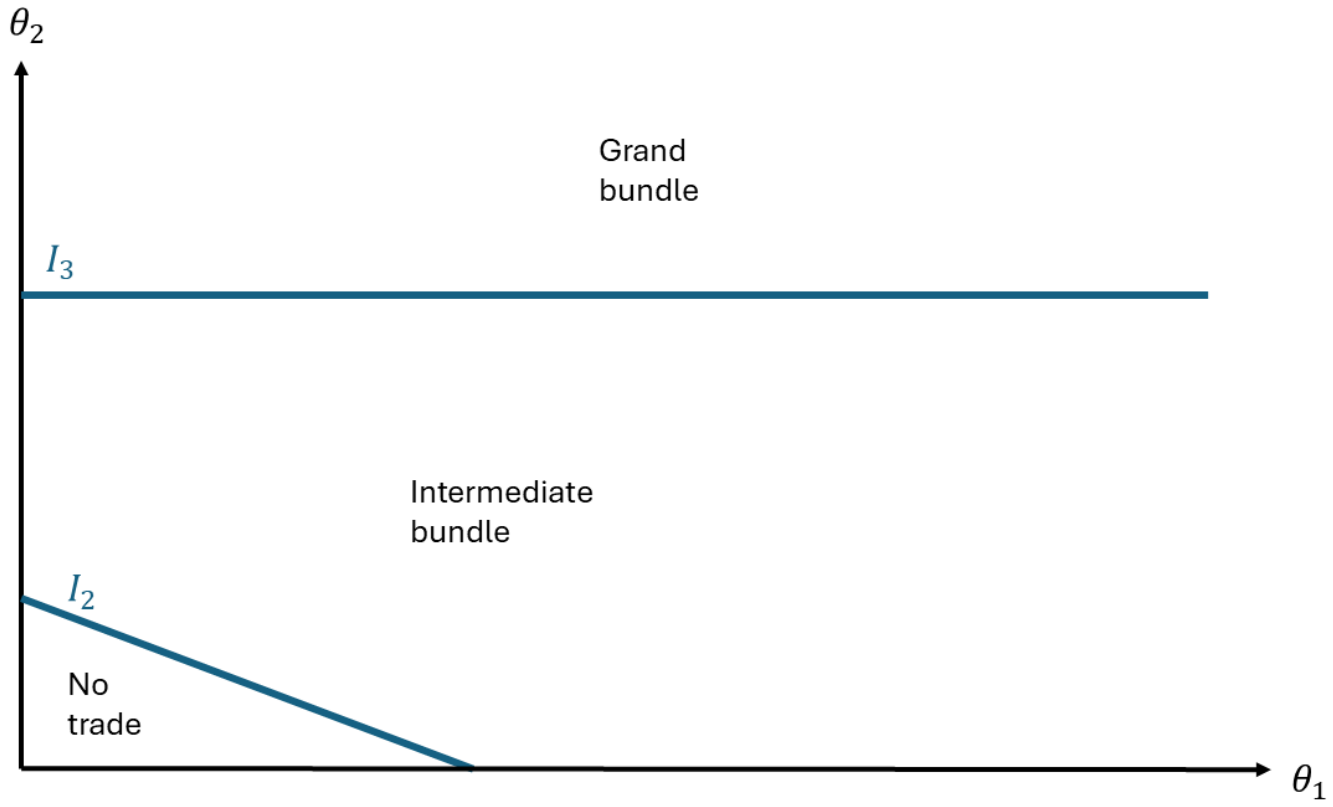

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- **Posted-requirement menu:** single $(1, p^\star, y^\star)$ (plus outside option).

Application 2: Money and ordeals

Theorem

There exists an optimal searchable menu that is a ***posted-requirement menu****.*

If the principal cares only about ***profit*** *($\alpha = 0$), then a* ***posted-price menu*** *is optimal among searchable menus.*

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- Proof: Similar idea but **no upper bound** on y!

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- A tax schedule is **progressive** if $T(\cdot)$ is convex.
- Type θ has **linear preferences** if payoff is $c - \alpha y$ for some $\alpha \in \mathbb{R}_+$; Θ is **rich** if it includes all linear preferences.

Application 3: Income taxation

Theorem

Any progressive tax schedule is a ***searchable menu*** *ordered by earnings.*

Conversely, if Θ *is rich, then any searchable menu must correspond to a* ***progressive tax schedule****.*

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- Provides a **normative foundation** for progressive taxation (see also Doligalski, 2019 and Vairo, 2025).

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- **Progressivity** $\iff$ **searchability** on rich type spaces.
- Provides a **normative foundation** for progressive taxation (see also Doligalski, 2019 and Vairo, 2025).
- Searchability is a necessary condition for the **tax perturbation** approach (Saez, 2001) to work!

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- Bigger problem: In a taxation problem with multi-dimensional allocation, “laissez-faire” is **not searchable**.
- Intuitively, agent’s problem is still tractable if it decomposes into a small number of separable decisions.
- We will relax our constraint by allowing choices from **multiple menus**.

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- We restrict attention to **finite menus**.

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- A **neighbor** of y changes exactly **one** component y_K to its predecessor or successor in $\preceq_K$.

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- Starting anywhere, successive strictly improving local moves reach a **global optimum**.
- With a single component menu, every searchable menu is grid-searchable.

Grid searchability: Characterization

Say that p has ***block-additive pricing*** if there exist functions $p_K : \mathcal{Y}_K \to \mathbb{R}$ for all $K \in \mathcal{K}$ such that

$$p(y) = \sum_{K \in \mathcal{K}} p_K(y_K) \,.$$

Theorem

Let $(\mathcal{Y}, p, \preceq)$ be a product menu with a strictly increasing p.

- *If Θ is rich and $(\mathcal{Y}, p, \preceq)$ is grid-searchable, then there exists an outcome-equivalent product menu with **block-additive pricing**, where each **component menu is searchable**.*
- *If $(\mathcal{Y}, p, \preceq)$ has block-additive pricing where each component menu is searchable, then $(\mathcal{Y}, p, \preceq)$ is grid-searchable.*

Application 1: Multi-product monopoly

Return to **Application 1**: $q \in [0, 1]^n$, additive values, and a full-support type density with finite first moments.

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Corollary

For any partition $\mathcal{K}$, there exists an optimal menu among grid-searchable menus with strictly increasing prices that has:

- ***block-additive pricing**;*
- *an **upgrade menu** within each block K, with at most $|K|$ priced tiers and the full bundle at the highest tier.*

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Coarsest partition: the original upgrade menu.

Finest partition: separate posted prices for each good.

Application 2: Money and ordeals

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For any partition and any grid-searchable product menu with strictly increasing prices, there exists a ***posted-requirement menu*** *achieving a weakly better outcome for the designer.*

- Under block-additive pricing, components containing only ordeals offer no gain relative to choosing zero ordeals.
- Only the component containing the productive dimension q matters $\Longrightarrow$ previous result applies.

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  - **Richness:** all linear preferences $c - \sum_i \alpha_i y_i$ are present.

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Corollary

*Suppose $\mathcal{K}$ is the **finest** partition. Then,*

$$T(y) = \sum_i T_i(y_i) \text{ and each } T_i \text{ is progressive}$$

if and only if the product menu is grid-searchable (for any discrete grid of income levels).

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Implication: Separate search across income sources requires **separable taxation**, with **progressivity within each source**.

Note: Coarser partitions can permit nonseparable taxes.

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- A fairly natural, economically meaningful restriction makes multidimensional screening **manageable**.
- Strong **predictions** about optimal mechanisms in applications.
- Searchability appears to be a **portable** notion.

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- (Grid-)searchability is **too restrictive** in complex non-linear environments.

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- **Still not easy** to get to closed-form economically insightful solutions (incremental nesting is not the nicest constraint).

  $\Longrightarrow$ **Search for alternative frameworks**

- Not clear **why** the mechanism designer should care about making the life of the agent easier (cf. criticism of simplicity in Li ⓡ Dworczak, 2024).

  $\Longrightarrow$ **Worst-case optimality foundation for searchability**

- (Grid-)searchability is **too restrictive** in complex non-linear environments.

  $\Longrightarrow$ **Other notions of search**

Appendix

A search-game formulation

Definition of search game:

Consider a finite ordered menu $x^1 \prec \cdots \prec x^m$.

- **Ascending search game:** Start at option 1. At each $s < m$, the agent chooses whether to:
  - **clinch** x^s and stop;
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- **Descending search game:** The same procedure in reverse.

Searchability and one-step simplicity

- Pycia and Troyan (2023): A strategy is **one-step dominant** if, at each on-path node, there is a plan for the immediate next node whose **worst outcome** is weakly preferred to the **best outcome from deviating now**.
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Proposition

*In a regular environment, a finite irredundant menu is **searchable** if and only if both its ascending and descending **search games are one-step simple**.*

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Intuition: Comparing clinching now with continuing and clinching the next option is enough to find an optimum.

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