

Optimal Redistribution via Income Taxation and Market Design

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Sir John Hicks Lecture

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Introduction

How to allocate scarce resources?

Introduction

Use markets.

Use markets.

(Smith, 1776)

Use markets.

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I Welfare Theorem

How to allocate scarce resources

**How to allocate scarce resources
in the presence of inequality?**

Introduction

Still markets.

Still **markets**.

II Welfare Theorem

How to allocate scarce resources

How to allocate scarce resources in the presence of inequality

**How to allocate scarce resources
in the presence of **inequality**
and **incomplete information**?**

Example

Example

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Example

Based on ideas in: Dworczak [Ⓙ] Kominers [Ⓙ] Akbarpour (2021)

Shout-out to: **Weitzman** (1977), **Spence** (1977), **Nichols and Zeckhauser** (1982), **Blackorby and Donaldson** (1988), **Condorelli** (2013)

Example

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- All agents have the same utility function

$$\log(w) + u \cdot x,$$

where x is the probability of having a house.

Example

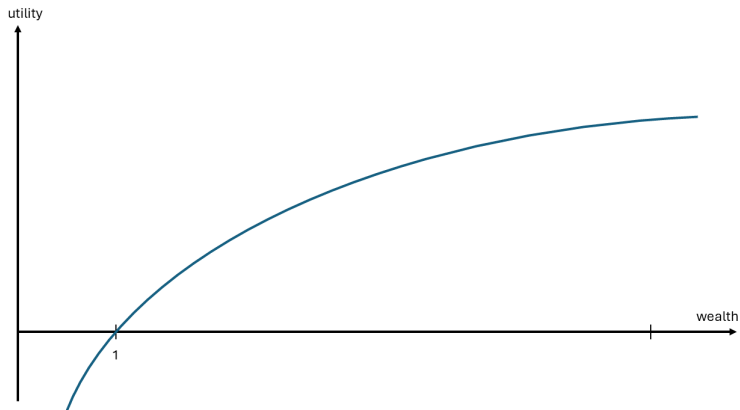
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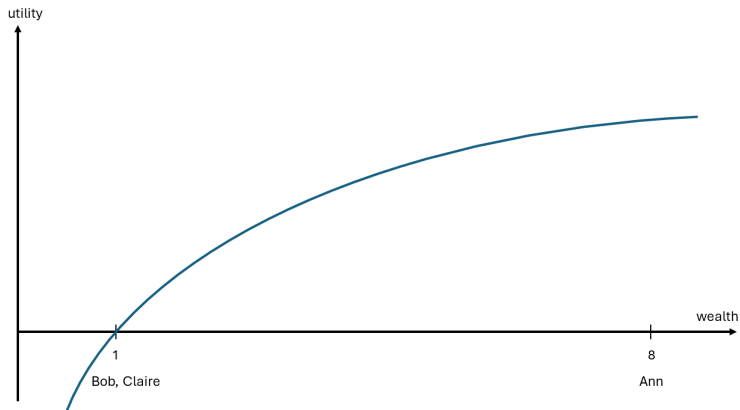
where x is the probability of having a house.

- Social planner aims to maximize the sum of utilities.

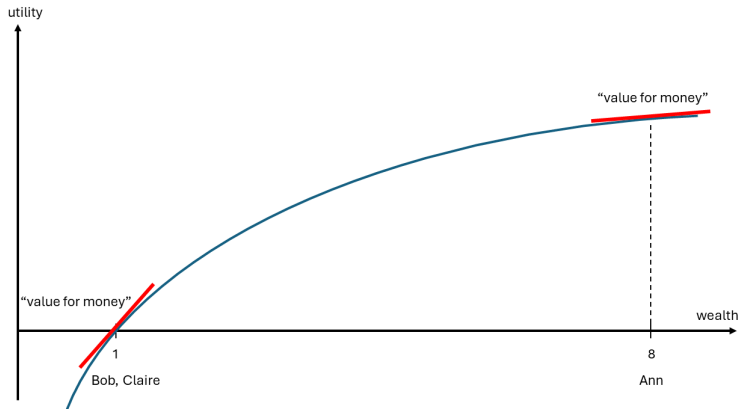
Example



Example



Example



Example

	utility for good	value for money	
Ann	6	$1/8$	
Bob	18	1	
Claire	24	1	

Example

	utility for good	value for money	willingness to pay
Ann	6	$1/8$	48
Bob	18	1	18
Claire	24	1	24

Example

	utility for good	value for money	willingness to pay
Ann	6	1/8	48
Bob	18	1	18
Claire	24	1	24

Equivalent representation of the problem:

	util. good	val. money	willing. pay	welfare weight
Ann	48	1	48	1/8
Bob	18	1	18	1
Claire	24	1	24	1

Example

	val. good	val. money	WTP	utility
Ann	6	1/8	48	
Bob	18	1	18	
Claire	24	1	24	

Mechanism #1: **Market equilibrium** (+ lump-sum transfers)

Example

	val. good	val. money	WTP	utility
Ann	6	1/8	48	
Bob	18	1	18	
Claire	24	1	24	

Mechanism #1: **Market equilibrium** (+ lump-sum transfers)

Market-clearing price: 24

House owners: Ann & Claire

Example

	val. good	val. money	WTP	utility
Ann	6	1/8	48	
Bob	18	1	18	
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Mechanism #1: **Market equilibrium** (+ lump-sum transfers)

Market-clearing price: 24

House owners: Ann & Claire

Lump-sum transfer: 16

Example

	val. good	val. money	WTP	utility
Ann	6	1/8	48	$6 - 1/8 \cdot (24 - 16)$
Bob	18	1	18	
Claire	24	1	24	

Mechanism #1: **Market equilibrium** (+ lump-sum transfers)

Market-clearing price: 24

House owners: Ann & Claire

Lump-sum transfer: 16

Example

	val. good	val. money	WTP	utility
Ann	6	1/8	48	5
Bob	18	1	18	$0 + 1 \cdot 16$
Claire	24	1	24	

Mechanism #1: **Market equilibrium** (+ lump-sum transfers)

Market-clearing price: 24

House owners: Ann & Claire

Lump-sum transfer: 16

Example

	val. good	val. money	WTP	utility
Ann	6	1/8	48	5
Bob	18	1	18	16
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	val. good	val. money	WTP	utility
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Bob	18	1	18	16
Claire	24	1	24	16

Mechanism #1: **Market equilibrium** (+ lump-sum transfers)

Market-clearing price: 24

House owners: Ann & Claire

Lump-sum transfer: 16

Total utility: 37

Example

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Ann	6	1/8	48	
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Ann	6	1/8	48	
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Mechanism #2: **Random allocation**

Example

	val. good	val. money	WTP	utility
Ann	6	1/8	48	
Bob	18	1	18	
Claire	24	1	24	

Mechanism #2: **Random allocation**

Price: 0

House owners: $\frac{2}{3}$ Ann & $\frac{2}{3}$ Bob & $\frac{2}{3}$ Claire

Example

	val. good	val. money	WTP	utility
Ann	6	1/8	48	
Bob	18	1	18	
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Mechanism #2: **Random allocation**

Price: 0

House owners: $\frac{2}{3}$ Ann & $\frac{2}{3}$ Bob & $\frac{2}{3}$ Claire

Lump-sum transfer: 0

Example

	val. good	val. money	WTP	utility
Ann	6	1/8	48	$2/3 \cdot 6$
Bob	18	1	18	
Claire	24	1	24	

Mechanism #2: **Random allocation**

Price: 0

House owners: $\frac{2}{3}$ Ann & $\frac{2}{3}$ Bob & $\frac{2}{3}$ Claire

Lump-sum transfer: 0

Example

	val. good	val. money	WTP	utility
Ann	6	1/8	48	4
Bob	18	1	18	$2/3 \cdot 18$
Claire	24	1	24	

Mechanism #2: **Random allocation**

Price: 0

House owners: $\frac{2}{3}$ Ann & $\frac{2}{3}$ Bob & $\frac{2}{3}$ Claire

Lump-sum transfer: 0

Example

	val. good	val. money	WTP	utility
Ann	6	1/8	48	4
Bob	18	1	18	12
Claire	24	1	24	$2/3 \cdot 24$

Mechanism #2: **Random allocation**

Price: 0

House owners: $\frac{2}{3}$ Ann & $\frac{2}{3}$ Bob & $\frac{2}{3}$ Claire

Lump-sum transfer: 0

Example

	val. good	val. money	WTP	utility
Ann	6	1/8	48	4
Bob	18	1	18	12
Claire	24	1	24	16

Mechanism #2: **Random allocation**

Price: 0

House owners: $\frac{2}{3}$ Ann & $\frac{2}{3}$ Bob & $\frac{2}{3}$ Claire

Lump-sum transfer: 0

Total utility: 32 (< 37)

Example

	val. good	val. money	WTP	utility
Ann	6	1/8	48	4
Bob	18	1	18	12
Claire	24	1	24	16

Mechanism #2: **Random allocation**

Price: 0

House owners: $\frac{2}{3}$ Ann & $\frac{2}{3}$ Bob & $\frac{2}{3}$ Claire

Lump-sum transfer: 0

Among all mechanisms charging a single price p , $p = 24$ is optimal.

Example

	val. good	val. money	WTP	utility
Ann	6	1/8	48	
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Example

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Mechanism #3: Allocate 1st unit by **lottery**, sell 2nd unit in **market**

Example

	val. good	val. money	WTP	utility
Ann	6	1/8	48	
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Claire	24	1	24	

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Market-clearing price for 2nd unit: 24

(Ann's indifference: $6 - 1/8 \cdot 24 = 1/2 \cdot 6$)

House owners: Ann & 50% Claire & 50% Bob

Example

	val. good	val. money	WTP	utility
Ann	6	1/8	48	
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House owners: Ann & 50% Claire & 50% Bob

Lump-sum transfer: 8

Example

	val. good	val. money	WTP	utility
Ann	6	1/8	48	$6 - 1/8 \cdot (24 - 8)$
Bob	18	1	18	
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Example

	val. good	val. money	WTP	utility
Ann	6	1/8	48	4
Bob	18	1	18	$1/2 \cdot 18 + 1 \cdot 8$
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Example

	val. good	val. money	WTP	utility
Ann	6	1/8	48	4
Bob	18	1	18	17
Claire	24	1	24	20

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Market-clearing price for 2nd unit: 24

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Total utility: 41 (> 37)

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Lump-sum transfer: 8

This is in fact the **optimal mechanism!** (subject to IC constraints)

Example

	val. money	WTP	utility—market	utility—hybrid
Ann	1/8	48	5	4
Bob	1	18	16	17
Claire	1	24	16	20

Example

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Intuition:

- Values for money (welfare weights) are **negatively correlated** with willingness to pay.

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- Designer can increase the utility of agents with high values for money by **reducing the price of the rationed option**.

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- Market **behavior** can be **used to screen**: Choosing the lottery reveals a high value for money.
- Designer can increase the utility of agents with high values for money by **reducing the price of the rationed option**.
- Mathematically, rationing results from Myerson's (1981) **ironing**.

Inequality-aware Market Design (IMD)

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- Redistribution in **simple (one- and two-sided) markets**:
DKA (2021), Reuter and Groh (2020), ADK (2024), Tokarski KAD (2023) (energy), ...
- Problems with **allocative externalities**:
Kang (2023), A Budish DK (2024) (Covid-19 vaccines), Pai and Strack (2025), ...
- Optimality of **costly screening** and allocating bads:
Kang and Zheng (2020), Yang DA (2024), D (2025), ...
- Interaction of redistributive mechanisms and **private markets**:
Kang (2023), Kang and Watt (2024a, 2024b), ...
- **Information-design** interventions:
Doval and Smolin (2024), Augias [Ⓘ] Ghersengorin [Ⓘ] Barreto (2025), ...

Inequality-aware Market Design (IMD)

Policymakers frequently **distort allocation of goods and services**, at least in part due to **redistributive concerns**:

- **Housing** (rent control; public housing; LIHTC in the US)
- **Healthcare** (public healthcare in Europe, Medicare and Medicaid in the US)
- **Food** (food stamps in the US; in-kind food transfer programs (rice, wheat,...) in developing countries)
- **Energy** (electricity in virtually all European countries at the moment; kerosene in India; energy subsidies in Latin America;...)
- **Covid-19 vaccines** (allocated for free in most countries)
- **Road access** (congestion pricing is still rare in practice)
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- ...

Desideratum: Understand the optimal design of such policies.

Question

**How to allocate scarce resources
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How to allocate scarce resources in the presence of **inequality and **incomplete information**?**

I have cheated a bit...

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 - Attempt to build a **bridge to public finance** literature.

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- But what if the designer can **control macro-level tools** to address inequality?
- What about **income taxation**? ← **public-finance approach**
- **This paper:**
 - Attempt to build a **bridge to public finance** literature.
 - Use modern tools from mechanism design to study the **interaction between IMD and income taxation**.

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Market-design perspective: **Taste heterogeneity is crucial!**
It is known that **AS theorem may fail** in this case:

Cremer and Gahvari (1995, 1998), Saez (2002), Kaplow (2008), Golosov et al. (2013), Gauthier and Henriët (2018), Allcott et al. (2019), Scheuer and Slemrod (2020), Schulz (2021), Hellwig and Werquin (2023), Ferey et al. (2024), Gaubert et al. (2025), Ahlvik et al. (2025),...

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Preview of results

Framework: Agents differ in **taste** and **ability**, choose **labor** and **consumption**; designer maximizes welfare over IC mechanisms.

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Result #1: Assuming separable utility functions, if **three strong assumptions** hold, then redistribution should be accomplished **entirely via income taxes**.

Result #2: If any of the three assumptions are relaxed, then **redistributive market design** ($\approx$ non-linear commodity taxation) also plays a role.

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Result #1: Assuming separable utility functions, if **three strong assumptions** hold, then redistribution should be accomplished **entirely via income taxes**.

Result #2: If any of the three assumptions are relaxed, then **redistributive market design** ($\approx$ non-linear commodity taxation) also plays a role.

- We provide **closed-form solutions** in a linear model with two goods, binary ability type, and continuous taste type.
- MD approach uncovers a **strong role of taste heterogeneity in motivating market-level interventions**.

Model

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Model

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- Allocation:
 - Labor supply: (pre-tax) **earnings** $z \in \mathbb{R}_+$
 - Consumption: vector of **consumption** $c \in \mathbb{R}_+^L$
- Each agent’s utility function takes the **separable** form:

$$v(c, t) - w(z, a),$$

where $v(c, t)$ is the utility of consumption, and $w(z, a)$ is the disutility of labor supply.

Model

- Goods c can be produced at **fixed marginal costs** $k \in \mathbb{R}_+^L$.

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- The planner uses social welfare weights $\lambda(t, a) \geq 0$, with average weight normalized to 1.

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- Planner **maximizes social welfare**

$$\int \lambda(t, a) U(t, a) dF(t, a)$$

subject to **incentive compatibility** and the **resource constraint**:

$$\int [z(t, a) - k \cdot c(t, a)] dF(t, a) \geq G.$$

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- We assume existence of solutions and strong duality.

When Interventions in Markets are Redundant

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Useful concept:

- Let m denote *disposable income*.

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- Let $v^*(m, t)$ denote utility type t gets from disposable income m (at prices equal to marginal costs):

$$v^*(m, t) = \max_{c \in \mathbb{R}_+^L} \{v(c, t) : k \cdot c \leq m\}.$$

When Interventions in Markets are Redundant

Useful concept:

- Let m denote *disposable income*.
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$$v^*(m, t) = \max_{c \in \mathbb{R}_+^L} \{v(c, t) : k \cdot c \leq m\}.$$

- Define the **marginal utility of disposable income** as

$$\frac{\partial v^*(m, t)}{\partial m}.$$

When Interventions in Markets are Redundant

Theorem

Suppose that:

1. *Taste does not affect the marginal utility of disposable income:*

$$v^*(m, t) = v_0^*(m) + v_1^*(t), \quad \forall(t, m).$$

2. *Welfare weights depend only on the ability type:*

$$\lambda(t, a) = \bar{\lambda}(a), \quad \forall(t, a).$$

3. *Ability and taste types are statistically independent:*

$$F(t, a) = F_t(t)F_a(a), \quad \forall(t, a).$$

When Interventions in Markets are Redundant

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4. The solution to the relaxed problem is **implementable in the original problem** by pricing goods at marginal cost.

When Interventions in Markets are Redundant

Theorem

Suppose that:

1. *Taste does not affect the marginal utility of disposable income:*

$$v^*(m, t) = v_0^*(m) + v_1^*(t), \quad \forall(t, m).$$

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Interaction of Income Taxation and Market Design

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(without Assumption 1)

Assumption 1 with two goods

- There are two goods: **numeraire** $n \geq 0$ (marg. cost 1), and a unit-demand **good with quality** $x \in [0, 1]$ (marg. cost k),

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- Assume k high enough $\implies n \geq 0$ must bind for some agents.

Simplified framework

- Framework: **linear utility, two goods, binary ability.**

- **Utility function** is:

$$n + tx - \frac{z}{a},$$

with $n \geq 0$ (numeraire), $x \in [0, 1]$ (quality), $z \in [0, \bar{z}]$ (earnings).

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- **Assumpt. 2 and 3 hold:** $\bar{\lambda}_H \leq \bar{\lambda}_L$ and $t \sim F$ (for both abilities)

Optimal mechanism without Assumption 1

Theorem

Suppose $\frac{1-F(t)}{f(t)}$ is non-increasing and $\frac{F(t)}{f(t)}$ is non-decreasing.

There exists an optimal mechanism in which:

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- High-quality option with tax ⇒ more surplus can be extracted from (some) high-ability agents.

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Contrast to original Atkinson-Stiglitz logic (example: **healthcare**)

- AS: More able agents consume more healthcare **only** because they have more income; better to redistribute income without distorting the allocation of healthcare.
- Our setting: Demand for healthcare stems from **both** income (ability) and need (taste); subsidies for low-quality healthcare improve redistribution because they **endogenously target low-ability agents who have a particularly high marginal utility of income**.

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- **Market-level interventions** are generally **part of an optimal response to inequality**.
- This is **not** to say that all market interventions motivated by redistributive concerns are justified...
- ... but it does mean that **traditional theoretical foundations opposing goods market distortions are insufficient**.

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- **More research** incorporating inequality into microeconomic theory is **needed**!

Appendix

▸ Extra results

▸ Concave utility

▸ Proofs

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 - The optimal mechanism can be implemented as a **means-tested subsidy for the good**.

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- Mechanism 1 is optimal iff the motive to redistribute across ability types is sufficiently strong.

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 $\implies$ Optimal x_L takes the form $x_L(t) = \bar{q}_L \cdot \mathbf{1}_{\{t \geq t_L^*\}}$.

Derivation of the optimal mechanism

- We have an outside-option constraint: $\forall t \in [0, \bar{t}]$,

$$\Delta + \int_0^t x_H(\tau) d\tau \equiv U_H(t) \geq U_L(t) \equiv \int_0^t x_L(\tau) d\tau.$$

- Fixing x_L , this is a linear mechanism design problem with a **type-dependent outside-option constraint** (Jullien, 2000)
- Dworczak and Muir (2024): This problem can be solved by adapting the **ironing technique** (Myerson, 1981).
 - $\implies$ The optimal (Δ, x_H) depend **linearly** on x_L .
- The problem of optimizing over x_L is linear:
 - $\implies$ Optimal x_L takes the form $x_L(t) = \bar{q}_L \cdot \mathbf{1}_{\{t \geq t_L^*\}}$.
 - $\implies$ Optimal x_H has at most three steps.

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- Suppose that u is a strictly increasing, strictly concave, continuously differentiable function.

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 - The goods allocation of high-ability agents with high enough taste type is distorted downwards.