Optimal Redistribution via Income Taxation and Market Design

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Introduction

How to allocate scarce resources?

Introduction

Use **markets**.

Introduction

Use **markets**.

(Smith, 1776)

Use **markets**.

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I Welfare Theorem

How to allocate scarce resources

How to allocate scarce resources
in the presence of **inequality**?

Introduction

Still **markets**.

Still **markets**.

II Welfare Theorem

How to allocate scarce resources

How to allocate scarce resources
in the presence of **inequality**

How to allocate scarce resources
in the presence of **inequality**
and **incomplete information**?

Example

Example

Based on ideas in: Dworczak ⓡ Kominers ⓡ Akbarpour (2021)

Shout-out to: **Weitzman** (1977), **Spence** (1977), **Nichols and Zeckhauser** (1982), **Condorelli** (2013)

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  Ann: 8, Bob: 1, Claire: 1.

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- All agents have the same utility function

$$\log(w) + u \cdot x,$$

  where x is the probability of having a house.

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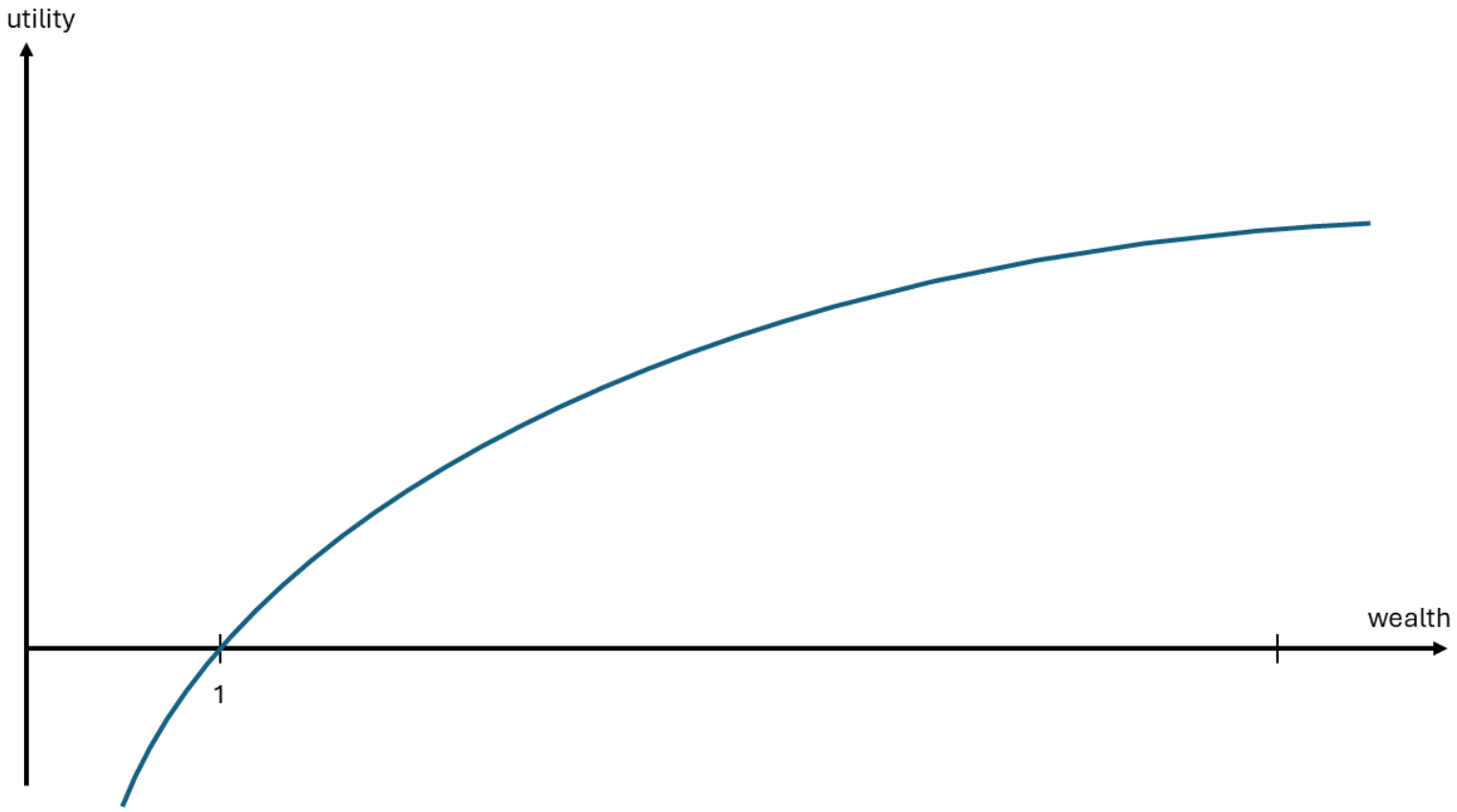
  $$\log(w) + u \cdot x,$$

  where x is the probability of having a house.
- Social planner aims to maximize the sum of utilities.

Example



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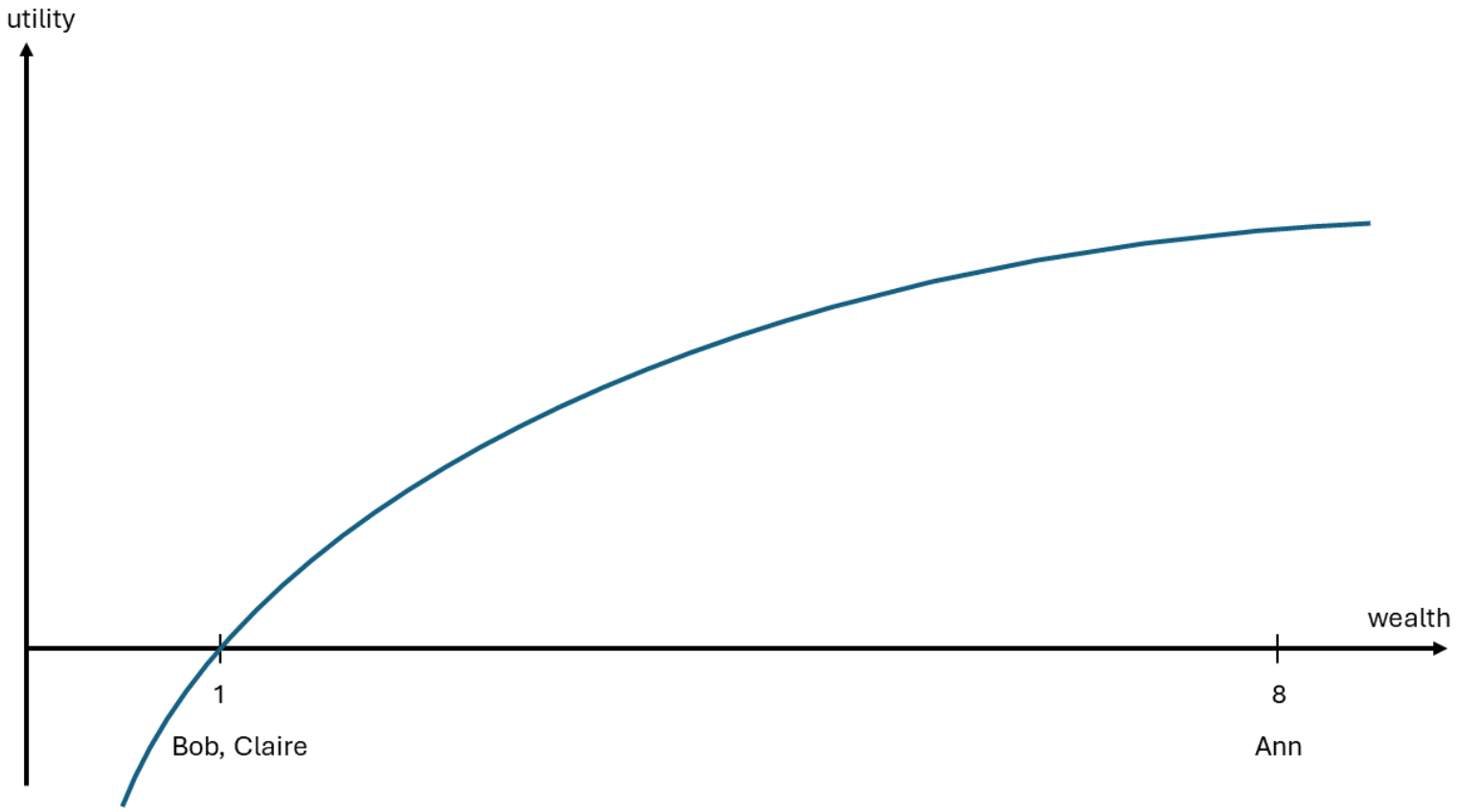

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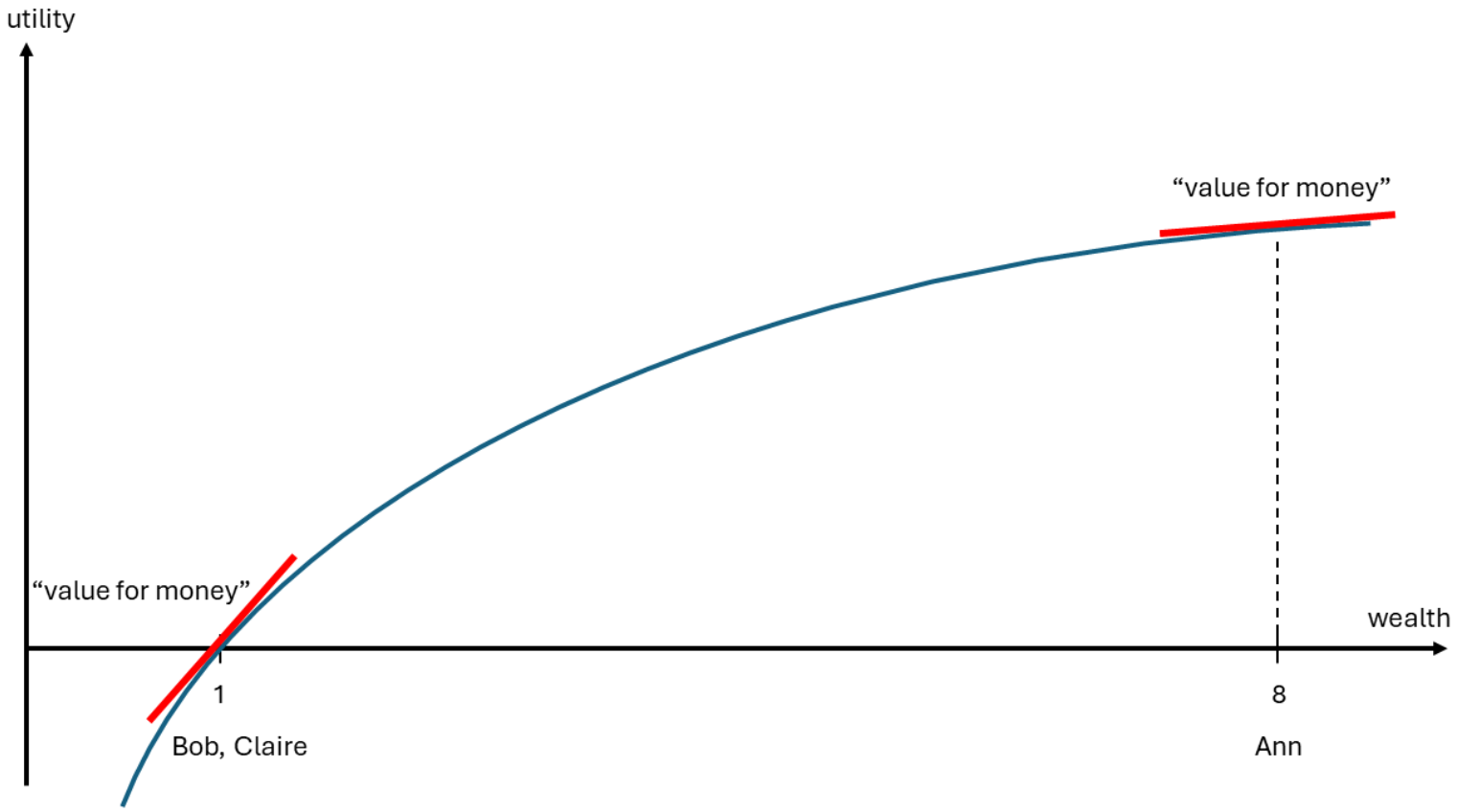
utility
"value for money"
"value for money"
wealth
1
8
Bob, Claire
Ann

Example

	utility for good	value for money	
Ann	6	1/8	
Bob	18	1	
Claire	24	1	

Example

	utility for good	value for money	willingness to pay
Ann	6	1/8	48
Bob	18	1	18
Claire	24	1	24

Example

	utility for good	value for money	willingness to pay
Ann	6	1/8	48
Bob	18	1	18
Claire	24	1	24

Equivalent representation of the problem:

	util. good	val. money	willing. pay	welfare weight
Ann	48	1	48	1/8
Bob	18	1	18	1
Claire	24	1	24	1

Example

	val. good	val. money	WTP	**utility**
Ann	6	1/8	48	
Bob	18	1	18	
Claire	24	1	24	

Mechanism #1: **Market equilibrium** (+ lump-sum transfers)

Example

	val. good	val. money	WTP	**utility**
Ann	6	1/8	48	
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Market-clearing price: 24

House owners: Ann & Claire

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Mechanism #1: **Market equilibrium** (+ lump-sum transfers)

Market-clearing price: 24

House owners: Ann & Claire

Lump-sum transfer: 16

Example

	val. good	val. money	WTP	**utility**
Ann	6	1/8	48	$6 - 1/8 \cdot (24 - 16)$
Bob	18	1	18	
Claire	24	1	24	

Mechanism #1: **Market equilibrium** (+ lump-sum transfers)

Market-clearing price: 24

House owners: Ann & Claire

Lump-sum transfer: 16

Example

	val. good	val. money	WTP	**utility**
Ann	6	1/8	48	5
Bob	18	1	18	$0 + 1 \cdot 16$
Claire	24	1	24	

Mechanism #1: **Market equilibrium** (+ lump-sum transfers)

Market-clearing price: 24

House owners: Ann & Claire

Lump-sum transfer: 16

Example

	val. good	val. money	WTP	**utility**
Ann	6	1/8	48	5
Bob	18	1	18	16
Claire	24	1	24	$24 - 1 \cdot (24 - 16)$

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Market-clearing price: 24

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Example

	val. good	val. money	WTP	**utility**
Ann	6	1/8	48	5
Bob	18	1	18	16
Claire	24	1	24	16

Mechanism #1: **Market equilibrium** (+ lump-sum transfers)

Market-clearing price: 24

House owners: Ann & Claire

Lump-sum transfer: 16

Total utility: 37

Example

	val. good	val. money	WTP	**utility**
Ann	6	1/8	48	
Bob	18	1	18	
Claire	24	1	24	

Example

	val. good	val. money	WTP	**utility**
Ann	6	1/8	48	
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Claire	24	1	24	

Mechanism #2: **Random allocation**

Example

	val. good	val. money	WTP	**utility**
Ann	6	1/8	48	
Bob	18	1	18	
Claire	24	1	24	

Mechanism #2: **Random allocation**

Price: 0

House owners: $\frac{2}{3}$ Ann & $\frac{2}{3}$ Bob & $\frac{2}{3}$ Claire

Example

	val. good	val. money	WTP	**utility**
Ann	6	1/8	48	
Bob	18	1	18	
Claire	24	1	24	

Mechanism #2: **Random allocation**

Price: 0

House owners: $\frac{2}{3}$ Ann & $\frac{2}{3}$ Bob & $\frac{2}{3}$ Claire

Lump-sum transfer: 0

Example

	val. good	val. money	WTP	**utility**
Ann	6	1/8	48	$2/3 \cdot 6$
Bob	18	1	18	
Claire	24	1	24	

Mechanism #2: **Random allocation**

Price: 0

House owners: $\frac{2}{3}$ Ann & $\frac{2}{3}$ Bob & $\frac{2}{3}$ Claire

Lump-sum transfer: 0

Example

	val. good	val. money	WTP	**utility**
Ann	6	1/8	48	4
Bob	18	1	18	$2/3 \cdot 18$
Claire	24	1	24	

Mechanism #2: **Random allocation**

Price: 0

House owners: $\frac{2}{3}$ Ann & $\frac{2}{3}$ Bob & $\frac{2}{3}$ Claire

Lump-sum transfer: 0

Example

	val. good	val. money	WTP	**utility**
Ann	6	1/8	48	4
Bob	18	1	18	12
Claire	24	1	24	$2/3 \cdot 24$

Mechanism #2: **Random allocation**

Price: 0

House owners: $\frac{2}{3}$ Ann & $\frac{2}{3}$ Bob & $\frac{2}{3}$ Claire

Lump-sum transfer: 0

Example

	val. good	val. money	WTP	**utility**
Ann	6	1/8	48	4
Bob	18	1	18	12
Claire	24	1	24	16

Mechanism #2: **Random allocation**

Price: 0

House owners: $\frac{2}{3}$ Ann & $\frac{2}{3}$ Bob & $\frac{2}{3}$ Claire

Lump-sum transfer: 0

Total utility: 32 (< 37)

Example

	val. good	val. money	WTP	**utility**
Ann	6	1/8	48	4
Bob	18	1	18	12
Claire	24	1	24	16

Mechanism #2: **Random allocation**

Price: 0

House owners: $\frac{2}{3}$ Ann & $\frac{2}{3}$ Bob & $\frac{2}{3}$ Claire

Lump-sum transfer: 0

Among all mechanisms charging a single price p, $p = 24$ is optimal.

Example

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Ann	6	1/8	48	
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Example

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Mechanism #3: Allocate 1st unit by **lottery**, sell 2nd unit in **market**

Example

	val. good	val. money	WTP	**utility**
Ann	6	1/8	48	
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Mechanism #3: Allocate 1st unit by **lottery**, sell 2nd unit in **market**

Market-clearing price for 2nd unit: 24

(Ann's indifference: $6 - 1/8 \cdot \mathbf{24} = 1/2 \cdot 6$)

House owners: Ann & 50% Claire & 50% Bob

Example

	val. good	val. money	WTP	**utility**
Ann	6	1/8	48	
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House owners: Ann & 50% Claire & 50% Bob

Lump-sum transfer: 8

Example

	val. good	val. money	WTP	**utility**
Ann	6	1/8	48	$6 - 1/8 \cdot (24 - 8)$
Bob	18	1	18	
Claire	24	1	24	

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Example

	val. good	val. money	WTP	**utility**
Ann	6	1/8	48	4
Bob	18	1	18	$1/2 \cdot 18 + 1 \cdot 8$
Claire	24	1	24	

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	val. good	val. money	WTP	**utility**
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Example

	val. good	val. money	WTP	**utility**
Ann	6	1/8	48	4
Bob	18	1	18	17
Claire	24	1	24	20

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Total utility: 41 (> 37)

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(Ann's indifference: $6 - 1/8 \cdot \mathbf{24} = 1/2 \cdot 6$)

House owners: Ann & 50% Claire & 50% Bob

Lump-sum transfer: 8

This is in fact the **optimal mechanism**! (subject to IC constraints)

Example

	val. money	WTP	utility—market	utility—hybrid
Ann	1/8	48	5	4
Bob	1	18	16	17
Claire	1	24	16	20

Example

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Intuition:

- Values for money (welfare weights) are **negatively correlated** with willingness to pay.

Example

	val. money	WTP	utility—market	utility—hybrid
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- Values for money (welfare weights) are **negatively correlated** with willingness to pay.
- Market **behavior** can be **used to screen**: Choosing the lottery reveals a high value for money.

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- Designer can increase the utility of agents with high values for money by **reducing the price of the rationed option**.

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Intuition:

- Values for money (welfare weights) are **negatively correlated** with willingness to pay.
- Market **behavior** can be **used to screen**: Choosing the lottery reveals a high value for money.
- Designer can increase the utility of agents with high values for money by **reducing the price of the rationed option**.
- Mathematically, rationing results from Myerson's (1981) **ironing**.

Inequality-aware Market Design (IMD)

- Market with **buyers and sellers** for an indivisible good:
  - DKA (2021), ...
- One-sided allocation with **different qualities and labels**:
  - ADK (2024), Tokarski KAD (2023) (energy pricing), ...
- Problems with **allocative externalities**:
  - Kang (2023), A Budish DK (2024) (Covid-19 vaccines), Pai and Strack (2025), ...
- Optimality of **costly screening** and allocating bads:
  - Kang and Zheng (2020), Yang DA (2024), D (2025), ...
- Interaction of redistributive mechanisms and **private markets**:
  - Kang (2023), Kang and Watt (2024a, 2024b), ...

Inequality-aware Market Design (IMD)

Policymakers frequently **distort allocation of goods and services**, at least in part due to **redistributive concerns**:

- Housing (rent control; public housing; LIHTC in the US)
- Health care (public health care in Europe, Medicare and Medicaid in the US)
- Food (food stamps in the US; in-kind food transfer programs (rice, wheat,...) in developing countries)
- Energy (electricity in virtually all European countries at the moment; kerosene in India; energy subsidies in Latin America;...)
- Covid-19 vaccines (allocated for free in most countries)
- Road access (congestion pricing is still rare in practice)
- ...

Question

How to allocate scarce resources
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I have cheated a bit...

IMD and Income Taxation

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- What if **income taxation** is available as a tool (the public-finance approach)?

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  - When (if at all) **should we distort goods markets** to effect redistribution when income taxation is feasible?

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- But what if the designer can **control other tools** to address inequality?
- What if **income taxation** is available as a tool (the public-finance approach)?
- **Main question for this project**:
  - When (if at all) **should we distort goods markets** to effect redistribution when income taxation is feasible?
  - What is the optimal **interaction between inequality-aware market design and income taxation**?

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Market-design perspective: We should allow for **heterogeneity in consumption tastes**! AS theorem may fail in this case:

Cremer and Gavhari (1995, 1998, 2002), Saez (2002), Diamond and Spinnewijn (2011), Golosov et al. (2013), Gauthier and Henriet (2018), Scheuer and Slemrod (2020), Schulz (2021), Hellwig and Werquin (2023), Ferey et al. (2024), Ahlvik et al. (2024), Gerritsen et al. (2025), ...

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Our paper

Framework: Agents differ in **taste** and **ability**, choose **labor** and **consumption**, designer maximizes welfare over IC mechanisms.

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Result #1: Assuming separable utility functions, if

1. taste type does not affect the marginal utility of income;
2. welfare weights depend only on the ability type;
3. taste type and ability type are statistically independent,

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Result #1: Assuming separable utility functions, if

1. taste type does not affect the marginal utility of income;
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then **redistribution** should be effected **entirely via income taxes**.

Our paper

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Result #2: If any of the assumptions 1, 2, or 3 are relaxed, then **market design also plays a role**.

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Result #2: If any of the assumptions 1, 2, or 3 are relaxed, then **market design also plays a role**.

- We provide **analytical solutions** in a linear model with two goods, binary ability type, and continuous taste type.

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- This allows us to characterize the **interaction** of inequality-aware market design and income taxation.

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- We provide **analytical solutions** in a linear model with two goods, binary ability type, and continuous taste type.
- This allows us to characterize the **interaction** of inequality-aware market design and income taxation.
- Take-away: Interventions in markets are **typically part of the optimal mechanism**.

Model

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- There is a unit mass of agents characterized by two-dimensional types: $(t,\ a)$, where t is "**taste**" and a is "**ability**."

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  - **numeraire** consumption good $c \in \mathbb{R}$,
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- Each agent's utility function is

$$U((c, x, z), (t, a)) = u(c) + v(x; t) - w(z; a),$$

where the functions $u,\ v,\ w$ are increasing.

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Model

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- The planner **maximizes social surplus**

$$\int \lambda(t, a)\, U(t, a)\, dF(t, a)$$

subject to **incentive compatibility** and the **resource constraint**:

$$\int [z(t, a) - c(t, a) - k \cdot x(t, a)]\ dF(t, a) \geq G.$$

Model

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subject to **incentive compatibility** and the **resource constraint**:

$$\int [z(t, a) - c(t, a) - k \cdot x(t, a)]\ dF(t, a) \geq G.$$

- We assume existence of solutions and strong duality.

Atkinson-Stiglitz result

When Interventions in Markets are Redundant

Atkinson-Stiglitz result

Useful concept:

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- Define the **marginal utility of disposable income** (whenever exists) as $\partial V(m, t)/\partial m$.

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Theorem

Suppose that:

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Any ***optimal mechanism*** *induces an* ***efficient choice of goods*** *(by pricing them at marginal costs), and can be implemented with an* ***income tax alone****.*

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4. Hence, the solution to the relaxed problem is **implementable in the original problem** (by pricing goods at marginal cost).

Interaction of Income Taxation and Market Design

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- Let $F_i(t)$ be the CDF of taste t conditional on ability type i, supported on $[0,\ \bar{t}]$, for $i \in \{L,\ H\}$.
- **Linear utility function** is:

$$u(c) + t \cdot x - \frac{1}{a} \cdot z \quad \text{where} \quad u(c) = \begin{cases} c & c \geq \underline{c}, \\ -\infty & c < \underline{c}, \end{cases}$$

where $\underline{c}$ is a "**subsistence**" level.

Relaxing Assumption 1

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Assumption 1 fails (subsistence constraint has bite):

$$\mu_H \left(1 - \frac{1}{a_H}\right) < G + \underline{c} + k.$$

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Suppose $\frac{1-F(t)}{f(t)}$ *is non-increasing and* $\frac{F(t)}{f(t)}$ *is non-decreasing.*

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  - high-ability agents with sufficiently high taste choose the **taxed high-quality option**.
- High-quality option with tax $\Longrightarrow$ more surplus can be extracted from (some) high-ability agents.

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- ... but there is a **first-order gain** since all types $t > k$ can buy a good with ϵ higher quality (inframarginal effect).

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Intuition under concave u: Extension

- Purchasing the good implies **higher marginal utility of income** $u'(c)$ relative to agents who do not buy.

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- Think of healthcare services as an example.

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Contrast to original Atkinson-Stiglitz logic (example: **healthcare**)

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- Our setting: High demand for healthcare could stem from **both** income (ability) and need (taste).
- Subsidies for low-quality healthcare services improve redistribution relative to income tax alone because they **endogenously target low-ability agents who have a particularly high marginal utility of additional income**.

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  - The optimal mechanism can be implemented as a **means-tested subsidy for the good**.

Relaxing Assumption 2 & 3

What drives the trade-off between mechanism 1 and mechanism 2?

- Because income reveals ability, using **income for** (third-degree) **price discrimination** in the goods market is (generally) useful.

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- Mechanism 2 makes the income tax **less progressive**, and uses the resulting **slack** to implement income-dependent prices.
- Mechanism 1 is optimal iff the motive to redistribute across ability types is sufficiently strong.

Formal result

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  $\implies$ Optimal x_L takes the form $x_L(t) = \bar{x}_L \cdot \mathbf{1}_{\{t \geq t_L^\star\}}$.

Derivation of the optimal mechanism

- We have an outside-option constraint: $\forall t \in [0, \bar{t}]$,

$$\Delta + \int_0^t x_H(\tau) d\tau \equiv U_H(t) \geq U_L(t) \equiv \int_0^t x_L(\tau) d\tau.$$

- Fixing x_L, this is a linear mechanism design problem with a **type-dependent outside-option constraint** (Jullien, 2000)

- Dworczak and Muir (2024): This problem can be solved by adapting the **ironing technique** (Myerson, 1981).

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  $\Longrightarrow$ Optimal x_H has at most three steps.

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  - **Means-tested subsidies** may sometimes be more effective than a strongly redistributive income tax.

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Questions?

Appendix

Relaxing Assumption 2

- Suppose that Assumptions 1 and 3 from the AS result hold:

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  - $\Lambda_L(t) \geq \Lambda_H(t)$, for all t (weaker than $\lambda_L(t) \geq \lambda_H(t)$ for all t).

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Under regularity conditions, and fixing all parameters other than $\bar{\lambda}_L$ (while adjusting $\bar{\lambda}_H$ to keep the mean welfare weight constant at 1), there exists a cutoff value $\bar{\lambda}_L^0 \geq 1$ such that mechanism 1 is optimal if $\bar{\lambda}_L > \bar{\lambda}_L^0$ and mechanism 2 is optimal if $\bar{\lambda}_L < \bar{\lambda}_L^0$.

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If the designer is primarily concerned with **redistribution across ability** types, it is optimal to redistribute as much as possible through the income tax.

Otherwise, it may be optimal to limit the role of income taxes to make income useful for **price discrimination** in market design:
$\implies$ income tax and market design are to some extent **substitutes**.

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- Let $\Lambda(t)$ be the average welfare weight on all types higher than t:

$$\Lambda(p) = \frac{\bar{\lambda}_L\, \mu_L(1 - F_L(p)) + \bar{\lambda}_H\, \mu_H(1 - F_H(p))}{\mu_L(1 - F_L(p)) + \mu_H(1 - F_H(p))}.$$

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  - $(\bar{\lambda}_L - 1)h_L(r) \geq (\bar{\lambda}_H - 1)h_H(r)$, for all t.

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Robustness of results to concavity in the utility function u:

- Suppose that u is a strictly increasing, strictly concave, continuously differentiable function.

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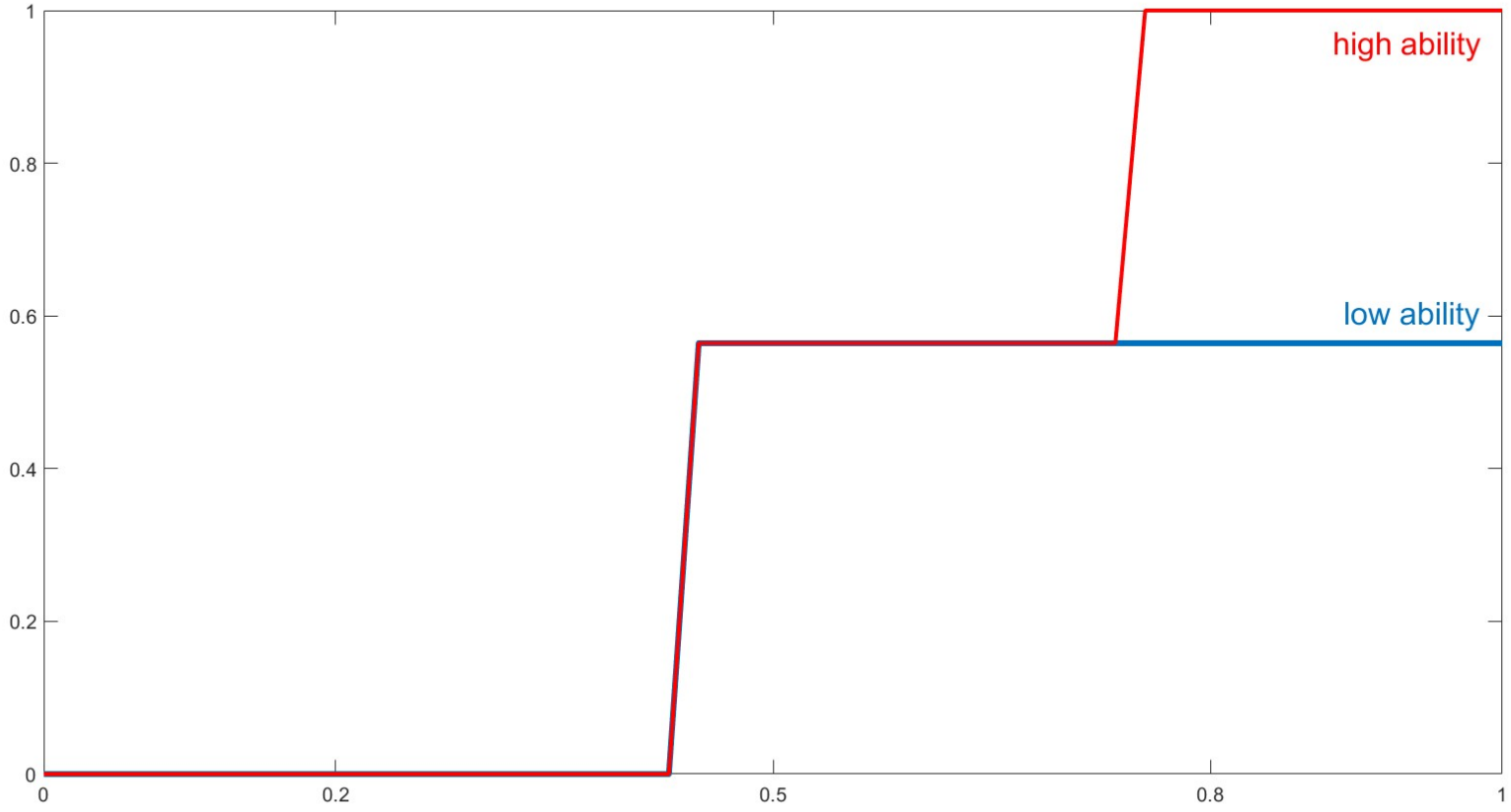
Relaxing Assumption 1



Goods allocation in optimal mechanism

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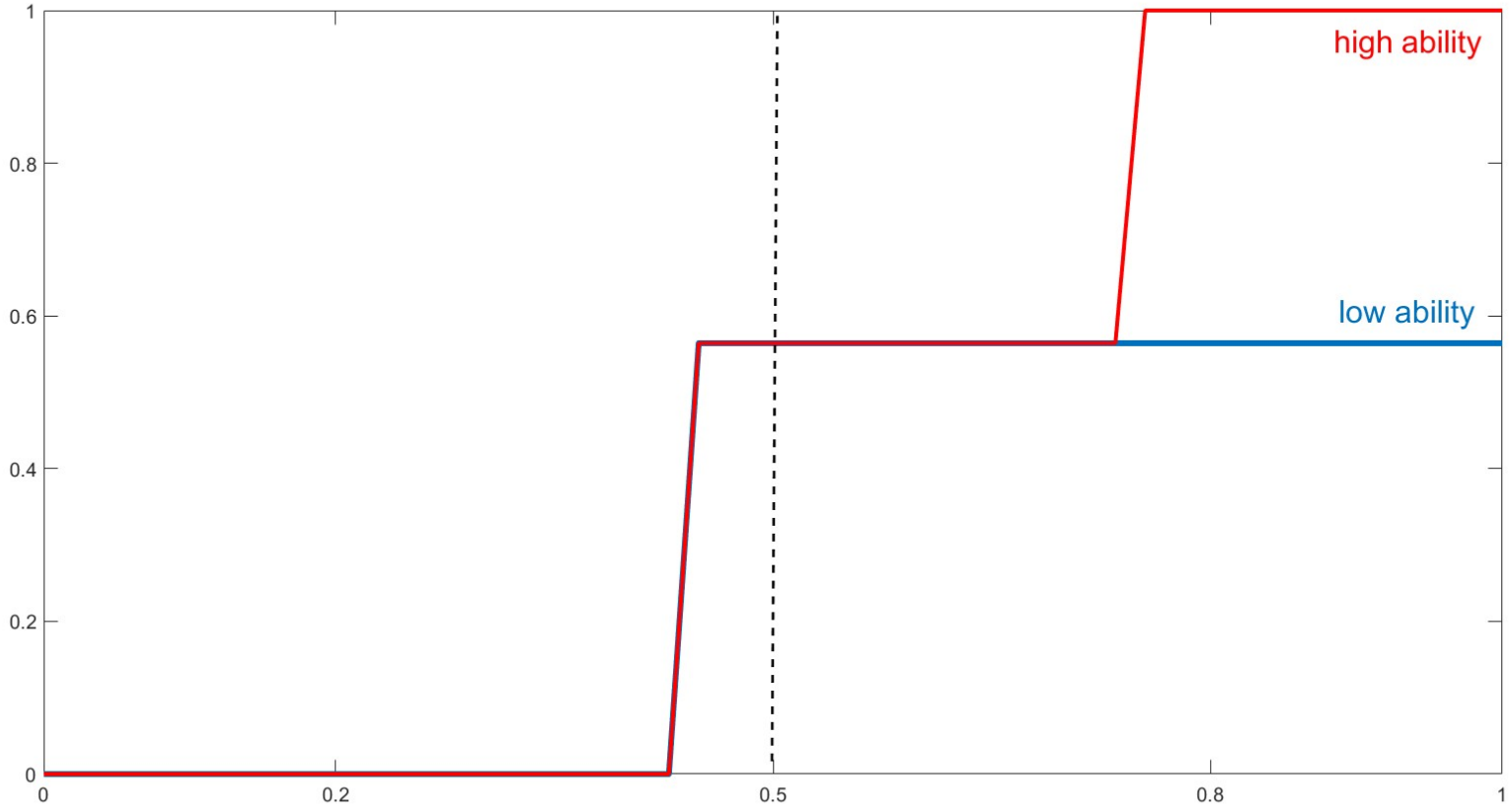

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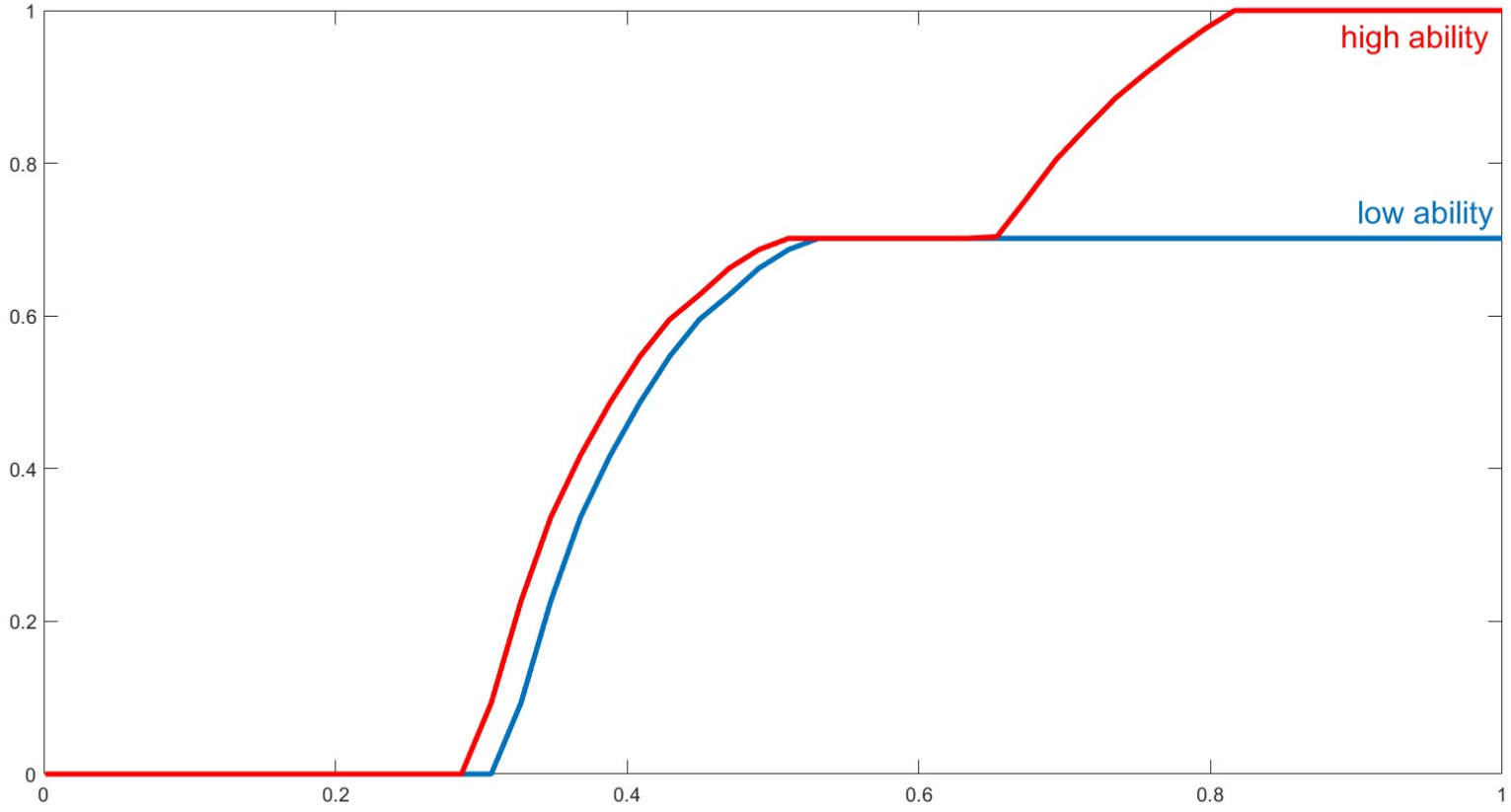

Goods allocation in optimal mechanism ($u(c) = \sqrt{c}$)

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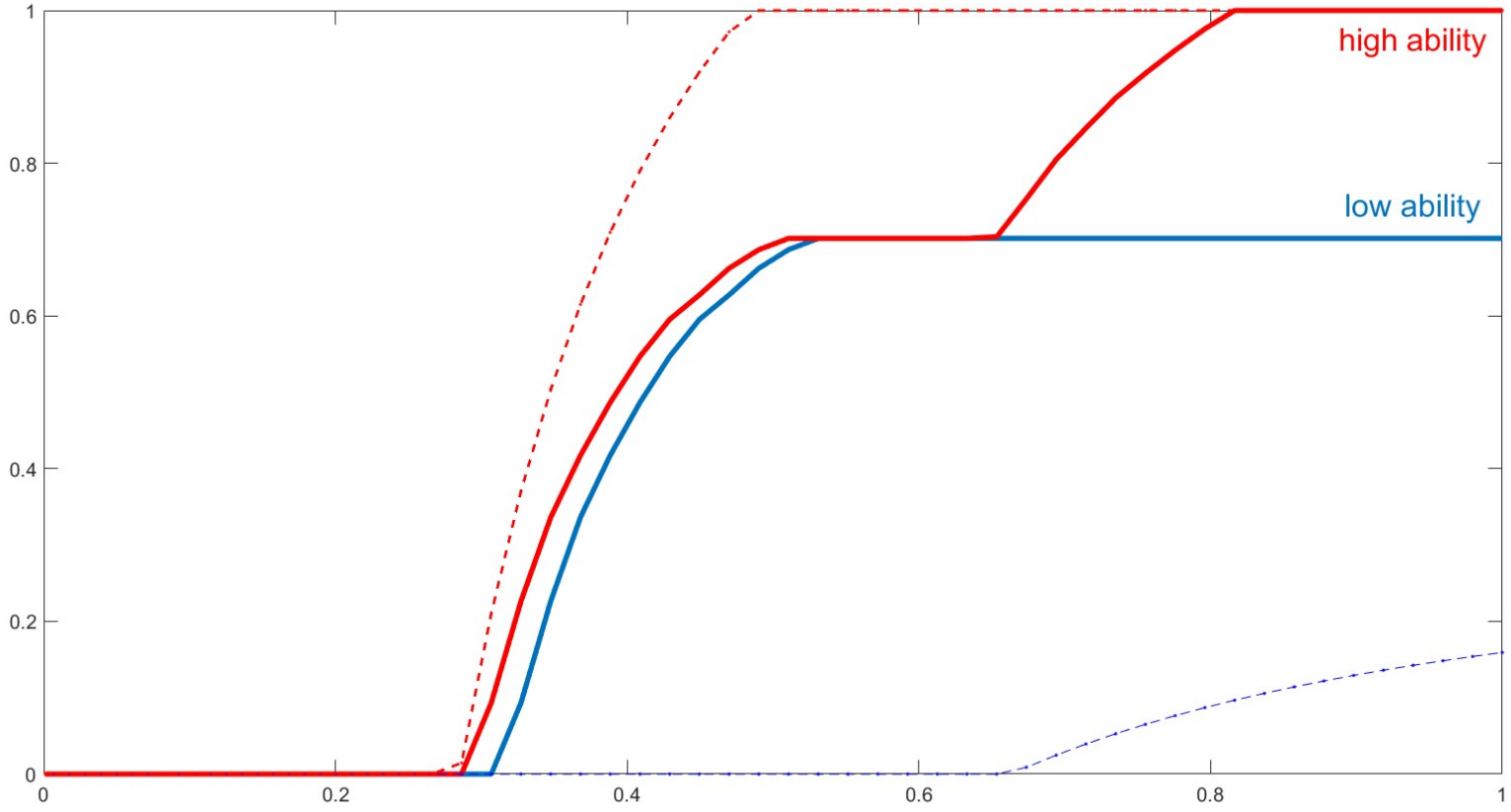

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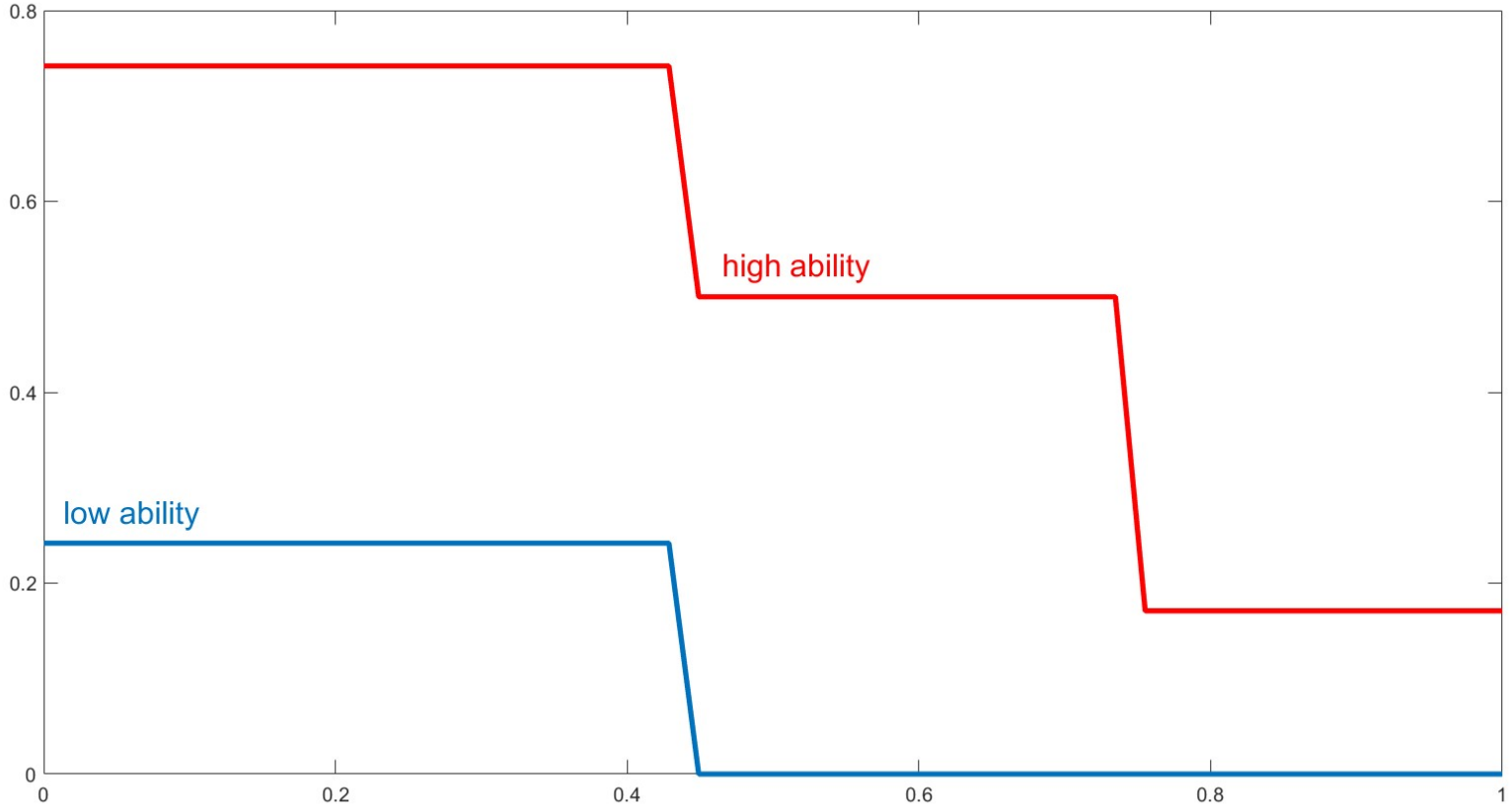

Numeraire consumption in optimal mechanism

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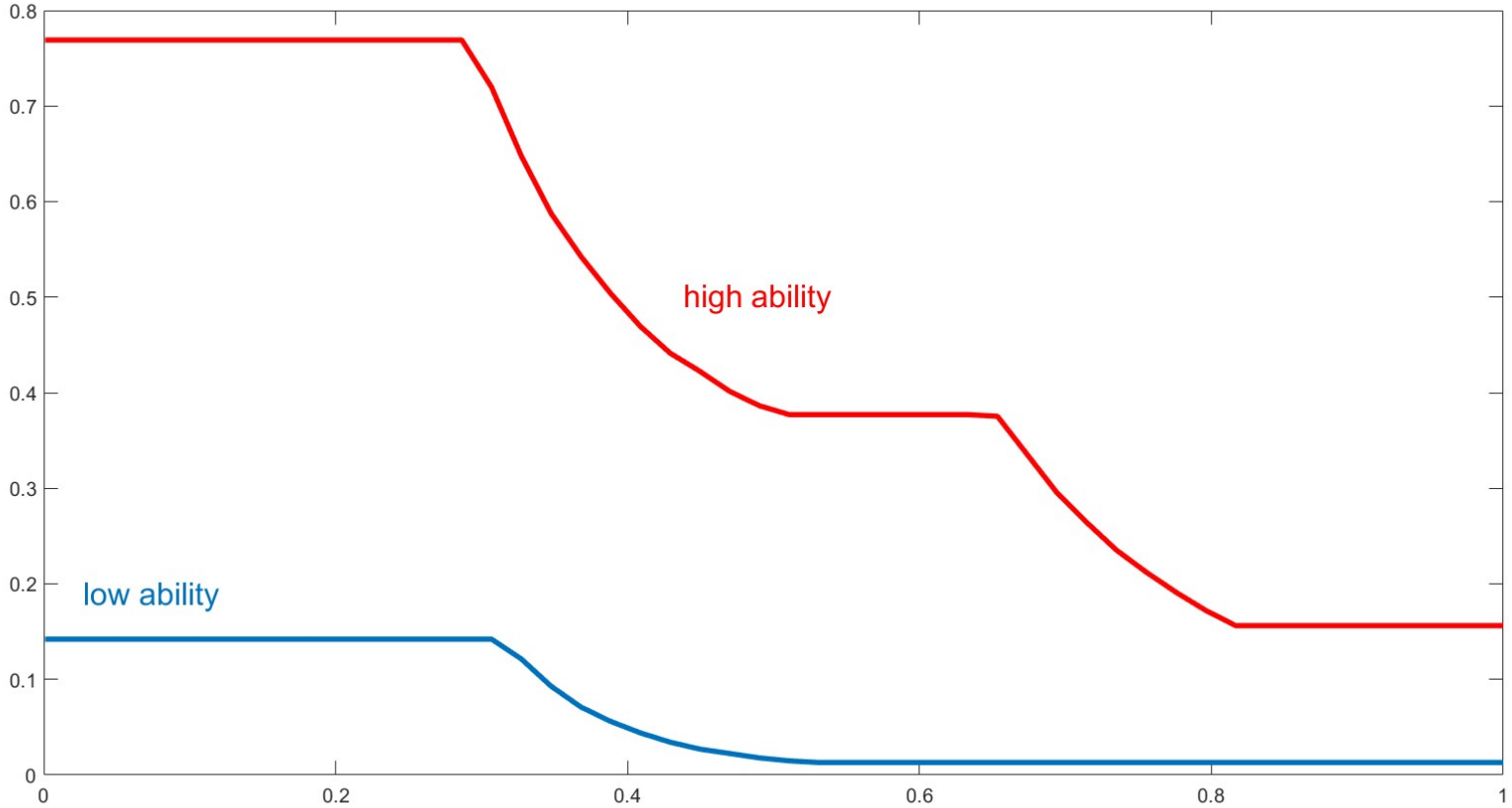

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